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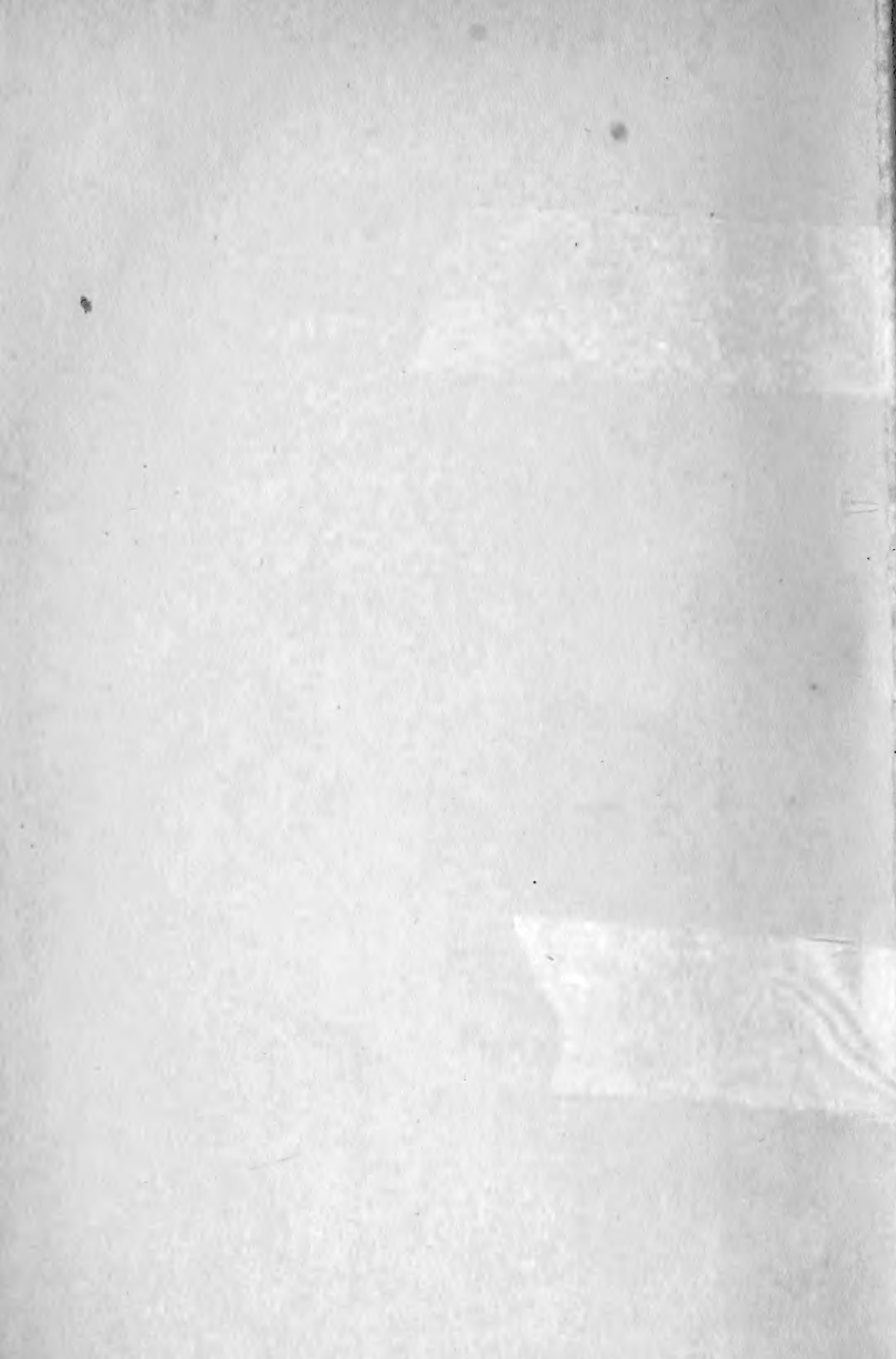
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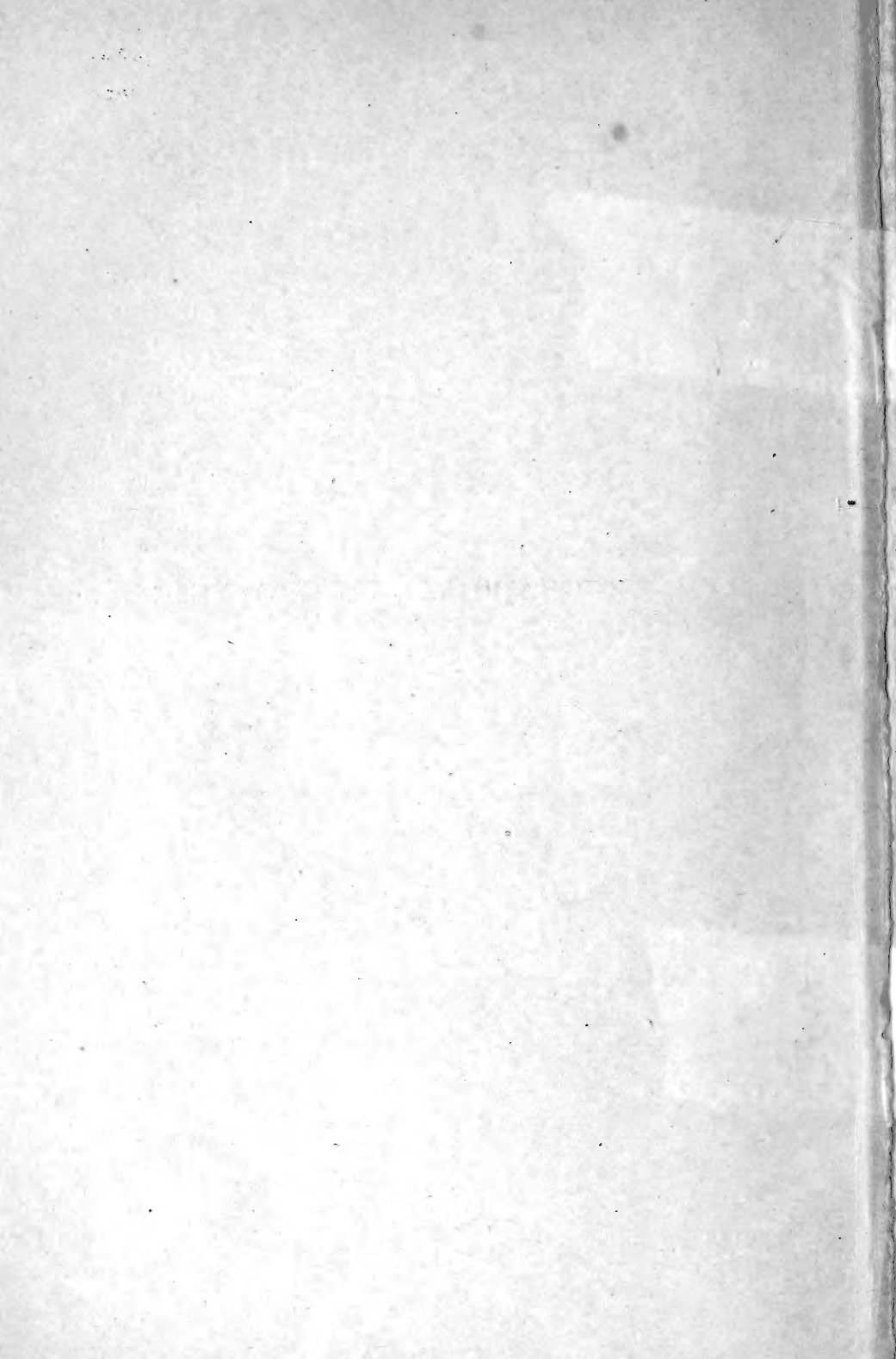


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HYDROGRAPHICAL SURVEYING



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HYDROGRAPHICAL SURVEYING

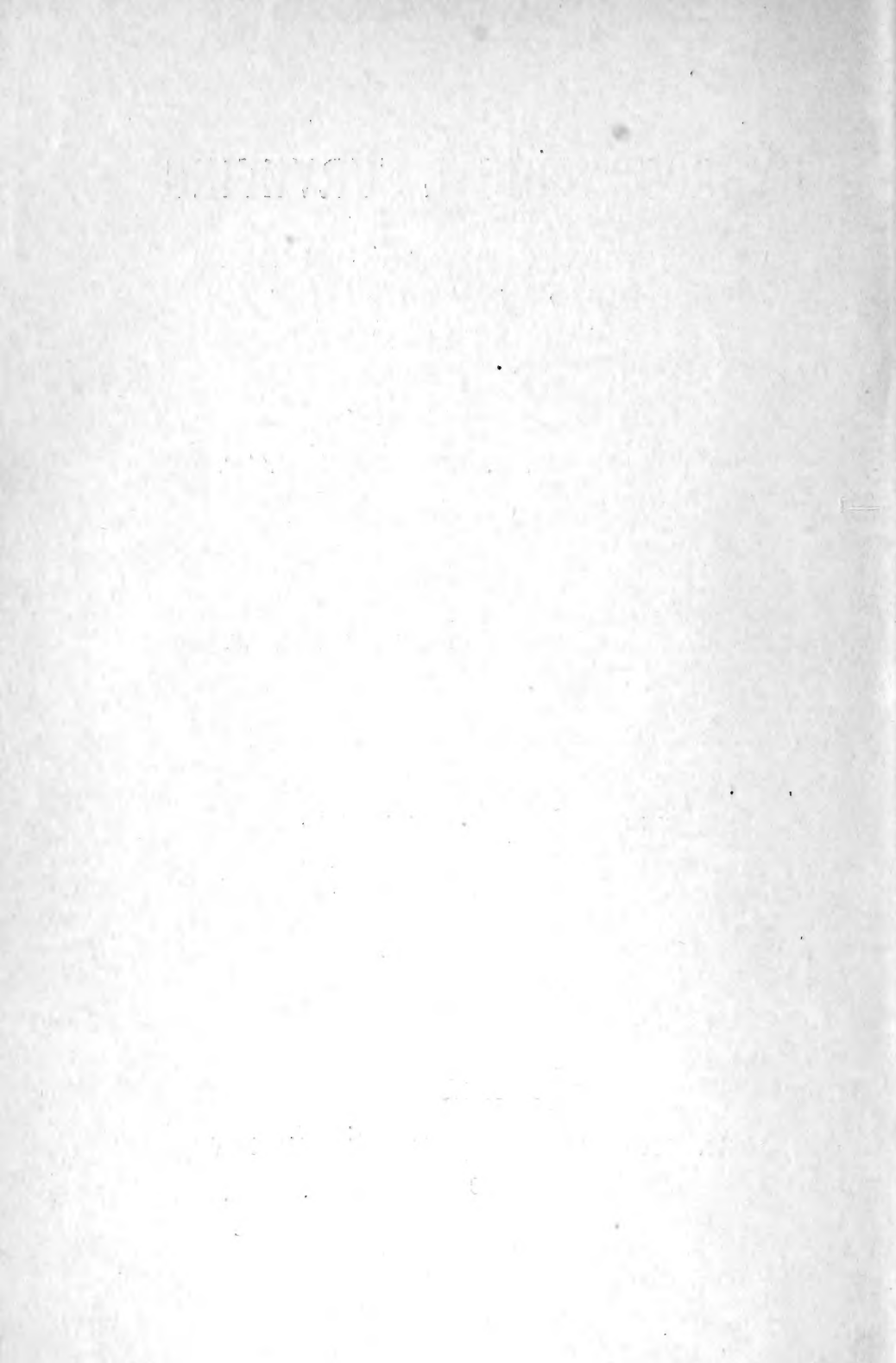
A DESCRIPTION OF MEANS AND METHODS
EMPLOYED IN CONSTRUCTING
MARINE CHARTS

BY THE LATE REAR-ADMIRAL
SIR WILLIAM J. L. WHARTON, K.C.B.
LATE HYDROGRAPHER TO THE ADMIRALTY

FOURTH EDITION, REVISED AND ENLARGED,
BY
ADMIRAL SIR MOSTYN FIELD, K.C.B., F.R.S.
SOMETIME HYDROGRAPHER TO THE ADMIRALTY

WITH DIAGRAMS AND ILLUSTRATIONS

LONDON
JOHN MURRAY, ALBEMARLE STREET
1920



PREFACE TO THE FOURTH EDITION

ANOTHER edition of "Hydrographical Surveying" being called for, the opportunity is used to make certain minor corrections, and to add, in the form of a supplement, some important improvements in connection with ship sounding and sweeping both by ships and boats. These have been effected during recent years through the ingenuity of the officers whose names are associated in the text with the various devices therein described.

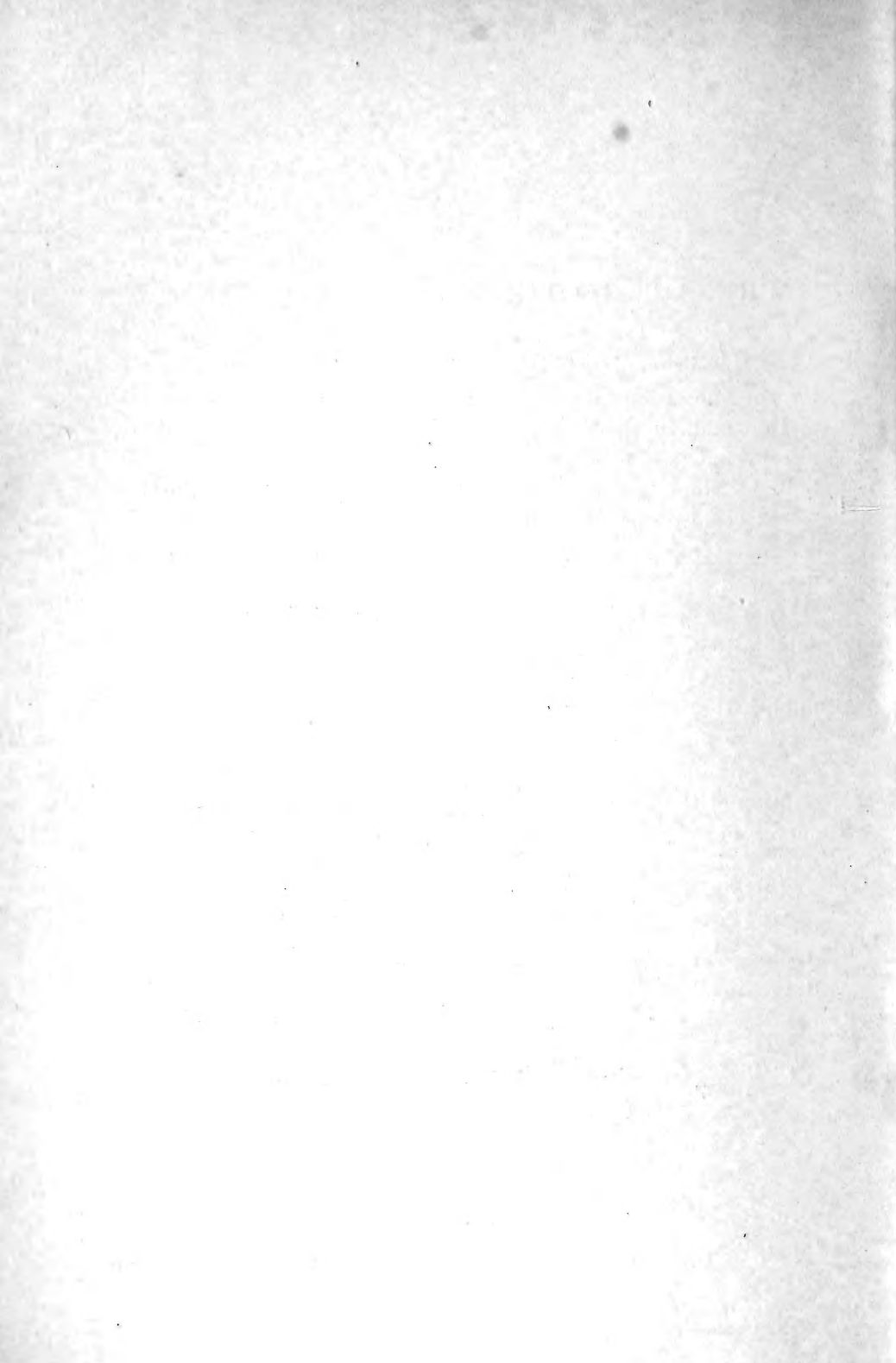
I am greatly indebted to the Lords Commissioners of the Admiralty and to the Hydrographer, Rear-Admiral Sir J. F. Parry, K.C.B., for access to official reports and permission to publish the information contained therein.

I have also gratefully to acknowledge the contribution by Rear-Admiral F. C. Learmonth, C.B., C.B.E., of a detailed description of the methods employed by him in the course of the survey of outlying banks in the North Sea.

To Colonel Sir Charles F. Close, K.B.E., C.B., C.M.G., Director of Ordnance Survey, I desire to express my thanks for the use I have been allowed to make of the articles on Rectangular Co-ordinates in his textbook on "Topographical Surveying."

A. M. F.

TWYFORD,
HANTS,
March 10, 1919.



PREFACE TO THE THIRD EDITION

THE lamented death of Rear-Admiral Sir William Wharton, K.C.B., F.R.S., having occurred since the publication of the last edition of 'Hydrographical Surveying,' at the request of the publisher I have undertaken the preparation of a new edition, rendered necessary by lapse of time and improvements effected in various directions.

Material alteration in the text of the former edition has been avoided as far as possible, but the work has been enlarged by the addition of some new features, including expedients connected with work in the field which have been found useful in practice. Most of these are, no doubt, familiar to the experienced surveyor, but they are not so obvious to younger officers unless their attention is called to them. Amongst other fresh matters referred to in this edition, the following may be mentioned : The use of photography for the reproduction of chart drawings on smaller scales, and the special style of drawing consequent thereon ; the introduction of chronographs for use in the field ; the development of the Pillsbury deep-sea current meter ; a new form of automatic tide-gauge, and its possibilities for use in deep water ; the application of the range-finder to surveying purposes ; an improved slipping apparatus for ship sounding ; and the practical effect of better knowledge of the nature of the slope of the bottom at different depths in connection with searching for vigias.

To officers in charge of surveys, it is hoped that the examples

of irregular triangulation may be found interesting and suggestive in dealing with cases requiring the exercise of ingenuity to overcome difficulties. The question of co-ordinating the results by triangulation with astronomical positions is also one that should have special interest for them.

I am indebted to Captain H. E. Purey-Cust, R.N., for his kind assistance in revising some of the additional matter prepared for this edition ; and to Rear-Admiral W. U. Moore and Captain T. H. Tizard, C.B., R.N., F.R.S., for the benefit of their advice and suggestions on certain points.

Through the courtesy of the manager of the *Times*, I have been enabled to make use of articles contributed by me to the tenth edition of "Encyclopædia Britannica," for which I have to express grateful acknowledgment.

A. MOSTYN FIELD.

DURRAN LODGE, EAST SHEEN,
October 27, 1908.

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HYDROGRAPHICAL SURVEYING

PRELIMINARY

THERE is nothing mysteriously difficult in the art of Hydrographical or Marine Surveying. For the ordinary details, no deep theoretical or mathematical knowledge is needed ; on the contrary, it is an eminently practical branch of the Naval Profession.

An aspirant to its acquirement should have a quick eye, should possess the ordinary good common sense that is necessary to secure success in all walks of life, but above all he must have a boundless capacity for taking pains in details at all times and seasons.

The advice, "*Surtout, point de zèle,*" does not apply to surveying. Without zeal, and the utmost keenness for the progress of the work, the attention and interest will soon fail ; and the necessity for constant application throughout long days, often extended into the night, will soon seem monotonous, and become a bore to one whose heart is not thoroughly in it.

Happily, it is a profession of volunteers, and the author's experience is, that in no branch of the public service can the juniors be more anxious to do their duty, not only to the letter, but to the utmost of the spirit, and to such as these no day seems long enough. To them, the interest is constantly kept up. Every day has its incidents. The accuracy of the work of each assistant, when proved, is an infinite gratification to him, and he has also the continual satisfaction of feeling that of all he does a permanent record will remain, in the chart which is to guide hundreds of his fellow-seamen on their way.

For any naval officer, then, who is really anxious to learn, the practical part of surveying will soon be mastered. It

will quickly become a labour of love, and the constant attention and trouble necessary will be merged in the interest taken in the work. Thorough honesty must always guide him, so that nothing may appear that is not known to be correct. Omissions there must always be, but let there be no sins of commission, that pains and care will prevent.

It is not of course suggested that all can become thorough good surveyors in all branches. One man will have a particular aptitude for astronomical observing ; another will have a natural talent as a draughtsman, that no efforts on the part of another can compete with, and so on ; but to become a good practical hand is within reach of all who seriously are desirous of being so, and will take the trouble to gain the necessary experience, without which all theory and book-teaching will be useless.

One crucial test of a surveyor's capability is his power of so planning and carrying out his work as to economise time. Even in a plan of an ordinary bay or harbour, which every naval officer should be able to make, the trained surveyor, by his experience of how to set about it, will accomplish it in a fraction of the time required by another. Nothing is more important than the knowledge of how to suit means to the end, and many hours are wasted by the anxious tyro in endeavouring to attain an accuracy in detail which cannot be utilised in the finished plan.

Inside of the broad principles of map making, marine surveying is made up of numerous dodges and details, for which there is nothing like practical exposition on the ground, and those who can get others to show them will need but little other help, but as in many cases this instructor will not be at hand, it is hoped that the following pages may sometimes supply the information required.

It may seem to many that some points remarked on are too insignificant to be heeded, but those who are acquainted with the work will know how much time is lost by inattention to, or ignorance of, these little things, and a young surveyor will be a very few days at work before he finds this out.

We assume our reader to have the ordinary knowledge of the sextant that all naval officers are taught, and that he is not entirely ignorant of the first principles of making a plan from a base by means of angles.

We write mainly for those who join the Surveying Service, and shall speak throughout as though we had the resources of an ordinarily fitted surveying ship at command.

We have endeavoured to take things in the order that they will generally come in the prosecution of a survey.

In work of the nature of Hydrographical Surveying, it is impossible to give directions as to how to undertake every detail. Ordinary means fail now and again from exceptional, local, or other circumstances, and ready resource in overcoming difficulties is one of the most important requisites in a nautical surveyor. To invent or improvise a method of doing a particular piece of work is a most satisfactory achievement when successful, but it is scarcely necessary to say that this can only come to the most naturally talented with experience.

The conditions under which modern surveys are usually carried on tend to confine the experience of many surveying officers to regular triangulation; this is apt to engender a certain want of resource when difficulties arise in the course of a survey requiring other methods.

The fact of being unaccustomed to irregular triangulation involving the use of the ship, produces a distrust of such methods arising from an imperfect realisation of the accuracy of the results obtainable therefrom.

This leads to a tendency to neglect the study of such means and their application to cases which not infrequently arise.

A considerable number of examples of irregular triangulation have been inserted in the present edition, for the purpose of giving to officers whose experience has been obtained in other directions some foundations upon which to build.

These examples, being based on practical experience, are illustrative of conditions occasionally to be met with. They do not pretend to be exhaustive, but it is hoped they may be helpful in suggesting various ways of overcoming difficulties.

Much time and unnecessary labour may often be saved by the judicious use of the ship. Recourse to regular triangulation in wooded country involves the occupation of numerous shore stations, and the attendant labour and loss of time in cutting timber, which might possibly be avoided by an intelligent appreciation of what is possible by looking at the problem from other points of view.

In the earlier days of hydrography, surveys were made on small scales, and covered large areas which were more or less imperfectly examined. A regular system of triangulation being generally inapplicable to such conditions, commanding officers were constantly stimulated to exercise their ingenuity, and the assistant surveyors became habituated to methods of fixing their points which demanded closer study of the geometrical problems involved than when dealing with work conducted on more regular lines.

There is not now the same incentive as formerly to improvise methods, owing to the larger scales of modern surveys and the necessity for that minute examination of the ground which is usually associated with regular triangulation.

Such procedure is not to be discouraged in its proper place, but stress is laid upon irregular triangulation, because the latter is sometimes neglected when it might have been used with great advantage in the saving of time without impairing the value of the results.

Rigid accuracy must always be insisted upon ; but, as a general rule, a degree of accuracy which aims beyond that which is necessary to plot points with precision on the scale on which the survey is carried out is waste of time.

It should be a cardinal principle in scheming a triangulation to avoid unnecessary multiplication of main stations ; much time is often wasted in making stations that serve no useful purpose, and this applies with greater force when wooded hills or mountains difficult of access are concerned. Of fixed points, and secondary stations on or near the coast, there should be an ample number ; and every conspicuous natural object should be accurately fixed.

Success depends chiefly upon the skill exercised by the officer in command in economising the time at his disposal, or on his judgment in evading the difficulties caused by contrary winds, bad weather, strong tides, an inhospitable population, and the natural features of the coast ; and upon the concentrated attention given by himself and his assistants to a multitude of small details.

The following pages will not be found to provide for every occasion, but will describe the ordinary and accepted modes of setting about work, giving examples of special cases illustrating difficulties that may be met with.

CHAPTER I

INSTRUMENTS AND FITTINGS

Sextants and Stands—Horizon—Theodolite—Station Pointer—Scales—
Straight-edges—Chains—Protractors—Pocket Aneroids—Heliostat—
Ten-foot Pole—Range-finders—Drawing-Boards—Weights—Transfer
Paper—Paper—Books—Chronometers—Marks—Boat's Fittings—
Lead-lines—Beacons.

In preparing for any surveying work, whether in a regularly fitted surveying ship or not, the first thing is to test all instruments and ascertain their errors. To do the former well, it is necessary to have an intimate knowledge of the points on which each instrument is liable to go wrong, which is only thoroughly to be learnt by experience; but a few hints will assist the beginner.

Errors of
Instru-
ments to
be ascer-
tained.

A thorough acquaintance with the construction of instruments will save many an hour, lost by one whose instrument has gone wrong while in the middle of his work, and spent in fruitless efforts to make out where the fault lies.

No instrument, not even engine-divided protractors, can be assumed to be without error, and are seldom found so, and though those errors may be small, in some cases they are of importance, and no work can be deemed satisfactory without the knowledge of how much correction should be applied, in such instances as it may be necessary to do so.

No Instru-
ment
Perfect.

We shall therefore commence by some observations on instruments, and on all materials and fittings required for conducting a regular marine survey, embodying in these such hints on using each instrument in general, as are likely to be useful, and also some on choosing them that are not mentioned by Heather in his work on Instruments.*

Contents
of
Chapter.

* "Mathematical Instruments." J. F. Heather, M.A. Lockwood and Co., London.

Heather's
Work on
Instru-
ments.

This useful work, which should be in the hands of every surveyor, goes so fully into the construction of instruments, and in most cases into the methods of ascertaining and, as far as may be, correcting their errors, that we shall refer the reader to it on most points, adding only certain practical suggestions that are not therein mentioned.

HADLEY'S SEXTANT.

It is not, perhaps, necessary to say much about the sextant, as so many works have already treated the subject ; but there are several practical points not generally mentioned, which may be of value in selecting a sextant with a view to the work of a nautical surveyor.

Besides those noted by Heather, then,—

1. One of the eye-pieces of the inverting telescope should have a high magnifying power, about 15 diameters, as contacts of the sun's limbs in observations with the artificial horizon are far easier made the larger the suns.

Dark Eye-
pieces.

2. Several dark eye-pieces should be provided, with neutral tint glass in them of different intensities. These should be fitted, not to *screw* on to the eye-piece, but ground conical, to slip on to a similar conically ground surface on the telescope eye-piece. These will be found very useful on cloudy days, as a little practice will soon enable the observer to substitute one shade for another in a fraction of a second, as clouds sweep on or off the sun, and many sights will thereby be saved. It is very important to have the suns in artificial horizon observations of the same brilliancy, and for this reason the hinged shades on the sextant should never be used for the purpose ; as, in the first place, they introduce error, and also, if the shades have to be altered to suit the varying brightness of the sun during the observation, the suns will be of different brilliancies, as these shades are never of the same tints.

By using the dark eye-pieces, the up-and-down piece,* when adjusted to equalise the suns, will bring the axis of the telescope nearly exactly in line with the edge of the silvered surface of the horizon glass, which is the best position for observing, and

* The up-and-down piece of a sextant is the portion that bears the collar for the telescope.

from which it must never be moved until the equal altitudes or other observations are complete. No matter what depth of shade is then used by shifting the dark eye-pieces, the two images will be of the same tint.

The darker the shade used the better. Beginners are very apt to use too bright suns.

If in observing with the sun the observer can accustom himself to use one eye for taking the observation, and the other for reading and setting the vernier, he will find it very convenient, and it will tend to keep both his eyes in good order.

3. It is very convenient for picking up the images in the artificial horizon, if the up-and-down piece is so placed as to enable the observer to look over it into the horizon glass.

Position
of Up-and-
down
Piece.

In many sextants the up-and-down piece is placed so close to the index glass that this is not possible, and regard should be had to this point.

4. An interrupted thread, to screw the telescope into the collar of the up-and-down piece, is a great convenience.

5. An extended vernier, *i.e.* a vernier whose divisions are twice the distance apart of those on the arc, will be found convenient for accurate observing.

6. A steel tangent screw will be found to last longer and work more evenly than a brass one.

The methods of ascertaining the index and other errors of Hadley's sextant, and correcting them, are so fully entered into by Heather, that they are here omitted, with the exception of the following remarks on the centring error :—

This very important error of the sextant cannot be corrected in the instrument, and it requires a considerable amount of labour to settle its quantity, which in an indifferent instrument may be quite sufficient to vitiate the result of any observations on one side only of the zenith.

Centring
Error.

The centring error, pure and simple, arises from the non-coincidence of the centres of the index arm and of the graduated arc, so that the vernier does not move truly along the arc, and the angle read off will not be correct. This error varies with the angle, and is generally greater as the angle increases, but the same result of error appears from the index arm becoming bent ; from any part of the frame receiving a blow which alters its shape ; from the flexure of the instrument from varying

temperature ; and from defective graduation ; but, as it is generally impossible to disentangle the errors arising from these different sources, they are all included in the one correction for centring.

Centring error is to be obtained by comparing the angle measured by the sextant with the true angle.

It is to be found roughly by measuring a series of angles carefully, by repetition, with a large theodolite, between well-defined objects on the horizontal plane at different angular distances, and then measuring the same with the sextant placed on a stand. The difference will be the centring error at each angle, index error being first applied.

The most accurate method, because it employs a large number of observations for the same, or nearly the same, angle, is by observation of pairs of circum-meridian stars in the artificial horizon, at various altitudes. Double the difference between the resulting latitude by each star, and the mean latitude, will be the centring error for an angle equal to the double altitude of that star, that is the angle actually measured by the sextant, index error being carefully determined and applied before working out.

The sign of the correction is easy to determine from a consideration of whether the altitude is too little or too great. Thus in north latitude, if stars south of the zenith give a latitude too great, their altitudes have been too little, and the correction for centring will be plus.

It is hardly necessary to say that every precaution must be taken to eliminate other errors, such as choosing stars of a closely similar altitude, unless the latitude is already accurately known ; determining the roof error of the horizon, or eliminating it by reversion ; carefully correcting the refraction for temperature, etc., and that it requires considerable accuracy of observation, and many sets, to arrive at a good result. The agreement, or otherwise, of the mean latitude by each pair will form an excellent test of the general accuracy of observation, and the agreement of the resulting centring errors by different observations at the same altitudes will enable the observer to judge of the truth of his final errors. Thus every careful set of observations for latitude affords a means of testing this error.

Centring error may also be obtained by careful measurement of the angles between stars. The correct apparent distances must be found in the same manner as in clearing a lunar distance; the true distance being first calculated from their declinations and right ascensions, but if stars in the same vertical plane can be chosen, the apparent distance can be arrived at by simple application of the refractions.

There are other methods, involving more calculation, which need not be described.

The centring error is determined at Kew Observatory for certain angles by fixed collimators, and is given on every Kew certificate, but it must be remembered that in any case it can never be considered as determined for good. Including, as it does, errors from so many causes, it does not remain perfectly steady, but its amount should be ascertained from time to time for any sextant which is to be employed for accurate determination of positions, for circumstances often prevent the use of methods whereby it as well as other errors are eliminated. For instance, a latitude may have to be obtained by altitude of the sun only, when, without knowledge of the centring error, it may easily be incorrect to as much as a minute, or even more.

As an example, the author's Troughton sextant had at 120° a centring error of $-20''$. After a fall and repair by the maker, it was $+50''$.

To find the error caused by the refraction, through non-parallelism of the sides, of the coloured shades. Errors of
Hinged
Shades.

Measure the diameter of the sun, with different combinations of the shades. Take out the pin which supports one set of the coloured shades, and replace the shades *reversed*, so that the face before next the index glass is now away from it. Remeasure the diameter of the sun with same combinations as before, and half the difference of the measurements of each set will be the error due to the shade reversed.

These errors can be neglected in sea observations, and if coloured eye-pieces are fitted as recommended above, the shades are not required when the artificial horizon is made use of.

A level fitted to revolve on the pillar on the Index bar on which the magnifying glass pivots, and working in the plane of the sextant, is a useful addition. It is set by actual experi- Spirit
Level on
Index Bar.

ment as follows, and clamped permanently. The sextant being mounted on its stand, observe the double altitude of the sun in artificial horizon, with the telescope pointing as nearly as possible to the centre of the artificial horizon. Make the suns exactly cover each other, and bring the bubble of the level into the centre of its run, clamping it securely. So long as the clamp is not loosened, the sextant being set to the double altitude of a star, if the sextant is turned truly vertically on its axis until the bubble is in the centre of its run, and moved bodily on its stand towards or from the artificial horizon until the telescope points correctly to the centre, then both direct and reflected images of the star will be seen in the field of the telescope.

Electric
Light.

A small electric light fitted on the arm carrying the magnifying glass, with wires leading to a dry battery or a Bichromate Battery, will be found a very great convenience. The light coming from a fixed point, there is less liability to errors of parallax in reading off the sextant, than if an ordinary lantern is used.

Rack and
Pinion.

This fitting for focussing the telescope enables the focus to be obtained with greater nicety.

SOUNDING SEXTANT.

This useful form of sextant is made of various sizes. It chiefly differs from the observing sextant in being generally lighter and handier, in having the arc cut only to minutes, and having a tube of a bell shape so as to include a larger field in the telescope.

All angles in the frame of the instrument should be rounded off, especially that at the zero end of the arc. Considerable injuries may result to the face of the observer when using the sextant in a boat in a lively sea, if this is not done.

The graduation of the arc should be plain enough to read without a magnifying glass.

The measurable angle should be as large as possible, *i.e.* about 140° .

The index glass should be large, so as easily to pick up objects.

The telescope should be of a high magnifying power and clear definition.

Good Tube
Invaluable.

These sextants are now supplied by the Hydrographic Office with two telescopes—one for ordinary use, and another, of aluminium, with a larger object glass for occasions when faint objects are required to be seen. The collimation of these large telescopes is however a delicate matter, and when accuracy is required, should be tested.

When in good adjustment, a sounding sextant so fitted is invaluable for star observations with a faint sea horizon.

RESILVERING MIRRORS.

On service, the mirrors of sextants, especially sounding sextants, frequently get dimmed by damp, and the surveyor must be able to resilver them himself.

A supply of tinfoil, of good quality, for this purpose, is one of the necessary stores. Mercury is always to be had. The operation has been frequently described, but it is perhaps better to repeat it.

Take a piece of tinfoil, a little larger than the glass to be silvered, and smooth it out on a perfectly flat surface, as a sheet of plate glass, or a thick smooth book-cover. This smoothing can be well done by a little pad of chamois leather, which can be kept for the purpose, or by the finger.

Drop a small bubble of mercury on to the foil, and by gentle rubbing with the pad, spread it over the former so that it shows a bright surface. Pour mercury on until the piece of foil is quite fluid, and brush any large spots of dross lightly off. Lay a piece of clean paper, long enough to handle easily, on the mercury, and the glass, previously well cleaned by means of spirits of wine, on the paper. Pressing on the glass with one hand, withdraw the paper with the other, slowly and steadily, and a pure surface will appear under the glass, the dross all coming away with the paper.

Incline the book, or whatever surface we have been working on, so as to let superfluous mercury run off, placing strips of tinfoil at the lower edge to assist in sopping this up.

After from twelve to twenty-four hours, the amalgam will be dry, and firmly adhering to the glass. Cut the edges care-

fully round with a sharp knife, and varnish lightly over, either with the clear stuff used by the instrument makers, or with varnish that can be made on board, by dissolving sealing-wax in spirits of wine.

The glasses of some sextants seem fitted on purpose to invite the damp to penetrate between glass and silvered surface. These will want protection by sticking thin strips of paper along the edges exposed, and well varnishing. In some cases a stopping of thick amalgam, placed between the glass and the frame at the back, where there is one, will answer well, and prevent any damp getting at the back of the glass at all.

Amalgamated
Mercury.

The mercury which remains will contain tinfoil in amalgam, and should be preserved in a bottle by itself, draining off the thick of the amalgam by a sharply twisted paper funnel. It can then be used again for resilvering. Care must be taken not to allow any of this to get into the artificial horizon bottles, as the smallest quantity of it will spoil a whole bottle of pure mercury, and the amalgam can only be removed by evaporation.

Notwithstanding, mercury containing tin in amalgam can be used for artificial horizon work, by carefully sweeping the surface after it is poured out, with a piece of paper. Some observers have gone so far as to prefer amalgamated mercury for this purpose, but we do not agree, except when used in connection with the amalgamated trough described on p. 15.

Horizon
Glass

In resilvering an horizon glass, only the portion required should be operated on, leaving one half clear. The edge of the foil must be sharply and smoothly cut before applying the mercury, and not the smallest nick or cut permitted to remain in it.

SEXTANT STAND.

Though a practised observer will get good observations in an artificial horizon, with a sextant without a stand, he will get them far better *with* one, and in all work where accuracy is aimed at, a stand should be used.

Unsteadiness of hand, to which all are so liable, from previous exertion, indisposition, and many other causes, is put out of the question by using a stand.

With star observations this is especially the case, as it is

extremely difficult to hold the instrument in the hand firmly enough to prevent a little vibration of the images.

Sextant stands should be lacquered, not bright, and should have large heads to the foot screws, so as to be grasped easily while observing.

The bearing which carries the sextant should be accurately fitted into the socket in the handle, and should be very slightly conical. If too much so, it is liable to jam.

The counterbalances are usually too heavy for an ordinary sextant. They should be of such a weight as to balance the sextant without the screws at the ends of the pivot being set up too taut. Sometimes one weight is enough, or as much lead can be taken out of each as is necessary to reduce the weights to balance. The weights are now sometimes fitted to slide in and out, thus allowing of adjustment.

There is a great advantage in having the bearing which carries the sextant cut square. The circular motion in this form of stand is given by means of a large disc which is controlled by a clamp and tangent screw with a very coarse thread. The tangent screw enables the sextant to be kept pointed accurately to the Artificial Horizon as the sun or star changes its altitude, without the necessity of actually touching the sextant or moving it on its bearing by hand. This arrangement avoids the liability to jam, and the consequent exertion of force which may cause the sextant to move with a jerk, and throw out of the field the sun or star, thus disturbing the tranquillity of the observer and possibly losing one or two observations in consequence. Sextant stands of this pattern are made by Messrs. Cary and Porter.

Improved
Sextant
Stand.

The threads of the foot screws of the sextant stand should be of fine pitch.

A small level may advantageously be fitted to the arm of the sextant stand, so adjusted that when the arm is horizontal, and consequently the plane of the sextant vertical, the bubble is in the centre of its run. This adjustment is easily made whilst observing the altitude of the sun in artificial horizon, and making the suns cover during the process. This level, used in conjunction with the other level fixed on the Index bar of the sextant, ensures a star whose altitude is known, being found in the field of the telescope, when

the latter is correctly pointed to the centre of the artificial horizon.

Stools for
Stand.

Small three-legged stools about 14 inches high, on which to place the sextant stand, should be made, and it will be found convenient to sink hollows in the top to correspond with the three foot screws to prevent slipping.

Other little hollows sunk in the top for the spare dark eye-pieces to lie in, will also prevent these falling off, and by placing them in regular order, any one can be at once picked up without delay, when it is requisite to change them.

Another similar stool for the observer will make him comfortable, a great point for good observing.

Observing
Stool.

A special fitting forming the top of the observing stool deserves mention. It consists of two wooden discs in contact. The upper disc is secured to a metal block, running on a screwed bolt of very coarse pitch, fixed in a deep groove running across the lower disc, working on bearings at each end and actuated by a small handle. The upper disc carries the sextant stand; the lower disc revolves on a pivot passing through the centre of the stool, and secured by a butterfly nut underneath. By turning the handle, the upper disc is moved as required backwards or forwards in the direction of the artificial horizon as the sun or star moves in altitude.

ARTIFICIAL HORIZON.

The glass in the roof should be of the best quality, and the faces of each plane accurately parallel.

A wooden trough to place inside the iron one is a convenience, as it raises the level of the mercury up to the height of the lower edge of the glasses in the horizon roof, a consideration where low altitudes have to be observed. The reduced area of mercury will not matter when observing the sun. When taking stars, the iron trough only should be used, as stars are more difficult to pick up, and its larger area will facilitate operations.

Three short wooden legs or buttons, fitted to the iron trough, will enable it to stand steadier on uneven ground than the four projections usually cast on the under side.

Horizon
Stand.

In connection with this, an artificial horizon stand is very useful. This consists of two iron plates; the lower one has

three short legs on which it stands firmly ; the upper one is pierced by three long large-headed screws, which serve as legs and fit into slight hollows on the lower plate. By adjusting these, the horizon laid on the upper plate can be levelled, when we have uneven ground. Four iron battens, screwed on to the upper plate so as just to permit the horizon roof to fit inside them, will prevent any wind getting to the mercury.

The horizon cover should be marked at one end, or side, and this mark should in most cases be in the same position with regard to the observer. Of this more is said under "Observations."

Mark on
Cover.

A new form of horizon is now being introduced, with the object of diminishing the waves set up in mercury by vibrations.

Amalga-
mated
Trough.

It consists of a circular shallow trough, of metal gilt. This is amalgamated, after getting the surface absolutely clean and free from grease, by wetting it with a few drops of dilute sulphuric acid, and then rubbing into it a drop of mercury until the whole surface is bright, when a very small quantity of mercury added will flow evenly and form a horizontal surface. The dross is wiped off with a broad camel's-hair brush.

In this shallow trough waves are killed almost instantaneously. The trough should be thoroughly washed on each occasion before being used.

This form of artificial horizon has proved itself to be invaluable on shaky ground, and is supplied to all surveying ships. The circular form has recently been abandoned, and the latest pattern is rectangular in shape. Care is necessary to see that the thin film of amalgamated mercury flows evenly over the surface of the trough, and completely covers it. The trough should be levelled by means of a spirit level before pouring on the mercury.

THEODOLITE.

The less a theodolite is tampered with by unpractised hands the better, but they must be adjusted from time to time, and little things are constantly wanting attention.

The adjustments are well described by Heather, but as it is very important to know them, they are here given,

in case the former work should not be at hand. The adjustments are—

1. Adjustments of the telescope, viz., for parallax and for collimation.

2. Adjustment of horizontal limb, viz., to set the levels on the horizontal limb to indicate the verticality of the azimuthal axis.

3. Adjustment of the vertical limb, viz., to set the level beneath the telescope to indicate the horizontality of the line of collimation.

Commence operations by setting up the theodolite as level as you can by eye, by moving the legs. See that the legs are firm, and everything tight. Set all levels as true as you can, by the parallel plate screws and vertical arc tangent screw.

Adjust-
ment for
Parallax.

Parallax is occasioned by the image formed by the object glass not falling exactly on the cross-wires.

First adjust the movable eye-piece until the cross-wires are sharply defined. Then obtain the proper focus for the object by moving the milled head on the telescope.

This will throw out the image of the cross-wires, and the eye-piece must be again adjusted, until cross-wires and object are both truly in focus.

This has to be done each time the theodolite is set up, and is therefore only a temporary adjustment. The others are more permanent.

Adjust-
ment for
Collima-
tion.

Collimation is effected by directing the telescope on some well-defined point, and bringing it to coincide with the intersection of the wires, with the level downwards.

Turn the telescope in the Y's, until the level is uppermost. If the object is still at the intersection of the wires, the collimation in altitude is correct.

If not, bring the wires half-way towards the object by turning the screws holding the diaphragm. Then reset the telescope by the tangent screw for the object, and bring the telescope round in the Y's to its former position, when any displacement still existing must be corrected in the same way, half by the diaphragm screws, and half by the tangent screw. After a few trials the error should be corrected.

Do the same with the telescope with level right, and level left, at right angles to its former positions in the Y's, for

azimuth error. When this is done, the cross-wires, while the telescope is slowly revolved, should remain over the object.

It should be noticed that in loosening one collimating screw and tightening the other, the arm of the "tommy" moves in the same direction for both screws; one screw should be loosened before the other is tightened.

Action of
Colli-
mating
Screws.

Using the inverting tube, if it is required to move the cross of the wires upwards, the upper screw must be loosened and the lower one tightened. Attention to these rules will save an inexperienced hand much trouble, and wear and tear on the threads of the screws.

The collar being tightened by its clamping screw, unclamp the vernier plate, and turn it round till the telescope is over two of the parallel plate screws. Bring the bubble of the level beneath the telescope to the centre of its run by turning the tangent screw of the vertical arc. "Turn the vernier plate half round, bringing the telescope again over the same pair of the parallel plate screws; and, if the bubble of the level be not still in the centre of its run, bring it back to the centre, half-way, by turning the parallel plate screws over which it is placed, and half-way by turning the tangent screw of the vertical arc. Repeat this operation till the bubble remains accurately in the centre of its run in both positions of the telescope; and then, turning the vernier plate round till the telescope is over the other pair of parallel plate screws, bring the bubble again to the centre of its run by turning these screws. The bubble will now retain its position, while the vernier plate is turned completely round, showing that the internal azimuthal axis, about which it turns, is truly vertical.

Adjust-
ment of
Horizon-
tal Limb

"If the bubbles of the levels on the vernier plate are now brought to the centre of their tubes, by means of the screws fitted for the purpose, they will be adjusted to show the verticality of the internal azimuthal axis.

"Now, having clamped the vernier plate, loosen the collar, by turning back the screw, and move the whole instrument slowly round upon the external azimuthal axis, and, if the bubble of the level beneath the telescope maintains its position during a complete revolution, the external azimuthal axis is truly parallel with the internal, and both are vertical at the same time; but if the bubble does not maintain its position, it shows

“that the two parts of the axis have been inaccurately ground, and the fault can only be remedied by the instrument maker.”*

Adjust-
ment for
Vertical
Limb.

To adjust for the vertical limb, the bubble of the level being in the centre of its run, reverse the telescope, end for end, in the Y's, and if the bubble does not remain in the same position, correct for one half the error by the capstan-headed adjusting screw at one end of the level, and for the other half, by the vertical tangent screw. Repeat the operation till the result is perfectly satisfactory. Next turn the telescope round a little, both to the right and to the left, and if the bubble does not still remain in the centre of its run, the level must be adjusted laterally by means of the screw at its other end. This adjustment will probably disturb the first, and the whole operation must then be carefully repeated. By means of a small screw, fastening the vernier of the vertical limb to the vernier plate over the compass box, the zero of this vernier may now be set to the zero of the limb, and the vertical limb will be adjusted for it horizontally.

Adjust-
ment for
Vertical
Plane.

The vertical limb should move in a truly vertical plane.

Any error can only be adjusted in the larger instruments, but every theodolite must be tested for it, as, if much error exists, the instrument requires alteration by the maker. It will introduce error into all angles to objects much elevated or depressed, and it is especially important for observations for true bearing to know that this adjustment is perfect.

To test it, direct the theodolite when horizontal to either the edge of a well-built wall, or still better, a steady plumb-line. The cross-wires, when the instrument is elevated and depressed, should still intersect the line.

If they do not, in 6-inch theodolites, the adjustment can generally be made by means of screws on one of the Y frames. In smaller theodolites we must accept the error, and take care not to use them for true bearings.

These adjustments completed, the instrument will be ready for work.

Points
liable to
Derange-
ment.

There are, however, a variety of small points on which a theodolite may go wrong while away in the field, and a knowledge of the general causes of these temporary derangements

* From Heather.

is very useful, and may prevent loss of a day's work, and much aggravation to all concerned.

The parts of a theodolite, especially in an old instrument, that most frequently get out of order, are the small screws which hold the milled heads of the tangent screws in their places. A young observer is often much bothered and puzzled by his instrument not coming back to zero, which may result from many things, but most frequently from one of the small screws above mentioned being loose. Screwing it up tight enough to prevent any play when the instrument is clamped, but not so tight as to make the tangent screw work hard, will often remove the difficulty.

Other causes of not coming back to zero are :—

1. Looseness of the sockets through which the tangent screws work, and which can be easily tightened by their screws.

2. Looseness of the fittings of the brass stand on the theodolite legs. There are many working parts here, and any of them are liable to get loose. The leverage on the brass plate that fits on each leg head is enormous, if the leg should be allowed to swing out in taking off the rings ; and as the screws that hold them on are small, looseness may easily take place here.

3. In an old instrument, the faces of the clamping plates may screw close together without clamping the instrument tightly. This is from the part that holds the instrument, and which gets all the friction, being much worn. The parts into which the clamping screw fits must be smoothly filed on their inner faces, so as to ensure the other parts coming into contact with the body of the instrument, before the faces of the clamping plates meet.

4. The upper plate will sometimes not revolve freely, but catches every now and then. This is from the piece of metal which clamps the two plates together either not fitting very well, or being dirty inside, or perhaps bent. Placing the finger underneath so as to press it up to the lower plate, whenever the plates are to be revolved, will ensure its working smoothly, and is a better thing for a young hand to do than to attempt to take it off. The same thing will happen to the reading-glass plate ; but here it is often the little screw underneath which is loose, and simply screwing it up will relieve it.

Lifting it with the finger will always assist it to run round easily.

5. The fact of the tangent screw being run out nearly to the end of the thread, will frequently account for a theodolite not coming back to zero if the thread is worn. This may occur in the case of the tangent screw of either the upper or the lower plate; in such a case the tangent screw should be turned until it nearly butts, the last few threads being probably less worn.

Putting in
New
Webs.

An operation the nautical surveyor has frequently to perform is replacing the wires, or rather cobwebs, of his theodolite telescope.

For this purpose catch a garden spider, as a house spider does not spin his rope taut enough. Having cut some holes, say 2 inches square, in a strip of cardboard about 3 inches wide, place the spider on it, and shake him off. As he throws out his web in falling, twist it up on the cardboard so as to cross the holes, and lay it on one side.

Having taken the diaphragm from the telescope, and scraped off the old balsam, lay it on the table and place the smallest drop of Canada balsam on its edges. With the aid of a magnifying glass, place the cardboard across it, in such a manner that the web will lie in the notches cut in the diaphragm, when it will adhere to the balsam.

A silk thread, exceedingly fine, is an excellent substitute for cobwebs, and is indeed superior in some respects, being more easily inserted and stronger.

Measuring
Angles
with the
Theodo-
lite.

Heather gives a good description of measuring angles with the theodolite, to which we will add, that,

Regard must be had to the purpose for which the angles are to be taken, in settling how many times, and in what manner, the angles shall be repeated. An error of one minute will make no perceptible difference when plotted, unless the line be very long, say 5 feet. All objects, therefore, that are simply to be plotted, and do not come into the triangulation, can be taken round once with zero at 360° , and a second round taken after with another zero, say 100° for convenience' sake, simply for the purpose of making sure that there are no gross errors, as no theodolite in adjustment should give an angle in error exceeding one minute.

Angles to main stations, however, will be very likely required to enter into the calculation, and the correctness of the plotting will any way depend on them. These must therefore be repeated, the number of times varying according to the degree of accuracy required.

One method of repeating angles is thus given in Heather, somewhat altered.

Having taken the first measurement, loosen the clamp of the lower plate, turn the theodolite bodily round until the telescope is directed upon the zero-object, and again clamping the instrument, perfect the bisection of the zero by the cross-wires by means of the slow-motion screw on the neck of the instrument. The index of the vernier, together with the coincident division of the limb, will thus have been brought from the position in which it was when the telescope pointed at the object to be measured, round to the previous position of the 360° .

Now release the upper or vernier plate (looking again at the vernier first to see it has not been moved), turn it until the telescope is again directed towards the object, clamp and perfect the bisection by the tangent screw moving the upper plate. The reading now on the vernier will be twice that formerly read off, or nearly so, and will be entered in the book under the former observation. This process can be repeated as often as required. The mean angle can be obtained by dividing the last reading (increased by as many 360° as the plate has revolved) by the number of observations, but it is better for our purposes to put down each individual reading. The difference between every two consecutive readings will give a value for the angle, and we can then see how they agree with one another. An example of this kind of repeating is given on p. 85.

The above method is perhaps the most accurate ; but, when many angles are to be taken, requires much time, and we shall arrive at a conclusion quite near enough for any hydrographical triangulation by taking all the angles in succession with the vernier set to 360° , and then, changing the degree of the zero to some even submultiple of 360° , as 90° , 180° , etc., take all objects again, repeating thus as often as necessary ; this will be found much quicker. 360° , 120° , and 240° divide the arc

Repeating
Angles.

First
Method.

Second
Method.

equally and give three readings, which is often sufficient. The other method can be reserved for taking single angles, as, for example, a flash from a distant station.

Reading
both
Verniers.

If both verniers are read, any error arising from bad centring should be eliminated for any given position of the plates.

For practical hydrographical purposes if one vernier is read with the zero in several positions, submultiples of 360° , it is as a rule sufficient.

Different to the sextant, the theodolite has no index error to apply to horizontal angles, but to the vertical arc there is a correction to be found and applied, which will be mentioned in discussing the method of ascertaining heights.

Coloured
Shades to
Eye-
pieces.

A theodolite for hydrographical purposes should be fitted with coloured shades to the eye-piece of the telescope for observing the sun for true bearing.

STATION POINTER.

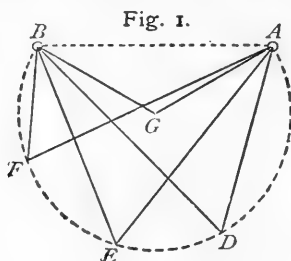
This useful instrument is of hourly service in nautical surveying.

Either in sounding, coast-lining, or topographical plotting, the position of the observer depends mainly on it.

Theory of
Station
Pointer.

The station pointer is used to plot a position on the chart, by means of angles taken *at* it, to other objects already fixed.

Its construction depends upon the fact that the angles subtended by the chord of the segment of a circle, measured from any point in the circumference, are equal. (Euclid III. 21.)



Thus, in the figure, the angles $A D B$, $A E B$, $A F B$ are all equal, so that if we have observed the angle subtended by $A B$, we know at any rate that we are somewhere on the cir-

two circles must be our exact position X, as it is the only one from which we could have obtained these two angles at the same time. See Fig. 2.

The station pointer obtains us this position X without the trouble of drawing the circles, as it is manifest that, if we have the angle A X B on one leg of the station pointer and B X C on the other, the only spot at which we can get the three legs to coincide with the points A, B, and C, will be X.

We place the station pointer, therefore, on the paper, bringing the chamfered edges of the three legs of the instrument to pass over the three points observed, and make a prick with a needle in the nick in the centre, which will then mark the spot.

A piece of tracing-paper on which the three angles are protracted will answer the same purpose, but, of course, this will entail more time, and in the open air will give trouble, as liable to be blown about by the wind. Nevertheless, this has often to be used, as when points are close together on a small scale, the central part of the station pointer will hide them, and prevent the use of the instrument.

A very useful instrument has been devised by Commander Cust for such occasions, and consists in a plate of transparent zylonite on which a graduated arc is engraved. The requisite angles are drawn on this with pencil, with the angles reversed, and the plate being turned over, so as to bring the pencil lines in contact with the paper to obviate parallax, is used as a station pointer.

Three
Circle
Method.

This method of fixing is generally known as the "two circle" method, but it is really the "three circle" method, for the circle drawn through the two outer points and the observer's position is also involved. A comprehension of this is of value.

Position of
"Points."

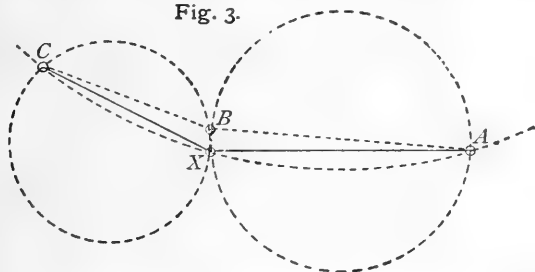
The chance of error in a fix varies greatly with the position of the three points with regard to one another and the observer.

It is in general sufficient to realise that the more rectangular the intersection of the two circles, the less chance there is of any error in the resulting fix, but there are cases where the fix is admirable though these circles are almost tangential, because the third and larger circle produces a rectangular cut.

With points and the observer's position placed as in Fig. 2, the two circles give a good intersection, and the fix is good.

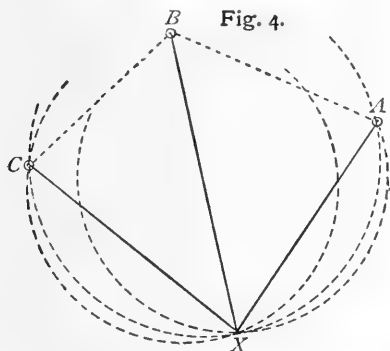
Let us, however, take the same points with the observer's position close to the centre object B, as in Fig. 3. We there see that the two circles are nearly tangential, but the third circle through the outer points and the observer, which the station pointer also gives us, cuts at a right angle, and as the position X cannot be off it, the fix is one of the best.

Fig. 3.



In such a case the whole angle between A and C should be observed, if not too large (as in our figure), as the accuracy of the fix depends entirely on this whole angle, and when so near to B a little movement may make considerable difference if B X A and B X C are separately measured.

Fig. 4.



Let us now take three points and the observer's position as in Fig. 4, using the same letters.

The angles we have observed give us X as the point of intersection. It is evident that it is difficult to localise this point exactly, as all three circles so nearly coincide as make it impossible to say where the precise point is at which they intersect, and, with the station pointer, we should find that we

could move the centre of the instrument considerably, without materially affecting the coincidence of the legs with the three points.

**On the
Circle.**

When X is so placed as to fall on a circle passing through the three points A B C, there will be no intersection whatever, as the two circles will coincide ; and we cannot tell where we are on the circumference of this practically single circle.

The nearer, therefore, we are to being on a circle, whose circumference will include all the three points and our own position, the worse will be what is technically called the fix, and this must always be guarded against in selecting objects to observe.

When one object is farther from us than the central one, we shall, as a rule, have a good fix ; but when the central object is the farthest, the two circles will begin to make a bad intersection.

**Sensitive
Circles.**

A circle is "sensitive" when the angle between the two objects responds readily to any small movement of the observer towards or away from the centre of the circle passing through the observer's position and the objects. This is notably the case when one object is very near to the observer and the other very distant ; but not so when *both* objects are distant. Speaking generally, the sensibility of an angle depends upon the relative distances of the two objects from the observer, as well as on the absolute distance of the nearer of the two.

The more rectangular the angle at which the circles intersect each other, and the more "sensitive" they are, the better will be the fix ; one condition is useless without the other.

**General
Rules.**

There is nothing in the whole range of surveying that requires so much attention and knowledge as the fix, and many are the errors which have crept into surveys from disregard of its conditions.

When moving along, as when sounding, and fixing from time to time, if both angles change slowly, the fix will be bad, for we must be moving nearly along the circumferences of both circles, and they must therefore nearly coincide.

In plotting the angles with the station pointer, the fix will be good if a very slight movement of the centre of the instrument throws one or more of the points away from the leg ; but

if this can be done without disturbing the coincidence of the legs and all three points appreciably, the fix is bad.

This is perhaps the most important thing for a beginner to remember and to practise, as it is a practical test involving no theory nor complications.

Theoretically, one of the best positions is inside the triangle formed by the objects, but in practice it is often impossible to observe the large angles incidental to this position.

Practically the beginner will find the following rules safe :—

1. Never observe objects of which the central is the farthest.

2. Choose objects disposed as follows :—

(a) One outside object distant and the other two near, the angle between the two near objects being not less than 30° or more than 140° . The amount of the angle between the middle and distant objects is immaterial.

(b) The three objects nearly in a straight line, the angle between any two being not less than 30° .

(c) As before remarked, that the observer is inside the triangle formed by the objects.

There are, nevertheless, cases where the middle object is very distant, when the fix will be good enough for many purposes, but such cases require a thorough grasp of the subject, and should not be adopted by the beginner unless forced to it.

1. The best fix is with one distant and two near objects, but the angle between the latter should not be less than 30° or 40° . The centre object must be either of the two *near* objects, and it is immaterial whether it be the nearer or farther of the two. The angle should be observed between the two near objects and between the distant object and the farther of the two near ones.

Choice of
Objects
for
Station
Pointer
Fixes.

2. With the centre object the nearest, a very good fix is obtained ; but if it is very much nearer than either of the other two, the *whole* angle should be observed and *one* of the others.

3. With the centre object the farthest, caution is necessary. If it is *very much* farther off than the other two objects, a good fix may be obtained if the observer's position lies well away from the circumference of the circle passing through the three points ; but this will not be the case if the observer is nearly on the line joining the two nearer objects.

General
Rule for
observing
Angles of
a Fix.

Size of
Angles
Admissi-
ble.

The angle should be observed between the two most distant objects and between the two nearest objects.

The size of angles admissible in a good fix depends on the position of the three objects. If two objects are equi-distant, the angle must not be small, for a slight error in the angle will make a great difference in the position; but if one object be much farther off than the other, a very small angle between these will suffice, so long as the third object is so placed as to make a fairly large angle.

An arrangement of the objects not yet considered, is when two of them are in line from the observer's position.

"Points"
in Transit.

This is technically called "transit," and no transit of known marks is allowed to take place without making use of it.

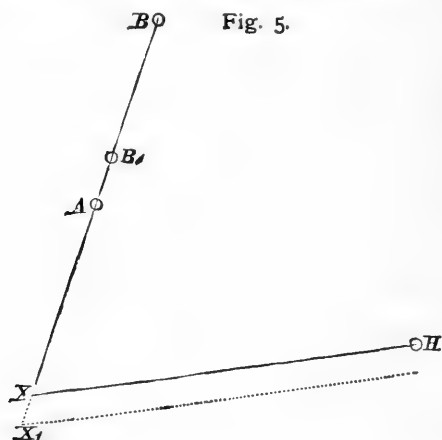


Fig. 5.

One angle to a third object is here enough to fix the position, which is one advantage, another being that if two angles are taken and placed on the station pointer, the coincidence of the position, as plotted by these two angles, with the transit line, gives an excellent check.

Here, Fig. 5, A and B are in line of transit (ϕ); H is a third object.

It will be evident that when the observed angle is on the station pointer, and the latter is placed with one leg coinciding with the line A B, we have only to move it up or down that line, until H coincides with the other leg, which gives us X.

Any other position, as X_1 , would not allow the leg to pass over H.

It will also be seen that the farther apart A and B are, the truer will be the direction of the transit line. If one object was at B_1 , the position pricked through at X might be a little right or left of the true transit line, without the deviation being visible on the leg of the station pointer.

Also, it will be seen that the angle to the third object should be as near 90° as possible, anything under 25° being inadmissible, as the angle of intersection at X would permit of a false position without detection.

When using this method, the distance of the third object should also be considered. It must not be too far, or both theoretically and practically, by reason of the imperfection of instruments, the fix may be in error.

These conditions are illustrated in Fig. 6, in fixing the position of a point O on the line DO, by the intersection of the line AO.

Effect of a Small Error in an Intersecting Angle.

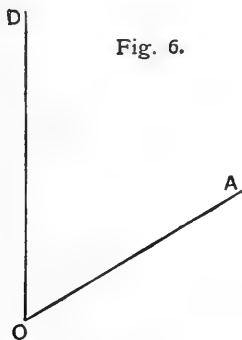


Fig. 6.

Resulting error of position on line DO due to a small error in angle DOA $\propto \frac{\text{dist. AO}}{\sin DOA}$,

and thus has its smallest value when $DOA = 90^\circ$ and when AO is short.

The principle here involved is of frequent application in deciding what cuts to accept when there is a slight discrepancy in the plotting of a point.

If the value of the resulting error in position when $DOA =$

90° be unity, and the error in the angle D O A and distance A O remain the same,

If D O A be changed to 30.0° error in position changes to 2.

| | | | | | | | |
|---|---|---|---------------|---|---|---|---|
| ” | ” | ” | 19.75° | ” | ” | ” | 3 |
| ” | ” | ” | 14.5° | ” | ” | ” | 4 |
| ” | ” | ” | 11.5° | ” | ” | ” | 5 |
| ” | ” | ” | 9.5° | ” | ” | ” | 6 |
| ” | ” | ” | 8.1° | ” | ” | ” | 7 |
| ” | ” | ” | 7.2° | ” | ” | ” | 8 |

which shows the rapid rate at which the error increases as the cutting angle diminishes after it is less than 30° . This is very important, and should be kept constantly in view.

**Choosing
a Station
Pointer.**

In choosing a station pointer, of which instrument Heather gives but a meagre account, the first important thing to look at is that the smallest angle to be read on the leg which will not come to zero, is as small as it should be. A well-planned modern station pointer should allow this leg to return to 3° or 4° , but old instruments frequently will not read less than 19° .

It is a great nuisance to find that the only angles you can take cannot be plotted by means of your station pointer, and the chance of this should therefore be minimised as much as possible.

Station pointers are generally made to allow the left angle to come to 0° .

When the angle on the right is too small to set, and the left angle is more than 90° , the difficulty can be got over by setting the small angle on the left leg and bringing the right leg round to the left until the required left angle is made between it and the left leg.

Another method when the above cannot be carried out, but care must be taken not to employ it when exact accuracy is required, is to set the angles on the station pointer reversed, *i.e.* the right angle on the left, and *vice versa*. Place the legs containing the larger angle on its “points,” with the centre near the supposed position, and make a prick. Move the centre a little, right and left of the first prick, keeping the points on, and make other pricks. A line drawn joining these pricks will form an arc of the circle for those “points.”

Now do the same for the small angle and its "points," and the intersection of the two small arcs will give the position required.

It is, in fact, projecting the circles by means of the station pointer.

Station pointers are made with brass, and with silver arcs; the latter are of course more durable, but for many purposes the brass are to be preferred. When sounding, or doing any work in the open, the reflection from a silver arc is often a bother, and hinders speedy setting of the vernier. The use of a reading glass is almost a necessity with silver arcs for this reason, and also on account of the fineness of the cutting; whereas, with the brass arcs, a surveyor with good eyes can set his instrument quite correctly without one, a great point in a boat.

For chart-room use the silver arc is to be preferred.

The nick in the centre of the instrument should be small, *i.e.* just deep and wide enough to admit of a needle fairly catching in it. The needle-pricker should always be used for marking the position; not a pencil-point, which soon wears blunt, and will not mark truly in the centre. The prick also remains, and can be seen under the figure with a reading glass, when inked in.

The prick should be on the continuation of the edge of the bar in which is the nick.

For ordinary soundings and field work, a station pointer of about 5 inches diameter of arc is most convenient. For ship sounding and chart-room work larger ones are supplied.

In testing a station pointer, the first thing is to see that the vernier of the leg which comes back to zero reads exactly 0° , using a magnifying-glass to read off accurately. If it does not, the screws which hold the vernier must be loosened slightly, and the vernier plate moved, until the arrow on the vernier corresponds exactly with 0° of the arc, and the $30'$ on the vernier with a division of the arc.

Testing a
Station
Pointer

Take either a large sheet of backed paper, or a white Bristol board, and mark out, by means of chords, lines radiating from a centre, and 10° apart. These lines must be very carefully ruled, and in Indian ink, as this sheet must be kept as the test of all station pointers and protractors, which should be from

Testing
Circle.

time to time examined by its means. Screwing on the lengthening legs, and placing the station pointer on this sheet, with the nick in the centre of the arc corresponding exactly with the prick in the centre of your testing circle, and putting weights on the central part of the station pointer, each leg can be in turn moved to correspond with the ruled 10° lines, and the reading of the vernier compared. The error at each 10° should be written on a small piece of paper in the form of a table, and pasted on the inside of the box.

If the legs of the instrument are exactly centred, the readings will either be correct, or the same amount in error all round, for each leg ; but as this is a degree of delicacy rarely attained, it will usually be found that the error varies for different positions of the leg. The verniers should be set to minimise the errors between 0° and 90° , which is the amount of angle most used in actual work.

The chamfered edge of the leg and lengthening piece should correspond exactly with the line in all its length ; if it does not, it is also a result of bad centring or bad fitting on of the lengthening piece, but a good instrument should not have this error in any appreciable extent.

It need scarcely be added that if the instrument is found very badly centred, it should be returned to the maker, or not be chosen if buying ; but when an instrument is sent to the other end of the world, you may have to make the best of it, and registering all the errors on the table, be careful to apply them when using the instrument.

Discretion
as to
applying
Errors.

The necessity for applying a small error depends upon circumstances, as, in some cases, the position of the points used will admit of a difference of several minutes in the angle, without any appreciable alteration of the position of the observer ; in others, it is necessary to be exact. As the surveyor gains experience, he will learn when to apply the error, and when not. At the commencement, he must always apply the error.

Caution
as to use
of Station
Pointer.

It may here be noted, that, if the points used to fix by are not correctly placed on the chart, the station pointer will not indicate anything wrong, *unless a third, or "check" angle, be taken and plotted.* This must always be remembered in using

a station pointer on a published chart, or the adoption of this instrument may have a disastrous result.

In the first place, the chart may be from a rough survey, and there may be absolute errors in the points on it ; and secondly, the distortion caused in printing with damp paper always changes the position of points, more or less, and with objects in certain positions, this alone may make an error in a station pointer fix.

In navigating, therefore, with a published chart, of the accuracy of which you are not certain, *always use a bearing*, as well as the sextant angles plotted by station pointers, or *use check angles to each fix*. If the result is to show that the points are not correct relatively to one another, use the compass only, as it is less likely to get you into trouble with a defective chart, for the reason that the non-intersection of three bearings will at once indicate something wrong, and the navigator will choose the points of danger in his course ahead to steer by, rejecting the others whose positions with regard to him are of little moment.

BRASS SCALES.

These must be examined by means of the beam compasses, to see that their divisions are correct, more especially the diagonal portion, as the makers are sometimes not careful enough. If a scale is found to vary, it should be rejected.

A brass scale should never be used for ruling, and *never be taken out of its box*. If it is, some day it will fall from the table, get bent, and its correctness is gone.

STEEL STRAIGHT-EDGE.

This must be examined to see if its edge is exactly straight, by ruling a very fine line, and reversing the straight-edge, when, either ruling another line over the first, or examining the coincidence of the edge with the line already ruled by means of a reading-glass, will prove whether it is perfect.

Placing steel straight-edges edge to edge is another method when there are more than one ; but great care must be taken with regard to the light if this is done, as it is difficult to

detect a small error if the light falls across. They ought, of course, to touch throughout their whole length.

A steel straight-edge must be kept very clean, and carefully wiped before using, or the paper will soon become very dirty.

If kept bright, care must be taken that no emery is allowed to touch the chamfered edge, or it will get so sharp in time as to cut the pencil, and even the fingers of the operator. When once clean, rubbing daily with a warm dry soft cloth will keep it so, with an occasional rub of emery in damp weather.

Straight-edges are now generally supplied nickelled.

MEASURING CHAINS.

To test measuring chains, which are generally 100 feet long, 100 feet should be accurately measured by beam compasses along a chalked line on a plank of the upper deck, and marked by nails driven in at intervals of 10 feet.

Always to
be re-
tested
when
used.

Before measuring a base, all the links of the chains should be examined, and bent ones straightened; the chains are then compared with this fixed length and the errors noted. The same reference should be made after measuring, and the mean of these errors applied to the distance measured.

It may be as well to note that, when the chain on comparison proves to be longer than 100 feet, the surplus is to be added to each length measured, and when it is shorter, subtracted.

Points of
Measure-
ment.

The length is to be measured from the *outer* side of one handle, to the *inner* side of the other. This is to allow for the necessity of having a pin to put in the ground at each length.

Each link is a foot long, and every tenth link is marked by a brass label, with as many fingers on it as there are tens of feet from the nearest end.

PROTRACTORS.

Protractors of all kinds must be tested for correctness of division by the same testing-sheet ruled for the station pointers.

This is especially necessary in the case of Bullock's pro-

tractors, which have extended arms, generally of very light construction, which a slight blow will bend out of the direct line. These sometimes admit of correction by means of screws, which is easily accomplished by placing the protractor on the testing-sheet, with the opposite verniers exactly coinciding with the same line, and adjusting the extending points until they also prick precisely on the line.

If the divisions of an ordinary protractor are found to be incorrect, there is of course nothing for it but either to return it to the maker to be re-cut, or to mark the errors at each ten degrees, or wherever necessary, on small bits of paper pasted on the protractor.

A boxwood or vulcanite protractor is easily kept clean by rubbing it over with a piece of india-rubber, but a brass or electro-plated one is very apt to dirty the paper in plotting. It is a good plan to carefully paste a piece of tracing-paper on the under-side of these, when a rub with the india-rubber before use will ensure cleanliness. The thinness of the tracing-paper will not interfere with correctness in laying off the angles.

Vulcanite protractors are admirable for field work, as they are light, easily read, and when made thick do not chip like boxwood ones.

Large brass circular protractors of 10 inches or so radius are very useful for laying off the secondary points of a survey, saving the time involved by using chords.

POCKET ANEROID BAROMETERS.

These are very useful when putting in the topography of a country, as they give sufficiently accurate results for minor heights, with but little loss of time; but for the more exact measurement of conspicuous hills, etc., they are but of little use, and the theodolite and sextant must be had recourse to.

In choosing a pocket barometer for the above-named service, therefore, it is not necessary that it should read very low, as it will be but rarely that nautical surveyors have to deal with the intricacies of land over a few thousand feet high. For this purpose, 25 inches is quite low enough, and the 5 inches of barometric range thus obtained can be so largely marked on

the dial as greatly to facilitate the reading to two places of decimals.

**Index
Needle.**

An important point for delicate reading is the construction of the index needle. This should be very thin towards the point, and turned with its edge at right angles to the plane of the dial. In this position it should admit of very accurate reading, and moreover assists the observer to hold the instrument at right angles to his line of sight, and thereby to avoid parallax errors. For this reason the point, though as thin and fine as possible one way, should be tolerably wide in the other, so as to show plainly by its apparent increase of width when it is being looked at from any direction but at right angles to the dial.

The point of the index needle should cover about half the graduation of the arc. If it is too short to reach to it, or so long as to project over it, it is not so easy to read accurately.

**Reading
an
Aneroid.**

In reading, the best position for the aneroid to be held is upright, on a level with the eye, the index being vertical. Read with one eye only, or parallax will creep in. Tap gently each time before reading, and turn the instrument flat, and then vertical again for a second reading, to prevent mistakes, tapping as before. In whatever position, however, the instrument is read the first time, it must be always held for all other readings on that day, the reason being that the weights of the different parts in such a delicately made little instrument have considerable influence on its free movement, and that this influence must be so disposed as to act in the same manner at each reading.

**Care of
Aneroid.**

The surveyor will of course never let the pocket aneroid out of his own possession, and will place it about his person in such a manner as to minimise chances of shocks or blows in scrambling through rough country, getting wet in rainy weather, or tumbling out of the pocket. To prevent the latter accident, always use a lanyard. The instrument should always be carried in its case.

If a small aneroid is not in regular use, the delicate internal parts, especially the chain, are liable to stick from oxydization, when at length taken up a height. It is therefore convenient, if an air-pump be on board, to place the instrument under the receiver from time to time, so as to keep all working parts in order.

HELIOSTAT.

This instrument, which is simply a mirror mounted in gim-bols, so as to turn and reflect the sun in every direction, is of great use. In a survey where many assistants are at work together, it saves an immense amount of time. Smaller beacons or marks can be erected, and the position of a theodolite station that has to be made on the side of a hill, or with dense foliage behind it, is at once made apparent to another observer, who has to take angles to it by the flash, which can be seen a long distance by the naked eye.

Most
Valuable
Adjunct
for Marine
Survey-
ing.

Some heliostats supplied are mirrors in gim-bols, mounted either on stands or in portable cases, with a spike to drive into the ground.

Fitting.

Neither of these forms is satisfactory, as in many places from which it is desired to use them they cannot be conveniently and firmly placed. Tripod legs of some description on which to place the mirror are best, and a movable arm working round the centre, and carrying an adjustable ring through which to direct the flash, will be found very handy. If the surveyor has to trust to placing some separate object, such as a stick or another tripod, a few feet from the mirror, by which to direct his beam of light, he will soon find himself in some position where there is no standing-ground for such object, as when his theodolite is on the top of a sharp hill, or on a steep coast-line under cliffs at the edge of the sea.

A better instrument is the excellent and convenient Galton's Sun Signal, now also supplied. This is fitted with a telescope, by looking through which and adjusting the mirror, a dim image of the sun is seen covering the object required to flash to. Nothing can be better adapted to the purposes of the nautical surveyor's work than this (when he is once accustomed to it, as at first it is a little awkward to manage), and when obtainable they should always be used. Care must be taken, however, that the instrument is in adjustment. This can be ascertained as follows : Place a board, with a sheet of white paper pinned on to it, about 50 yards off. Direct the sun signal flash on to it, and looking through the telescope, screen and unscreen rapidly with the hand the direct flash from the mirror. If the circular image formed by the direct

Galton's
Sun
Signal.

flash on the sheet is not coincident with the image of the sun as seen through the telescope, take off the cap at the end of the tube and adjust with the screw that will be found underneath.

A very workable arrangement can be fitted on board any ship as follows :—

Instru-
ment
easily im-
proved
on Board.

A blacksmith will soon make a frame which will convert an ordinary looking-glass into a perfect instrument for surveying work, as it must be remembered that we do not propose to use it for talking, and therefore do not require the extreme accuracy in directing the beam necessary in the military heliograph.

The sketch annexed shows a looking-glass fitted in this manner by a ship's blacksmith. The standard can be made of any height as convenient ; about $2\frac{1}{2}$ feet is a good length. In soft ground the end of the legs can be pressed into the earth, and on rocky ground stones placed against the legs will hold the instrument steady. The arm, *m*, of light iron, is carried separately, and slips over the shaft of the standard, clamping where required with a screw.

Into a circular socket in head of standard shaft, the leg of the frame holding the mirror is shipped ; this is also to be tightened by a retaining screw.

The mirror, which can be of any size from 2 to 6 inches or more in diameter, revolves on its retaining screws, as an ordinary toilet-table glass, and can be held in any position by tightening these screws.

The ring, of flat wood, is made as light as possible, so as to exert less strain in wind. Across it are nailed crossed strips of copper, with a white cardboard disc, about 1 inch in diameter, fastened to their centre.

The rod that carries this ring slips up and down in a hole at the end of the arm, and is clamped by a retaining screw.

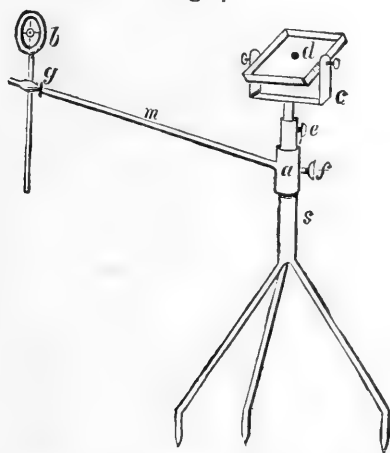
In the centre of the back of the mirror, a hole of about $\frac{3}{4}$ inch diameter is scraped in the tinfoil, being careful to leave a sharp edge. A similar hole is cut out of the wooden back of the glass frame. This we shall call the "blind spot."

Using the
Heliostat.

To direct the flash to an object, bring the mirror vertical, and looking through the hole in the centre, revolve the arm until in the direction of the object nearly, clamp it, and adjust

the disc rod as nearly as may be, for elevation or depression. Then, slightly loosening the screw clamping the arm, finally adjust the latter, so that the object, as regarded through the hole in the mirror, is obscured by the white cardboard disc in the centre of the ring. By turning the mirror so that the dark shade caused by the blind spot is thrown on to the disc, the flash will be truly directed, and must be kept so by slight alterations of the position of the mirror, which should therefore be clamped only sufficiently to hold it steady, and yet

Fig. 7.



LOOKING-GLASS AS FITTED BY BLACKSMITH FOR HELIOSTAT.

a, Sliding collar carrying arm *m*, revolving round *s*; *b*, wooden ring, painted black, with cross-wires and white cardboard centre, sliding vertically by means of rod through arm *m*; *c*, iron frame to hold mirror, fitting into socket in top of standard *s*; *s*, iron standard with fixed tripod legs; *d*, blind spot in mirror; *e*, screw for clamping mirror frame; *f*, screw for clamping arm; *g*, screw for clamping ring rod.

admit of gentle movement. The shadow of the blind spot should be slightly smaller than the disc, so as to ensure having it truly in the centre of the latter.

The mirror must be of the best glass, with its faces parallel, or the shadow of the blind spot will be very indistinct when the mirror is at a large angle, and also the beam of light will be dispersed before it has traversed many miles.

It is well to have the mirror a fair size, say 6 inches square, as in practice it will be found generally necessary, in order to

Best Glass
Necessary

Size of
Mirror.

save time, after once adjusting the flash, to leave a bluejacket to keep it on, while the surveyor is taking his angles ; and although a man will soon pick up the knack, a larger mirror will allow for eccentricities on his part, and also, on a dull day, a faint flash will be detected from a large mirror, where a small one would not carry any distance.

Power of
Penetra-
tion of
Flash.

On a bright day, a flash from a 3-inch by 2-inch mirror has been seen fifty-five miles and more.

In hazy weather, angles have been got when the place from which the flash was sent was entirely invisible ; and thus whole days have been saved by this simple contrivance.

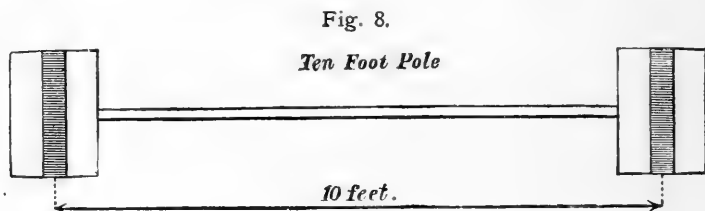
Only those who have spent hours, or even days, in straining their eyes to see a distant mark can appreciate the value of a heliostat.

TEN-FOOT POLE.

For coast-lining, a pole of measured length is often required, to get distances by observing the angle subtended by it.

A convenient form is as follows :—

Two oblong wooden frames, about 18 inches by 2 feet, are made as light as possible, and covered with canvas. These will fit, by means of sockets at the back, on to the ends of a pole, and copper pins passed through socket and pole will keep them



at a certain fixed distance apart. Ten feet is a convenient length for transport.

The face of the canvas on the frames is painted white, with a broad vertical black stripe in the centre, and the 10 feet will be measured from centre to centre of the black stripes.

In measuring with a sextant the angle subtended by such a pole, the image of one stripe will be brought to cover the other stripe.

A table of distances, corresponding to the angle subtended by the length of the pole used, should be in each assistant's possession for reference on the spot.*

Fig. 8 represents a 10-foot pole.

It is important in using a 10-foot pole to have regard to the direction from which the light falls on it; when the sun is low and behind the pole, it is difficult to obtain satisfactory results.

In order to get a longer base without the inconvenience of an unwieldy pole, the wooden frames may be connected by a wire cord instead of by a pole, which when stretched taut gives a distance of exactly 30 feet between the vertical stripes on the two frames.

Held firmly by two men, with the cord between them kept quite taut, one stands fast and the other walks slowly backwards and forwards, to enable the maximum subtended angle to be measured either by sextant or by theodolite.

RANGE-FINDERS.

The Barr-Stroud Navigational Range-finder is the one best adapted for surveying purposes, and is likely to prove a very useful adjunct to the equipment of a surveying vessel.

DRAWING-BOARDS.

In a surveying vessel it is convenient to have a considerable number of these, and of various sizes, so as to fit all scales. The largest may be about 29 inches by 25 inches. The size of which most will be wanted will be about 27 inches to 25 inches by 20 inches to 22 inches. There should be some smaller ones, 22 inches by 16 inches.

Lightness, combined with sufficient strength not to warp, is the requisite, and seasoned wood is therefore necessary. White pine, $\frac{3}{4}$ inch thick, is as good as anything, though the smaller boards may be made of thinner mahogany.

Duck covers to fit the boards are necessary for field work, when the work is plotted at the time, to carry them in, and prevent rubbing and wetting from rain.

* Appendix, Table R.

If it is intended to do most of the detail plotting in the field and boats, there should be about three or four boards to every assistant in the survey.

WEIGHTS.

The weights supplied by the Stationery Office are of iron, flat, oblong, and covered with leather.

Drum-shaped leaden weights to supplement these, covered with duck or baize, will be found very handy. These can be of various sizes and weights, to suit all requirements. Cylinders $2\frac{1}{2}$ inches in diameter and $1\frac{1}{2}$ inches in height are an average size, and can be cast on board in a wooden mould.

A few others heavier are useful, and some flat weights, $2\frac{1}{2}$ inches in diameter, and $\frac{1}{8}$ inch thick, are good for keeping down small tracings.

TRANSFER PAPER.

This must be made, not bought, as the stuff sold by stationers always has some oily material in it.

On to a damp sheet of tracing-paper scrape finely some blacklead, and rub it well in with the hand, a little at a time, allowing it to dry between each application. Rub off the loose particles before rubbing in more. The blacklead is only to be applied on one side of the tracing-paper. It must be done as evenly as possible, so as to ensure uniformity in the tint, but this is assisted by a good rub with a soft cloth when the sheet is finished. Two or three applications will be sufficient.

A lump of blacklead for this purpose is supplied to all surveying vessels, but the lead from a soft pencil will answer as well.

It is a dirty process to the operator, but in a few hours enough can be made to satisfy the requirements of the survey for a long time.

MOUNTING PAPER.

The consideration of what to plot the intending chart on must be undergone before commencing work. Two methods are in use.

To stretch, and firmly paste or glue the paper, on a drawing-board or table, where it must remain until the chart is complete; or to plot on a piece of drawing-paper mounted on holland or calico, and simply flattened out before use.

The advantage of the former is that the paper remains flat, and free from wrinkles or movement, the whole time work is being done on it; but for many reasons, the latter is most convenient for ship work. If many sheets are under weigh at one time, which frequently occurs in an extended survey and large staff, they take up less room, and interfere less with one another, when several persons are working at one table, than when sheets mounted on boards are used, and they are easier put out of the way. If the plotting sheet is very large, and formed of many pieces of paper, which it must often of necessity be, it is very difficult to stretch such a paper, and it would take up the whole of the table, where it would have to be placed, as a board of sufficient size would be very inconvenient in a ship.

Loose
Sheets
best with
Large
Staff.

The drawback is the constant stretching and taking up of the sheet, with the variation in temperature and dampness of the air, which is undoubtedly a source of annoyance in plotting long lines, as the radii measured the day before, or even a few hours before, will frequently be found so much altered in length as to necessitate remeasurement.

This variation of the sheet may also produce distortion; but with good paper, well mounted, it will be nearly the same for all parts of the chart, and the distortion is so slight as to be of no practical inconvenience, and it is a question whether more distortion is not produced when, in using the other method, the stretched sheet is finally cut off the board when finished.

Distortion.

The fact is, that a material on which to draw charts free from the possibility of stretching and distortion has yet to be discovered, and we must put up with these inconveniences as long as we use the paper of the present day.

Taking one thing with the other, then, the author recommends the use of loose mounted sheets for general ship work.

Paper, whether mounted or not, in damp climates rapidly gets into a useless condition, and this even in hermetically sealed tins.

Care of
Paper.

The stock of paper should be kept in tins in the driest place in the ship, which is probably in or near the engine-room.

The mounting of the backed sheets supplied from the Stationery Office is usually very well done, and it saves time to use these ; but as it may be necessary for the surveyor to mount sheets himself, the method will be described.

Mounting
Loose
Sheets.

The holland or calico on which the paper is to be mounted, and which must be in one piece and larger than the board, must be lightly dampened. It is then stretched over the board and tacked to the edges, care being taken to stretch it equally and squarely with the woof and warp. Rub plenty of strong paste into it with the hand, and see there are no lumps left. The sheet of paper must be well dampened with a sponge on both sides, taking care to *dab* only, on the side on which the work is to be done, and not *rub* with the sponge. The sheet is then carefully lifted by the four corners, one edge laid on the holland while the rest is kept clear of it, and the paper gently rubbed on to the board with a soft handkerchief, the paper being gradually lowered, so as to allow air bubbles to escape. It will take two people to do this, and it must be done with great care. It must be left to dry by itself, and no hot sun should be allowed to get to it, so that it may dry evenly.

Joining
Sheets.

If the plotting sheet is to be formed of more than one piece of paper the edges of the paper which will overlap must be fined down. This is done in the first instance by scraping with a sharp knife, having drawn a line on the paper where the overlapping will come, and then finishing off with ink eraser. The piece that is to be uppermost must be scraped on the under-side only, and the undermost one on the upper side, so as to make, in fact, a scarp. This will lessen the appearance of a joint, and the inconvenience of ruling lines hereafter over it.

Sizes of
Drawing-
Paper.

Drawing-paper is made of the following sizes :

| | | | | | |
|-------------|----|----|----|----------------------|------------------|
| Demy | .. | .. | .. | 20 inches by | 15 $\frac{1}{2}$ |
| Medium | .. | .. | .. | 22 $\frac{3}{4}$ „ „ | 17 $\frac{1}{2}$ |
| Royal | .. | .. | .. | 24 „ „ | 19 $\frac{1}{4}$ |
| Super Royal | .. | .. | .. | 27 $\frac{1}{4}$ „ „ | 19 $\frac{1}{4}$ |
| Imperial | .. | .. | .. | 30 „ „ | 22 |
| Elephant | .. | .. | .. | 28 „ „ | 23 |

| | | | | |
|-----------------|----|----|----|------------------|
| Columbier | .. | .. | .. | 35 inches by 23½ |
| Atlas | .. | .. | .. | 34 „ „ 26 |
| Double Elephant | .. | .. | .. | 40 „ „ 27 |
| Antiquarian | .. | .. | .. | 53 „ „ 31 |
| Emperor | .. | .. | .. | 68 „ „ 48 |

Atlas, Double Elephant, and Antiquarian are most used in chart-making, and are the sizes supplied by the Hydrographic Office.

For all rough work, as sounding sheets, ordinary field work, etc., drawing-cartridge is used. It is quite good enough, and does not entail such expense as the use of an indefinite quantity of hot-pressed drawing-paper.

This cartridge-paper is mounted on the drawing-boards by Field Boards. being wetted, and rubbed on to the well-pasted board in the manner described above for mounting on holland. It will dry quite flat. When this paper is done with, it must be floated off the board, which will cause it to distort and contract considerably by the time it again dries; but as all the work on it will have been beforehand transferred to the tracing, this does not so much matter. The paper must be kept, however, as a record, for which it is just as valuable.

Where a field board is wanted for delicate coast-line or other intricate work, a sheet of Atlas can be mounted, or white Bristol board used. The latter has many advantages, and can be *tacked* on to a board to keep it flat. If chart pins are used with thick Bristol board, they will not hold for long, and will give much bother by constantly falling out.

The Atlas paper, being good, may be mounted on to the board by merely pasting the edges, as described above. It can then be cut off without much distortion. If cartridge is treated this way, it is very apt to tear, being of a loose texture.

BOOKS.

Blank books of various forms and dimensions for boat work, field work, etc., are supplied from the Admiralty. These for the most part require no forms ruled in them; but there are a few purposes for which it is very convenient to have ruled forms bound up.

All such are now supplied by the Hydrographic Office, and it is not necessary to specify them. They are all enumerated in the Instructions to Surveyers. It is very necessary to record observations in these books in such form that they can be hereafter consulted as records.

CHRONOMETERS.

About the care of the chronometers little need be said. Full instructions are issued by the Admiralty on the subject, to which reference can be made, and Captain Shadwell's "Notes on Management of Chronometers"* contains all that can be said on the subject. The box or "room" for the chronometers is now made after a fixed plan, the principle of which may be said to be that the solid block on which the chronometers rest, and which is, when practicable, bolted to the beams beneath, not the deck, can receive no blow or shock other than those communicated through the ship herself, which is done by surrounding it with a bulkhead, with a clear space between. Vibrations are lessened as much as possible by the interposition of sheets of india-rubber in building up the block, and by padding the partitions in which each chronometer rests with soft cushions.

The lid of each box is removed, and a general lid covers the whole.

Uni-
formity of
Tempera-
ture.

A sheet of fearnought is laid over the chronometers, and has flaps cut over each one, so that they can be uncovered in turn, for purposes of comparison or winding. This is to assist to keep the temperature uniform, and also deaden the ticking of other watches when comparing.

Winding.

Winding is performed at the same hour daily, and comparing also. There is no necessity that these hours should be identical; but it is generally the practice that they should be. If they are both done at an early hour, there is more chance of the same officer being on board to do it, which is of importance.

Always wind until the mechanism is felt to butt, to ensure the watch being fully wound up.

* "Notes on Management of Chronometers and Measurement of Meridian Distances," by Captain C. Shadwell, C.B. Potter, London, 1861.

To prevent butting too sharply, the turns can be counted, which will warn the winder ; but if winding is done delicately, this is scarcely necessary. When, however, any other but the officer in charge of the chronometers winds them, he should do this ; to enable him to know the number of turns each watch requires, a piece of paper with the information can be pasted on each box.

The watch has to be reversed to wind, and it must be eased gently back when the operation is complete, not allowed to swing back. This daily reversing of the watch is said to be a good thing, as it distributes the oil in the bearings.

Accurate comparing only comes, like most other things of the kind, by practice. The comparing of watches is gone into at p. 300.

A comparison book and chronometer journal are kept ; the former being used to enter the comparisons at the time with their checks, etc., the latter as a fair book for a permanent record, and contains rates, and all data for noting the performance of each watch. Records.

A maximum and a minimum thermometer are placed in the chronometer-room, and the reading of their indices is taken and recorded in the comparison book, at the time of comparing.

This is of importance when it is intended to take account of change of rate from changes of temperature, and, in any case, will enable us to estimate how far our endeavours to maintain a uniform temperature in the room are succeeding.

MARKS.

It is of course necessary in making a survey of any description to have fixed objects, which are first plotted on to the sheet, and are technically known as "points." These vary, according to the description and scale of the survey, from mountain peaks, whose actual summits may be of considerable area, to thin staves.

It is a great saving of time to the nautical surveyor to find plenty of natural marks, as peaks, conspicuous trees, houses, church spires, etc., anything, in fact, which can be defined and recognised from the different directions it may be necessary to see them ; but it is rare to find a sufficient number of these, Natural
Marks

properly placed, to be able to altogether do away with putting up his own marks for the details of the survey, and it is of these we now speak.

White-
wash.

Whitewash is the great friend of the surveyor. It has the advantage of being portable, showing generally very well, being cheap, and obtainable all over the world ; it cannot disappear by being blown over, or by being stolen or knocked down by jealous inhabitants, who very naturally do not understand what the meaning of different objects dotting their shores may be. Whitewash, therefore, is used wherever practicable, as on rocky cliffs and points, or tree-stems, angles of houses, etc. ; and is also used to whiten other objects put up by the surveyor, as cairns, canvas, etc., where either there is no solid substance to whitewash, or it is necessary to see the mark from every direction, which it is evident cannot be the case when a cliff face is whitewashed, for example.

The nature of these marks must vary according to locality, and the distance it is necessary to see them.

Cairns.

Where there are stones, nothing is better than a cairn. It is rather a question as to whether a cairn on a hill-top is better whitewashed or not. If the sun strikes on it, or there are higher dark hills behind, it shines like a star ; but on a dull day, against the sky, a white cairn will be so much the colour of it, that a dark object will show better.

A cairn on the beach should certainly be whitened.

Tripods.

Where there are no stones, tripods of rough poles or stakes, about 8 feet long, round which a bit of old canvas about 6 feet long, whitewashed, can be laced with spun-yarn, will be found good. The poles are easy to carry in the boat, and can be taken up hills without difficulty ; they are easily taken down, and can be used over and over again. From their tripod form, they stand well in high wind, though it is as well to give them spun-yarn stays. The conical shape of the mark affords a capital object, and rough poles of the kind required can be got anywhere.

Where bamboos can be got they are very useful, from their lightness, to carry up hills, either to form tripods as just described, or as flag-poles.

Pieces of wide coarse white calico are useful for temporary marks, as where it is desired to see a station from another

station, when there is not sun to use the heliostat, and it is not requisite to have a very large mark left for future use.

A whaler forms a good mark boat, with her mast stepped and a bamboo lashed to it fitted with tail-block and halliards to hoist a black flag 16 feet square, which can be lowered if the weather renders it necessary. Mark
Boats.

A 40-pound anchor and 100-pound sinker lashed round the stock, with $\frac{1}{4}$ inch chain moorings, will hold satisfactorily. A roller fitted to project over the stem is necessary for weighing.

On a very flat low shore, where boat sounding has to be carried out a long distance, flagstaffs with large flags must be set up. Care should be taken in some parts of the world that these flags are not national ones, or anything that can be mistaken for such, as difficulties have frequently occurred through such being hoisted. As the large old flags obtained from dockyards are always of this type, they should be cut up, and resewn with such an arrangement of colours as shall denote nothing. Black flags will be generally seen farthest, but the colours of red and white intermingled also show well; the nature of the background has to be considered. Flags.

A flagstaff 80 feet high may be erected without much difficulty by using a stout barling spar as a lower mast, with its heel well sunk in the ground, and secured by long guys. Another barling is then swayed up and lashed as a topmast, to which is added a bamboo with tail-block and halliards attached.

A bamboo may also be lashed to the topmost branches of a tree, and a flag hoisted on it, showing well above the foliage.

On coasts lined with bush, like mangrove swamps, for instance, square pieces of canvas, whitewashed and laced to the boughs, will be found to show very well. Canvas.

In lacing them on, care must be taken to place them so that they will show as far round on both sides as possible, and always to have the lower part more to the front than the upper, as they will thereby catch more sun.

Whenever canvas is used, it is well to cut holes in it, sewing them round sufficiently to prevent tearing in the wind. This will make the canvas valueless for fishermen or natives of any kind, to whom, in all parts of the world, a good piece of stuff is a prize.

In preparing for a surveying cruise, therefore, provision of material for marks must not be forgotten.

Height of Masthead. This should be accurately measured perpendicularly from truck to water-line, and to hammock netting or rail, by tricing up the measuring chain.

Scaffoldings. If circumstances require it, a scaffolding 80 or 90 feet high may be erected on which to mount a theodolite. An outer scaffolding is necessary for the observer to stand on, and an inner one quite separate from the other for the theodolite.

The heads of the poles forming the four corners of each scaffolding are inclined inwards towards each other at an angle of about 10 degrees from the vertical, and they must be suitably braced and tied to give rigidity. It is, however, but very rarely necessary to resort to such expedients in Hydrographical Surveying.

Light-houses. The glare of a powerful light may be observed accurately at a distance from which the light itself is below the horizon. The wires of the theodolite may be seen at night by slightly illuminating the object glass of the telescope through the reflection of the light from a bull's-eye lantern upon a piece of white cardboard held in a suitable position.

BOATS' FITTINGS.

It is impossible to lay down any dogmatic rules for fitting boats for surveying work, as so much depends on individual tastes and requirements of the locality ; but a few points may be noted which have been found generally useful, and a list of articles which are always being wanted can be added as some sort of guide.

Steam Cutters. Steam cutters are, of course, the best boats for general sounding, as the engine never tires. The additional work that can be done with steam cutters at command is enormous, as they not only do their own work, but tow the pulling-boats to and from their stations, and thereby save many hours that would otherwise be spent in beating up or pulling backwards and forwards to the ship morning and evening.

A little table may conveniently be fitted in the stern-sheets of a cutter, as there is plenty of room. The stern-sheet canopy, which will generally have to be in place when sound-

ing, to prevent the spray injuring instruments, books, board, etc., should not be too high, so that the officer standing in the stern-sheets may be able to take his angles over it.

Fittings for a small wire sounding machine are necessary.

Steam cutters for surveying work should be fully rigged, as accidents will happen, especially as the boiler gets old, and it is awkward to find oneself broken down with the ship miles off, and probably out of sight, and nothing but a fore-sail, which is the present service-fitting for these boats. For this reason, unless working near the ship or in close harbours, the masts and sails should always be in the boat. Lumber irons should be fitted to carry these high up, so as not to interfere with the wash-streak of canvas, nor take up necessary room in the boat.

The usual fitting of a steam cutter, a canvas turtle-backed canopy forward, is inconvenient, as the leadsmen cannot then stand in the bows, which is the best place for him. The canvas wash-streak must therefore be carried to the stem, and the stanchions on the bows must be higher than those amidships, to allow for plenty of pitching in a head sea.

It will be found absolutely necessary to use the bow and stern lockers for stowing gear, men's clothes, etc.; but care must be taken that the lids are screwed down, whenever the boat is at work in open water.

Leadsmen's chains should be fitted on each side of the steam cutter, to enable two leadsmen to work when occasion requires it.

Steam cutters must have little skiffs of some kind, to tow as tenders for landing, as the boat is too heavy and draws too much water to be beached, and should always be kept off the ground, for fear of strains with the heavy boiler in her. When only sounding, the tender is of course not needed.

For pulling-boats, whalers will be found most generally useful; they employ fewer men, and have quite enough room in them.

The simpler the sail the better, as it may be often up and down, but a mizen is very useful.

Fixed wash-streaks forward and aft will keep out much water.

Crutches with a long shank, which will raise the crutch

2 inches, may be found useful in some types of whalers, as, with gear along the middle of the boat, and the low gunwale of an ordinary whaler, the loom of the oar cannot be depressed enough with a service crutch, when in broken water, for the blade to clear the tips of the waves.

Lockers, built in bow and stern, are useful for keeping gear and instruments in. These should be canvased over, for unless built of two thicknesses of wood, which is heavy, the tops will soon leak after a few months' hot sun. The top of the bow locker raises the leadsman, so that he can throw his lead well ahead, and he should have his foremost awning stanchion shipped, as a support in rough water.

The awning should be cut at the after-thwart, so as to enable the afterpart to be tipped, when it is necessary to stand up to take angles.

A waterproof sheet spread over a line between the two awning stanchions, and covering the stern-sheets, with the four corners secured outside the gunwale, forms a convenient tent for the protection of the sounding-board in showery weather.

A plank secured across the stern-sheets serves to mount a small wire sounding machine when required.

Cutters

Cutters should also be fitted with bow and stern lockers, and a table can be arranged in stern-sheets if thought necessary. No other special fitting is required beyond those given below for all boats.

FITTINGS AND GEAR FOR ALL BOATS USEFUL FOR GENERAL SURVEYING WORK.

Keel
Bands.

All boats should have stout iron keel bands. With the constant grounding and running over rocks, inevitable in surveying work, these save the boats enormously. With coppered steam cutters these must be of brass, fastened outside the copper sheets.

A galvanised-iron reel, under one of the foremost thwarts, to hold a 100-fathom line.

Sounding
Davit.

A small galvanised-iron davit, with snatch block to place sounding line in when in deep water, so that several men

can assist in hauling up the lead. In steam cutters it will be found handy for the leadsman always to use this.

A *Massey's* patent log, with the stray line between fan Patent and clock lengthened, so that the latter may be fastened outside Log. the gunwale for convenient reference, while the fan tows in the water behind.

A small galvanised-iron nun buoy, with a light chain and Buoy. weight to moor it by, is useful when sounding out a shoal patch.

The boat-hook should be marked in feet, with the marks Boat- slightly cut and painted. This is useful for sounding in hook. shallow water, and many other purposes.

A box containing some spare tins of preserved meat, in Spare Pro- case of accident detaining the boat beyond her time of return visions. to the ship.

The following list of stores may be handy :

General
Stores.

Lead lines, 100 fathoms, 1.

Lead lines, 25 fathoms, 2.

Leads, 11 pounds, 2.

Leads, 7 pounds, 1.

Anchor and cable. Latter should have a short ganger of chain in case of sharp rocky bottom, and be *always* made fast to crown, and stopped to the ring, in case of fouling.

Masts and sails.

Spare oar.

Awnings and stanchions.

Water barricoe.

Small portable ditto for carrying on shore, 2.

Axes, handy billy, 2.

Bag of lime.

Whitewash brush.

Box for arming for lead

Tin pannikins.

Bag for biscuit.

Old canvas for mark.

Canvas cases for rifles. (It is convenient always to have a rifle in the boat.)

Ensign, answering pennant, and signal book.

Tramping barricoes, 2.

**Ammuni-
tion.**

Ammunition case, containing—

| | |
|----------------------------|-----------------------|
| 3 rockets with rope tails. | 20 blank cartridges. |
| 2 long lights. | 50 ball cartridges. |
| 1 handle and primers. | 20 pistol cartridges. |
| 1 portfire. | 2-feet of slow match. |

If a boat meets with an accident, this ammunition will come in handy to attract attention after dark.

**Carpenter's
Stores.**

Carpenter's bag, containing—

| | |
|-----------------|-------------------|
| Hammer | Fearnought. |
| Nails of sorts. | Lead. |
| Chisel. | Tallow. |
| Bradawl. | Strips of copper. |
| Gimlet. | |

**Boat-
swain's
Stores.**

Boatswain's bag, containing—

| | |
|-----------------|-----------------|
| Marlinspike. | Palm. |
| 2 sail-needles. | Bits of canvas. |
| Twine. | Spun-yarn. |

LEAD-LINES.

The first thing in a newly commissioned ship is to get the lead-lines well stretched.

Until this is done it is only loss of time to mark the multitude of lines wanted for surveying.

As soon as the ship leaves port, tow seven or eight hundred fathoms of the new line astern, with a heavy lead on it, for some days. The line will then have got to its normal length, but lead-lines will always want remarking from time to time.

**Marking
of Lead-
Lines.**

The following system has been recently adopted in marking lead-lines, both for ship and boat use, no other system being now used by H.M. surveying vessels:

| <i>Marks.</i> | | | | | | <i>Fathoms.</i> |
|----------------------------------|----|----|----|----|----|-------------------|
| Leather, one tail | .. | .. | .. | .. | .. | 1, 11, 21, 31, 41 |
| „ two tails | .. | .. | .. | .. | .. | 2, 12, 22, 32, 42 |
| Blue | .. | .. | .. | .. | .. | 3, 13, 23, 33, 43 |
| Leather, four tails | .. | .. | .. | .. | .. | 4, 14, 24, 34, 44 |
| White | .. | .. | .. | .. | .. | 5, 15, 25, 35, 45 |
| Green | .. | .. | .. | .. | .. | 6, 16, 26, 36, 46 |
| Red | .. | .. | .. | .. | .. | 7, 17, 27, 37, 47 |
| Blue and White | .. | .. | .. | .. | .. | 8, 18, 28, 38, 48 |
| Red and White | .. | .. | .. | .. | .. | 9, 19, 29, 39, 49 |
| Leather, with hole | .. | .. | .. | .. | .. | 10 |
| „ „ „ and two tails | .. | .. | .. | .. | .. | 20 |
| „ „ „ „ blue | .. | .. | .. | .. | .. | 30 |
| „ „ „ „ four tails | .. | .. | .. | .. | .. | 40 |
| „ „ „ „ white | .. | .. | .. | .. | .. | 50 |
| Green | .. | .. | .. | .. | .. | 60 |
| Red | .. | .. | .. | .. | .. | 70 |
| Blue and White | .. | .. | .. | .. | .. | 80 |
| Red and White | .. | .. | .. | .. | .. | 90 |
| Leather, with hole, and one tail | .. | .. | .. | .. | .. | 100 |

In addition to the above marking, every half fathom up to 20 fathoms is to be marked “*Yellow*,” and every intermediate 5 fathoms above 50 fathoms is to be marked with “*One knot*.”

Lead-lines are to have, in addition, “one knot” inserted at 1, 2, 4, and 5 feet of each fathom, for a sufficient length of line so as to ensure that at least 40 feet (reduced) may be measured at high water springs—*e.g.*, given spring rise of 27 feet, $27 + 40 = 67$ feet, therefore line is to be marked in feet to 12 fathoms.

BEACONS.

Floating beacons are frequently of great service; they can be moored in almost any depth.

These are now generally made on board. A useful and convenient form is depicted in Fig. 9, which pretty well explains itself.

The heads of the two 27-gallon casks should be filled up

flush, and the planks above and below are screwed to the heads, the pole passing through the centre of each plank by a hole cut for the purpose. The planks can be hollowed out to fit the heads of the casks for further security.

Three casks can also be used if only small ones are available, by fitting the planks in triangle, with another plank across, through which the pole passes.

The strop for weighing should be of wire, which keeps well open from its own stiffness, and facilitates hooking on for hoisting in.

A slip by which the cable is attached to the mooring span assists in weighing the beacon.

Use a small kedge and light chain for anchoring, except in water, say, over 60 fathoms deep, when hemp or wire rope should be employed, with some chain next to the anchor to take chafe. Hemp is apt to chafe through, even though the precaution of "keekling" it may have been adopted. Its use, therefore, should be avoided unless the depth renders it necessary.

Beacons have been anchored in 3,000 fathoms by means of sounding wire, and weight of 100 pounds.

In water of from 20 to 100 fathoms, about $1\frac{1}{2}$ times the depth is necessary for the length of mooring rope. In deeper water, less.

This beacon will float nearly upright, and will carry in moderate weather a flag 12 feet square, of calico, which is lighter than bunting, and will be visible from the ship 10 miles, with a 30-foot bamboo. Black, with other colours to distinguish one beacon from another, is recommended.

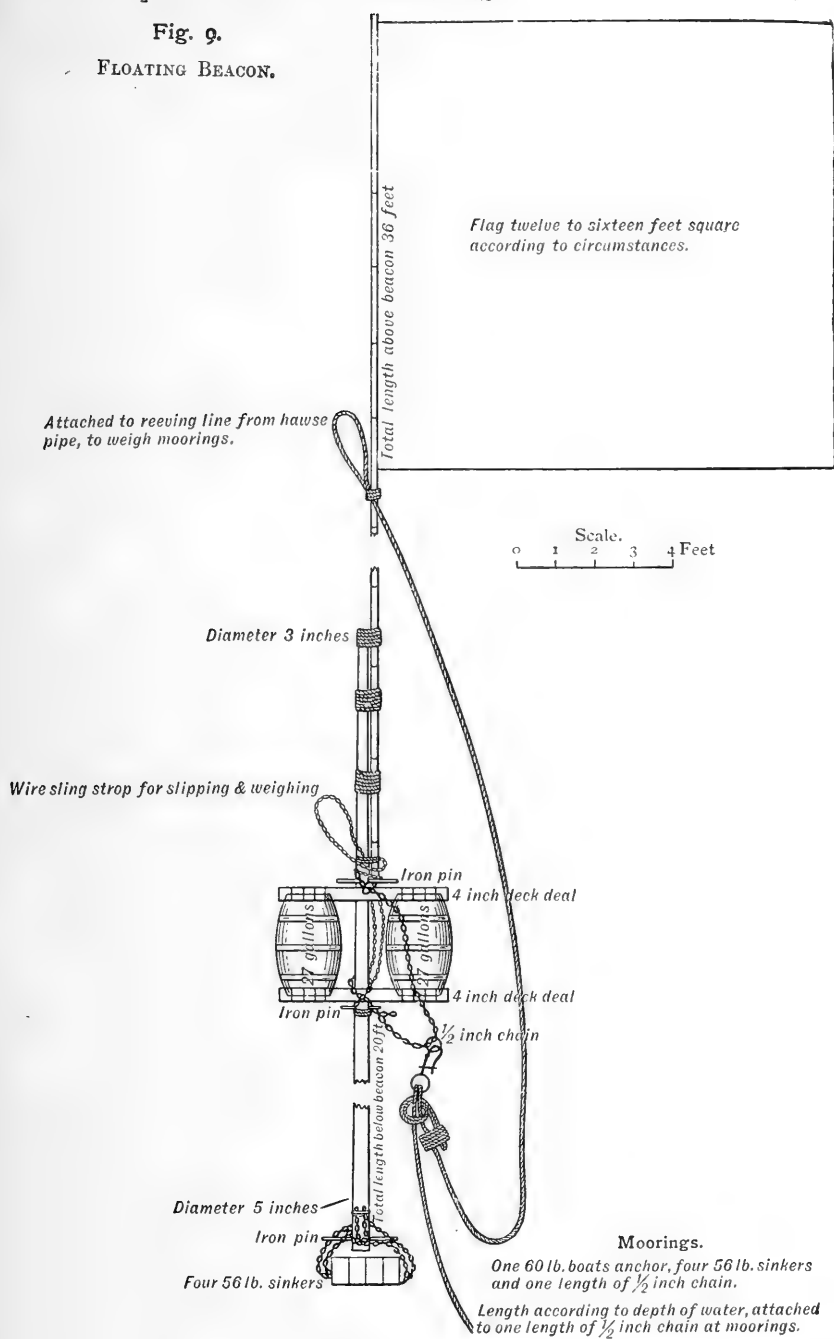
A piece of signal line should be fitted along the luff enclosed in several folds of the calico, and the flag is stopped to the bamboo round this.

Another form of cask beacon is made by woulding three casks to the central spar with rope, which tautens when wet, but the beacon above described is more quickly fitted when the parts are ready beforehand.

Slipping a beacon is best accomplished from the mainyard, but the foreyard can be used. The anchor being over on the weather side, and beacon lowered into the water, slip by means of a large well-greased toggle, as the moorings tauten.

Fig. 9.

FLOATING BEACON.



Weighing is accomplished best from the foreyard. Having hooked to the weighing strop, run the beacon up, and having made fast a line from the hawse pipe to the wire strop at the upper end of mooring chain, knock off the slip, when the beacon can be landed inboard, and the mooring run up to the hawse pipe.

Care is necessary to approach the beacon from the proper direction relatively to the wind and current, so that the ship will lie in the direction of their resultant, and thus avoid bringing undue strain on the moorings.

Fixed
Beacon.

A fixed beacon can be erected in shallow water of from 2 to 3 fathoms by constructing a tripod of spars of about 45 feet long. The heads of two of them are lashed together, and the heels kept open at a fixed distance by a plank about 27 feet long nailed on about 5 feet above the heels of the span. These are taken out by three boats, and the third tripod leg lashed in position on the boats, the heel in the opposite direction to the two others. The legs are weighted, and a gantline block lashed to the fork. The two first legs are let go first together, and the tripod hauled into position by guys. Weights can be added by slipping them down the legs, and the guys secured to anchors.

A vertical pole with bamboos can now be added, its weighted heel on the ground. It is placed by a jigger from the fork, to which it is afterwards lashed, and guys taken from the lower part to the tripod legs. A block and halliards from the bamboo permits a calico flag 14 feet square to be hoisted.

CHAPTER II

A MARINE SURVEY IN GENERAL

THERE is a great variety in the methods that can be employed in making a marine survey, so much so that the task of describing any general scheme of operations is by no means easy.

Under ordinary circumstances, it is often a good plan to begin the survey of a coast from the shelter of some harbour, of which it will eventually be necessary to make a plan on a larger scale, and to extend the triangulation outwards from thence.

The base, which is measured primarily for the purpose of the plan, being connected directly with the main triangulation of the coast survey, is afterwards utilised for calculating the long side upon which we shall begin plotting the coast survey. In case of the weather becoming unfavourable for the main triangulation outside, a harbour plan to fall back upon prevents the loss of time that might otherwise ensue under those circumstances; this is specially the case when a large staff of assistants has to be considered.

In the survey of a particularly wild, stormy, and exposed coast, such as the southern coast of Terra del Fuego, the general plan adopted by Captain Fitzroy, of H.M.S. *Beagle*, was to measure a base, and to survey, from the shelter of a harbour fixed astronomically, as far afield as practicable, fixing points to the utmost limit in every direction. Then, running for the next harbour, a similar survey of it and the vicinity was executed. The harbour surveys were afterwards connected by sketch surveys, the ship being fixed on the points already plotted, and all theodolite shots to intermediate points utilised as far as practicable. The use of steam now enables this sort of work to be carried out somewhat differently and more expeditiously.

If circumstances permit the use of floating beacons, the accuracy of such work is much increased.

Different
Kinds of
Surveys.

In the first place, Marine Surveys may be divided into three heads :

1st. Preliminary or Sketch Surveys.

2nd. Surveys for the ordinary purposes of navigation.

3rd. Detailed Surveys.

The boundaries between these are by no means strongly marked, although each differs considerably from the other, and a finished sheet as sent home is not unfrequently a combination of all three, comprising pieces of work done after very different fashions, according to needs and circumstances.

Sketch
Surveys.

A preliminary survey does not pretend to accuracy. The time expended on it, and the means used, cannot ensure it, and it only represents what our second name for it indicates, a sketch. A sketch survey will be founded on a base of some kind, but this will generally be rough, and in some instances, as in many running surveys, will depend solely upon the speed of the ship as far as it can be ascertained by patent log ; so that the whole affair from beginning to end is only a rough approximation.

The necessity for sketch surveys may be said to be getting less and less every year. Most parts of the world have their coasts mapped at any rate as far as this ; but there still remain portions of our globe of which the coast-lines are not marked at all, or are extremely hazily delineated, and to these any sketch survey will be an improvement.

Ordinary
Surveys.

The second head comprises the majority of charts now published, and many of those in course of construction in the present day, *i.e.*, they are constructed on such a scale, and with such limitations of time, etc., as to make it impossible either to show small details of land or sea, or to be perfectly certain that small inequalities of the bottom, or detached rocks, may not exist, unmarked. Everything, however, shown in such a survey should be correct, and it is only in its omissions that it should be imperfect.

Detailed
Surveys.

A detailed survey is accurately constructed from the commencement, on a scale large enough to admit of close sounding, and time is given up to working out all minutiae.

Detailed surveys are mostly confined to the more civilised

shores of the world, where there is much trade, and to such ports, harbours, and channels as are largely used in navigation.

The necessity for these surveys increases to an enormous extent every year, with the prodigious strides trade, more especially trade by means of steam vessels, is taking.

A steamer works against time ; her paying capabilities largely depend on her getting quickly from port to port, and captains will take every practicable short cut that offers, and shave round capes and corners in a manner to be deprecated, but which will continue as long as celerity is an object. A channel which a sailing vessel will work through in perfect safety, from the obvious necessity of keeping a certain distance off shore, for fear of failing wind, missing stays, etc., will be the scene of the wreck of many a steamer, from the inveterate love of shortening distances, and going too near to dangerous coasts only imperfectly surveyed. Better charts will not cure navigators of this propensity, but will save many disasters by revealing unknown dangers near the land.

Time, and the comparative scarcity of marine surveys, do not permit of keeping up to the rapid advance required in this style of survey ; and unless the countries of the world interested in ocean traffic largely increase their expenditure on these matters, it seems as if charts will get farther and farther behind requirements as years roll on.

Having settled of what description a chart is to be, there is still much diversity in the method of undertaking the details of it. The extent of the work, whether simply a plan of limited extent, or a large piece of open coast ; the scale on which it is to be done ; the nature of the coast and sea ; the time and means at disposal ; the number of assistants ; will all be considered in determining exactly how to set about the work.

Detail
always
Variable.

All this makes it very difficult to lay down rules for marine surveying. Experience alone can dictate what should be done in each particular instance. Though a plan may be produced, the time employed, and the result of the labour expended, will greatly vary according as to whether the work has been undertaken in the right way or not, apart from any personal qualities of the assistants, and nothing but the possession of the true surveying "knack," combined with experience, will point out this right way.

Triangulation.

All surveys are, however, alike in this respect. They are, as it were, built up on a framework of triangles of some kind, the corners of which are the main "points" of the chart, and to obtain this framework is always the first thing to do, and how to set about it the first thing to consider.

The construction of this "triangulation," as it is termed, is of various kinds; ranging, from the rough triangles obtained in a running survey, where the side is obtained by the distance it is supposed the ship has moved, and the angles are sextant angles, taken on board from a by no means stationary position, to the almost exactly formed triangles of a detailed survey, when carefully levelled theodolites observe the foundation of a regular trigonometrical network, which covers the whole portion to be mapped.

The term "triangulation" would seem to infer that this system of triangles would be always apparent; but in surveys irregularly plotted, and when working on a sheet previously graduated, it will seem that there is no triangulation, and in the strict sense of the word there is none, but the framework of the chart is still built up on the system of triangles, and it is difficult to find any other name for the process.

For the present we will speak only of the second and third kinds of surveys, leaving Sketch Surveys to be described separately.

The system employed in Ordinary Surveys and Detailed Surveys is the same, and they really only differ in the scale of the chart, and the amount of time that is spent on them, especially with regard to closeness of soundings.

In a detailed survey, time must be subservient to the necessity for exactness, and for exploring every foot of the ground.

In an ordinary survey, judgment has to be exercised as to how far we must be satisfied with what we can get for triangulation, and how much time we can spend on details.

It is by no means necessary in an ordinary survey to observe the angles at each corner of the triangles. The happy fact that the sum of the three angles is 180° enables us to manage whenever we have two of them, though it is, of course, more satisfactory to actually observe all three for the more important triangles.

The accuracy necessary in many details of a chart depends very much upon its scale. Over-accuracy is loss of time. Any time spent in obtaining what cannot be plotted on the chart is, as a rule, loss of time ; but it cannot be too strongly impressed upon the young nautical surveyor, that his work should be as correct as his scale will allow. Nothing should be put down of which he is not sure, and it is no loss of time to repeat angles to prevent mistakes. It is better to be over-accurate than to err in the opposite direction, and experience will soon show him when he must be very exact, and when a little latitude is permissible without interfering with the result.

Accuracy
Neces-
sary.

The accuracy of the main triangles of a chart is most important ; everything depends on them, and if they are incorrect, nothing will be satisfactory afterwards.

The general plan of a survey may be said to be this :—

General
Plan of
Survey.

1st. A base is obtained, either temporary, as in the case of an extended survey ; or absolute, as in a plan. This is the known side of the first triangle.

2nd. The main triangulation, that is, the establishment by means of angles of a series of positions, at a considerable distance apart, from which, and to which, angles are afterwards taken, to fix other stations. These are the corners of our framework, and are known as the “ main stations,” the two ends of the base being the first two, on which everything is built.

3rd. The fixing by means of angles from these main stations of a sufficient number of secondary stations, and marks, to enable the detail of the chart to be filled in between them. In most cases angles will be required to be taken from the marks themselves as well.

4th. All these points, or those embracing a sufficient area to work on, being plotted on the chart, they are transferred to the field boards, either by pricking through the plotting sheet with a fine needle, or, what is a better way when carefully done, by making a tracing of them on tracing-cloth, and pricking through that on to the boards.

5th. Each assistant then has a certain portion told off to him to do. It must depend upon circumstances, but as a rule it is more satisfactory to have the coast-line put in first,

and the soundings taken when this has been done. The topography, or detail of the land, can be done at any time.

6th. Each piece of work is inked by the assistant on his board, with all detail, and when complete, is carefully traced on the above tracing of the "points." All bits are thus collected together, and the total is retransferred to the plotting sheet by means of transfer paper, and inked in as the finished chart.

These details must not be taken as unalterable. Some prefer plotting everything on to the same original sheet, and when a surveyor is by himself, or with one assistant, he would probably do this, but the method described is calculated for a number of assistants, and has been found to work well.

It is not absolutely necessary to get a base before starting a plan. Circumstances may make it imperative to wait a day or more for this, and in the meantime, a distance between two stations, to be finally measured, can be assumed and plotted, and the whole system of triangulation built upon this. But it must be remembered that no heights can be measured by means of angles, until a scale is obtained.

If an extended survey, and plenty of hands, some will carry on the triangulation and marking in advance, while others are putting in the detail, and sounding the part already marked.

The deeper soundings will be taken from the ship, to a sufficient distance or depth off shore.

When to
obtain the
Astronomical
Positions.

It will depend upon circumstances when the astronomical observations for latitude and longitude are taken. If only an isolated plan is being done, the observations to fix some definite point on it can be taken at any time.

When an extended survey is in progress, that has been commenced on a measured base, they can also be taken when convenient. In this case the final scale of the chart will mainly depend upon the observations taken at either extremity of the chart, and they must consequently be done very carefully. Circumstances of weather, time of year, etc., will therefore influence the choice of the best time for these.

Sometimes an extended survey will be originally plotted on a base obtained by the astronomical positions, and in this

case, of course, they will be the first thing to undertake. At any rate, it will nearly always be convenient to obtain a true bearing at once, in order to have the meridian of the chart placed squarely on the paper from the commencement.

These separate steps in a survey will now be described in detail, following the order in which they will generally come, as far as can be done.

CHAPTER III

BASES

By Chain—By difference of Latitude—By Mast-head Angle—By Angle subtended by known length—By Range-finder—By Measured Rope—By Sound.

Different
Kinds of
Bases.

BASES for marine charts or plans upon which to build the triangulation are obtained in several ways, according as circumstances permit and accuracy requires.

1. By means of the 100-feet chain or steel tape supplied for the purpose.
2. By difference of latitude, or difference of longitude.
3. By measuring with a micrometer or sextant the angle subtended by a known length, as two poles a measured distance apart, the ends of a long pole, or the masts of a ship.
4. By range-finder.
5. By a measured rope, as a lead-line ; or by the wire from a sounding machine.
6. By sound.

CHAINED BASES.

The ground for a base, to be measured either by chain or rope, must be as level as can be found. Its length will be partly determined by the extent of the work to depend on it, varying from say 9,000 feet to 1,000 feet, or even less for a small harbour.

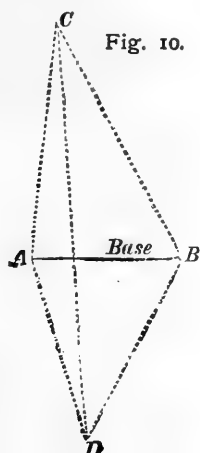
While it is certainly convenient to measure the base in one straight length, if convenient ground can be found, it is by no means necessary.

If several short lines making angles with one another are measured with the angles carefully observed, the terminal points being visible from one another, the resulting distance

calculated between the terminal points should be just as correct as if it had been measured direct.

It is seldom that a chained base can be found, even for a small plan, long enough to plot from directly, *i.e.*, the measured length when protracted on the paper would be generally so short, that by placing that on the sheet first, and making it the starting line, errors would be sure to creep in, in increasing the size of the triangles, any little error being multiplied. It therefore is usually necessary to extend the base, as it is termed.

This consists simply in calculating a sufficient number of triangles, conveniently arranged, to obtain a side long enough



to form a good start, so as to plot *inwards* as much as possible, when any little errors will be diminished, instead of increased.

As a commencement of this process, the base to be measured should, if possible, be placed so that there are two stations, one on each side of it, which can be used for the first triangles and consequent extension of the base.

Here, Fig. 10, A B is the measured bas, C and D the two first stations. Angles are observed at A, B, C, D. The other two sides in the triangles A B C, A B D being found, C D can be found in both the triangles A C D, B C D, which will check the result, and C D will be the extension of the base for further triangulation.

Of course this desired convenience will not always be found, but it is a thing to look out for.

It is by no means necessary to measure a long base, provided that convenient triangles can be found for extending the base by calculation. If the angles of these are of the necessary number of degrees, and they are carefully observed with theodolites, a short base, measured on flat smooth ground, will give a truer result than a longer one measured over inequalities. With a sextant survey it will be well to have as long a base as possible.

Planting
Staves for
measur-
ing by.

The ground having been walked over to ascertain its fitness, and the base stations (the ends of the base) being so placed that they see as much as possible on all sides: set up the theodolite at one end, and at the other a flagstaff or another theodolite, and let a man plant staves (boarding pikes make good ones) exactly in line between the two stations, giving him the position for the first two or three, by looking through the theodolite directed to the other station. After these are in place, he can plant the others in line by guiding himself by them.

Method of
measur-
ing.

Having the staves placed and in line, begin to measure from one end. If two persons are to measure, begin from opposite ends. A man is required for each end of the chain. The man at the foremost end of the chain carries ten pins, and the surveyor attends with his book to see the chain fairly placed in line between the staves, and to note down each length of chain measured. Do not let the men stretch the chain too tight, but it must lie straight on the ground between the two ends.

The chain being down for the first length, measuring from under the centre of the theodolite, put a pin in the ground, at the foremost end, *inside* the handle, and touching the flat side. Make a mark in the note-book, and walk on together, the man at each end lifting the chain as much as he can, until the hindermost comes to the pin. He must then place the *outside* of his handle so as to touch the pin. Another pin is put in at the foremost end inside the handle, the second note made in the book, the first pin taken up by the hindermost man, and on you go.

The lengths are best noted by strokes, crossing every fifth over the four, as in ordinary tallying.

Check at every ten lengths by the number of pins. When the tenth stroke is made, the foremost man should have no pins left in his hand, and the other man should have nine, the tenth having been just put in.

The odd feet and inches in the last length are measured by counting the links, which are each a foot long.

In walking forward, take care that the hinder man does not overwalk the former, or the chain will have a bight dragging on the ground, links will catch in something and get bent, and the error of the chain will be very different when retested, to what it was before landing.

The number of times a base must be measured depends on circumstances. If for a harbour plan, only twice, if they agree to a foot or two, will be sufficient. For a survey of greater extent, three or four times will be more satisfactory, unless the two first measurements agree very well. Repetition
Necessary.

Perfectly level ground can seldom be found, and the surveyor must make an allowance for inequalities by his judgment, which will be, of course, always subtracted from the measured length. Inequalities.

The chain must be tested for length, before and after measuring the base, to ascertain the error.

BASE BY DIFFERENCE OF LATITUDE.

When two stations are available from twenty to thirty or forty miles apart, visible from one another, and bearing not more than two points from the meridian, having also a few intermediate points visible from both, a very good base can be got by latitudes, and careful true bearings.

The base will then be $\text{diff. lat.} \times \sec.$ Mercatorial bearing.

Similarly, if the stations bear nearly east and west from each other, the diff. long. may be obtained by chronometer or rockets. The true diff. long. by observation is converted into spherical diff. long. from which the departure is found. The length of base will be $\text{Dep.} \times \text{Cosec.}$ Mercatorial bearing.

By means of the intermediate points, triangles can be calculated down to a workable length of side for fixing marks.

Where no smooth ground for measuring a base can be found,

and we want our scale to be near the truth from the first, this method is valuable.

The only drawback to it is the effect of local attraction on the pendulum, or, in other words, on the mercury in the artificial horizon. With high land behind a station and a deep sea in front it may result that there will be considerable error; and the difference in the distance between the terminal points of the survey, if it covers much ground, as deduced from such a base, and as determined from the observations at either end, may be much more than if starting from a measured base.

In order to minimise these effects on the observed diff. lat. or diff. long., the conditions at both base stations should be similar as far as possible.

The base stations should be as far apart as possible, in order to minimise the effect of any errors in the astronomical observations.

The observation spots would not necessarily be actually at the base stations, which latter would probably be situated on elevated summits at some little distance, in order to command distant views.

In such cases, each observation spot would be connected with its corresponding base station by a subsidiary triangulation, a short base being measured for the purpose.

The ship, at anchor off the observation spot, frequently affords a convenient means of effecting the connection by a mast-head angle base and simultaneous angles.

If possible, in the case of a base by diff. lat., the observation spots should be east or west of the elevated base stations; and in the case of a base by diff. long. they should be north or south of the base stations. This reduces the amount of the correction to be applied.

BASE BY MAST-HEAD ANGLE.

This consists in measuring, with a micrometer or sextant, the angle between the mast-head of the ship and the hammock netting, or some other fixed line on the ship's side; not the water-line, as that varies.

The vertical circle of a theodolite being only marked to minutes, unless it be a much larger one than is generally available, is not sufficiently accurate for this.

It is well to use two sextants to check errors, and read them both on and off the arc.

The height of mast-head above the line must be accurately known to give a good result.

Working out a right-angled triangle gives the distance required. A table should be formed of the distances corresponding to different angles of the mast-head of the ship, as this will be frequently used in sounding banks.

Two stations on shore, visible from each other with the ship nearly midway between them, give the best conditions for obtaining a long base by this method. The ship's truck is shot up simultaneously by theodolite from each station, and at the same time the angles of elevation are observed by sextant. It is conveniently done by mast-heading a flag at regular prearranged intervals, dipping ten seconds after, at which signal the observations are made simultaneously. The distance between the stations is determined by the mean of a number of separate observations. A 30-foot bamboo lashed to the mast-head is useful to give additional height. It is not necessary for the ship to be moored or even at anchor.

Base by
Mast-head
Angle
from Two
Stations.

BASE BY ANGLE OF SHORT MEASURED LENGTH.

Where the ship is not available, a base for a small plan can be obtained by measuring the angle between two well-defined marks placed in the ground at a carefully measured distance apart, or that subtended by the ends of a long pole.

This must also be done with the sextant or micrometer.

If staves in the ground are used, care must be taken that they are at right angles to the required base. Similarly, if a pole is used, care must be taken to hold it at right angles to the observer, which can be ensured, either by having a pointer nailed on to the centre of the pole projecting at right angles, and which must be directed towards the observer by the man holding the pole in both hands horizontal, or by simply waving the pole, held in this position, backwards and forwards gently,

when the observer will register the largest angle he observes as the correct one.

The angle observed should not be smaller than 1° , which with a distance of 20 feet will give a base of over 1,100 feet. It would be better, however, if practicable, to get a base by means of a longer distance, and a larger angle than this, when a very trustworthy result will be obtained; or to be content with a shorter base, and extend it by angles, as already described, to a longer working base.

Measurements must be made on and off the arc, and it would be well to use more than one sextant.

Small lengths of this kind may also be measured by a micrometer, but a sextant will give just as good results, and is in a ship always handy.

No appreciable error will be introduced by taking distance = length of pole \times cot. angle.

BASE BY RANGE-FINDER.

The writer has had no practical experience with this instrument for surveying purposes, but it is likely to prove useful in a variety of ways, and might be used to obtain a base when it is not convenient to measure it by more accurate methods.

The 9-foot Barr-Stroud range-finder should give a more reliable result than that obtained by sound. At a distance of about 2 miles, under favourable conditions of light and weather, and using a well-defined object, such as a lighthouse or flag-staff, the probable error of the mean of a number of observations by a skilful observer may be expected not to exceed 1 per cent. of the distance measured. By placing the ship midway between the two base stations, a longer base may be measured without increasing the percentage of error.

The 9-foot range-finder may be used effectively up to a distance of about 4 miles, but the probable error rapidly increases with the distance.

The Navigational range-finder, $31\frac{1}{2}$ inches in length, is portable, and can be used without a stand. It is unsuitable for the measurement of a base of greater length than about 1,000

yards consistently with a percentage of error equal to that of the 9 feet range-finder at 2 miles.

MEASURED BASE BY ROPE.

Measuring by a rope is of course not accurate. It is difficult to avoid stretching it more at one time than at another, and if it gets wet, it alters its length considerably. If measuring over ground where it is sure to get wet, it will be better to wet it well beforehand. Test it in that condition, and keep it well wet all the time of measuring.

BASE BY SOUND.

This consists in counting the interval of time which elapses between the sight of a flash of the gun and the arrival of the sound of the explosion, the gun being at one end of the required base, and the observer at the other.

Recourse is had to this method of obtaining a base when no flat ground can be found on which to measure. Its accuracy is not great by any means, but, if the final scale of the chart is to depend on astronomical positions, it is quite near enough for working out details such as heights, small parts measured with 10-foot pole, etc.

Final
Scale
should not
depend on
Base by
Sound.

Its value is much increased by observing from both ends, which should always be done if possible, and a surveying vessel should have two small brass Cohorn mortars supplied for this purpose, which can be sent away in a boat, and tumbled overboard without damage.

Useful
Hints.

The ship is often used at one end of a base by sound, especially in work amongst small islands, and it is also necessary sometimes to have a boat at the other, but if at any rate one shore station can be obtained it will be better. If choice of direction can be had, measure with the wind across the base, as, though the error from increase and decrease of velocity is eliminated by measurement from both ends, the sound may be difficult to hear against the breeze, if at all strong.

For either end choose positions for the hearers as much out

of the wind as possible, as it is the whistling of it in the ears which disturbs the receiver more than anything.

A base of 3 miles is a very good length, but the surveyor will generally not have much choice in this matter. Needless to say, on a calm day the sound will be heard farthest and easiest, but the choice of days is seldom possible in practice. If we waited for the best opportunity for every detail of survey, it would never get on, and the utmost that can be done is, when there is alternative work for which the day or opportunity is more suited, to take that in hand.

Signal to
be made.

The guns from the two ends should be fired alternately, at regular intervals, and at some preconcerted signal, as dipping from the ship a flag visible from both stations, which should be hoisted a minute or half a minute before as warning, or re-hoisting a dipped flag steadily, the gun being fired as the flag reaches the mast-head. It is distracting to the receiver to be waiting an indefinite period for the flash.

Watch to
be used.

A chronometer watch is the best, beating five ticks to the two seconds. An ordinary watch, which beats nine ticks in the same period, goes at such a pace as to be rather confusing, especially when not in practice, though, if the observer is used to the process, he will measure as accurately with an ordinary watch, and possibly more so.

Prepara-
tion for
counting.

When awaiting the flash, hold the watch to the ear and count to yourself—nought, nought, nought, etc., continually, keeping time with the ticks; you will then be ready to commence—one, two, three, etc., as soon as you see the flash or smoke of the gun.

If going to use a telescope to watch for the warning signal, tie the watch over the ear with a handkerchief, which will leave both hands free.

Count only up to ten or twenty, and mark off each ten or twenty by putting down a finger of the unoccupied hand, or by some such means.

Repeti-
tions.

If time allows, three or four measurements should be made each way, or more if they do not agree with one another. A signal must be arranged to ask for more than the number previously settled, if it be wanted.

Calculat-
ing the
Mean.

In meaning the result, the arithmetical mean is not strictly correct, as the acceleration caused by travelling with the wind

is not so great as the retardation caused in the opposite direction, as in the latter case the disturbing cause has clearly acted for a longer period. The formula used is—

$$T = \frac{2 t t^1 *}{t + t^1}$$

when T is the mean interval required,
t the interval observed one way,
*t*¹ the interval the other way.

The mean interval thus found, multiplied into the velocity of sound for the temperature at the time, will give the required distance.

The velocity of sound varies considerably, and an accurate law for all its causes of variation has not yet been discovered. The main cause is, however, temperature, and for this it can, to a certain extent, be corrected. Velocity
of Sound.

The most trustworthy experiments made show that sound travels about 1,090 feet in a second of time, at the temperature of 32° Fahrenheit, and increases at the rate of 1.15 foot for each degree of temperature above the freezing-point, decreasing in the same proportions for temperatures lower than 32°.

This is the only correction that can be made, and a base measured in the manner described, with these data, will give an approximation sufficiently near for all practical purposes.

As an example, let us suppose A and B the two ends of the base to be measured. Example
of Base by
Sound.

At A have been observed

44 beats with watch beating 5 beats to 2 seconds.

45

44

Mean 44.33 beats = 17.732 seconds.

81 beats with watch beating 9 beats to 2 seconds

82

83

Mean 82 beats = 18.204 seconds.

Mean at A = 17.968 seconds.

* See Appendix F.

At B have been observed

85 beats with watch beating 9 beats to 2 seconds

87

88

Mean 86.66 beats = 19.238 seconds.

47 beats with watch beating 5 beats to 2 seconds

47

48

Mean 47.33 beats = 18.932 seconds.

Mean at B = 19.085 seconds.

$$\text{Then working } T = \frac{2 t t^1}{t + t^1}$$

we get $T = 18.500$ seconds.

Temperature is 80° , at which velocity of sound is 1145.2 feet per second.

This, multiplied into the interval, gives 21,197 feet for the length of our base.

The temperature must be taken in the open with the thermometer shaded from the direct rays of the sun, but not in too cool a spot, or it will not give the true temperature of the free air.

Ship
between
Two Base
Stations.

Guns fired from the ship, either under weigh or at anchor, placed about midway and exactly in a line between the two base stations, give a long base ; provided the wind is blowing right across the line of the base, satisfactory results may be obtained. The base for the survey of the western portion of the Straits of Magellan was obtained in this manner, no other method being practicable.

Ship and
Single
Theodo-
lite
Station.

Another very useful method of using the ship to obtain a base by sound, in connection with a single theodolite station on shore, may here be mentioned. The ship, not necessarily at anchor, is placed at about three miles from the theodolite station, and in the most favourable position that circumstances will allow for giving a good cut to a conspicuous object on shore, which is also visible from the theodolite \triangle , and at a considerable distance from it ; the receiving angle at that object, between the line from the ship and that from the theodolite \triangle being not less than 30° . At the instant of firing each gun the ship

is "shot up" by the observer at the theodolite \triangle , and the observer in the fore-top at the same instant measures the angle between the base \triangle and the conspicuous object. This object thus practically becomes the other end of the base, the ship being eliminated altogether; a separate value is deduced for the shore base, corresponding to each gun and each different position of the ship at the moment of firing. If the wind is blowing across the line joining the theodolite \triangle and the ship, it will not be necessary to land a mortar for return signals; but it is preferable to do so. The formula

$$T = \frac{2 t t^1}{t + t^1}$$

is of course applicable to this case.

The following details for carrying out the above may be Details useful :

1. A single gun from the ship to indicate "Prepare." Flag hoisted simultaneously at the fore, at the dip.

2. Flag will be mast-headed one minute ten seconds before firing each gun.

3. Flag will be dipped ten seconds before firing.

4. Observe the flag by the theodolite as it is dipped on each occasion; note the interval between flash and report, and read off theodolite directly afterwards.

5. Flag will be mast-headed as each gun is fired.

6. At the seventh gun flag will be hauled down.

7. If the shore observer wishes ship to repeat, he will fire the mortar one minute after ship's flag is hauled down.

8. Two minutes after hauling down flag it will be rehoisted at the dip.

9. Flag will be mast-headed, and kept up one minute.

10. Flag dipped. The mortar is fired from the shore ten seconds afterwards.

11. Flag is rehoisted on mortar being fired.

12. At the seventh discharge of the mortar the flag will be hauled down.

13. If the ship wishes shore to repeat, she will fire a gun and hoist flag at the dip, and the operation will then be repeated as before.

CHAPTER IV

THE MAIN TRIANGULATION

General—Making a Main Station—False Station—Sketch—Convergency
—Calculation—Co-ordinating Astronomical Observations with Triangulation.

Definition. THE main triangulation has been already defined as “the establishment by means of angles of a series of positions, from which and to which, angles are afterwards taken, to fix the secondary points of the survey.”

Main Stations. All positions from which angles are taken, with the intention of fixing other objects, are called “stations,” the symbol for which is \triangle , but the ones with which we are immediately concerned, that is, the first and important positions, are distinguished as the “main stations,” and these collectively form the “main triangulation.”

The first object of main stations is to see other main stations, and with this in view their positions are chosen accordingly ; but angles to everything useful, secondary stations, marks, etc., are, of course, taken as well.

Secondary Stations. Secondary stations are those from which angles are taken solely to fix the smaller marks and details, etc., of the survey. They will be nearer together than the main stations, and may often be perforce so placed as to be useless for any other object.

Points. All objects fixed and plotted on the skeleton chart are known as “points.” A “point” may be a main station, a secondary station, or simply a mark ; but when fixed and plotted on the sheet, with the intention of using them in the survey, they are one and all spoken of by the generic term of “points.”

Main triangulations may be divided into two kinds : “calculated,” in which the triangles are all worked out, so that the

length of any side, or the distance between any two main stations, can be found; and "plotted," in which the main stations are simply the first points laid down on the paper. Varieties of Triangulations.

A calculated triangulation is used in any detailed survey, in plans, or whenever from circumstances it is convenient to have different parts of the same survey on separate sheets, which can therefore afterwards be put together in the engraving, without any fear of their not fitting into one another. Calculated Triangulations.

Bases for plans, on a larger scale than the rest of the chart, can often be taken out of a calculated main triangulation without measuring separate small lengths.

Plotted triangulations may further be subdivided into "regular" and "irregular." Plotted Triangulations.

A plotted regular triangulation will be when triangles have been obtained which could, if requisite, be calculated trigonometrically. As, however, a calculated triangulation is of great service as a record, and for future resurveys, it is expected to be furnished with every chart.

It is more satisfactory that triangulation should be regular if possible, but it very much depends upon the nature of the coast to be surveyed, in what manner it can be carried out.

In many extended surveys, where, for instance, the land is low and densely wooded, or perhaps bordered by reefs to a great distance from the shore, a regular triangulation is hardly possible, or would entail so much loss of time as would not justify its being undertaken. Irregular Plotted Triangulations.

The main points must be plotted in these cases by all sorts of means. The ship enters largely into the scheme, and frequently boats also. Stations may have to be fixed solely by angles observed at them. True bearings are freely used in the construction of the chart, and any regular system of triangles disappears.

A large proportion of existing charts have been, and many more are now being, constructed, by means of irregular plotting.

A survey can often be commenced with a regular triangulation, when it will be found necessary, after having advanced a certain distance, to have recourse to irregular means to fix main stations.

Here it is, when ordinary rules and systems fail, that the

skill of the chief of the survey is shown in overcoming these difficulties in the readiest and best method, and these are the circumstances on which we can give the fewest hints. Such as we do mention will be found in the next chapter on Plotting. In the present one we shall confine ourselves to regular triangulations.

Triangu-
lated
Coast Sur-
vey.

It is necessary to make a preliminary run over the ground to note suitable positions for main and secondary stations on prominent headlands, islands, and summits not too far back from the coast. If no former survey exists, a rough plot should be made by compass and patent log. A scheme must now be formed for the main triangulation, with the object of enclosing the whole survey in as few triangles as possible, regard being paid to the limit of vision of each station due to its height, to the existing meteorological conditions, to the limitation imposed by higher land intervening, and to its accessibility.

The triangles decided upon should be well conditioned, taking care not to introduce an angle of less than 30 degrees. A smaller angle is permissible, however, when the two longer sides of such a triangle are of nearly equal length, and when in the calculation that will follow one of these sides shall be derived from the other longer side and not from the short side.

In open country the selection of stations is a comparatively easy matter. In country densely wooded the time occupied by a triangulation is mainly governed by the judicious selection of stations quickly reached, sufficiently elevated to command distant views, and situated on summits capable of being readily cleared of trees in the desired direction. An all-round view is, of course, desirable, but not always attainable.

The object of secondary stations is to break up the large primary triangles into smaller ones, dividing up the distances between the primary stations into suitable lengths. They are selected with a view to greater accessibility than the latter, and should therefore usually be near the coast and at moderate elevation.

Upon shots from these secondary stations will depend the position of the greater number of the coast-line marks presently to be erected and fixed as the detailed survey of each section of the coast is taken in hand. The nature of the base to be

used, and its position in order to fulfil the conditions specified under the head of "Bases," must be considered, the base, when extended, forming a side of one of the main triangles. If it be possible to obtain the astronomical positions before commencing the survey, a measured base is unnecessary.

It is immaterial at what part of the survey the base is situated, but if it be near one end, a very satisfactory check on the accuracy of the triangulation is obtained by comparing the length of a side at the other extreme of the survey, derived by calculation through the whole system of triangles, with the length as deduced from a check base measured in its vicinity.

It is generally a saving of time to measure the base at some anchorage or harbour that requires a large-scale plan. The triangulation involved in extending the base to connect it with the main triangulation scheme can thus be utilised for both purposes, and while the triangulation is being calculated and plotted, the survey of the plan can be proceeded with.

True bearings are observed at both ends of the survey, and at as many main Δ s near the coast as possible, the results being subsequently compared.

Astronomical observations for latitude are obtained at observation spots near the extremes of the survey. The meridian distance is run between them two or three times, and the observation spots are connected with the primary triangulation. They are usually disposed at intervals of from 100 to 150 miles. Thus, errors due to a triangulation carried out with theodolites of moderate diameter do not accumulate to any serious extent.

If the survey be a very extended one, intermediate observation spots afford a satisfactory check, by comparing the positions as calculated in the triangulation with those obtained by direct observation.

A measured base is not essential. The length of any side in the triangulation being assumed as equal to a certain number of "units," the lengths in "units" of the other sides may be determined therefrom.

Measured
Base not
Neces-
sary.

Calculating the distance in units from the observation spot at one extreme of the survey to that at the other extreme, and comparing the number of "units" thus found with the distance in miles between the two observation spots derived

from their astronomical positions, the value of the unit is obtained, and thence the value in miles of each side in the triangulation.

Comparing true bearings taken at one end of the survey with those taken at the other end, by applying the necessary angles and convergences throughout the triangulation, and taking the mean if they differ, the most probable value of the true bearing of any particular side is obtained, and thence the true bearing of one observation spot from the other.

Comparing this true bearing with that obtained from the observed astronomical position of the observation spots, a check is afforded on the accuracy of the triangulation and astronomical positions, but there is no check as to scale ; this, however, is of less importance than the check in bearing.

Great
Accuracy
in First
Triangles.

The angles of the first few triangles in a triangulation, commenced on a measured base, will require to be extra carefully observed, and the theodolite must be carefully placed exactly on the spot of the mark which will distinguish the station. For, as we shall be increasing our distances in each triangle, until sides long enough to carry on the triangulation without further enlargement are arrived at, any little error in an angle will give a larger error in the resulting side. These first triangles will nearly always require to be calculated, as already remarked under the head of "Bases," in order to get a side long enough to plot from, whatever it may be the intention to do afterwards.

Errors in
Small Tri-
angles.

When the sides are short, any error in placing the theodolite accurately over the centre of the \triangle , or directly under the flag marking the \triangle , produces a larger error in the angles to stations close to than in the angles to those at a greater distance. It is necessary to pay great attention to the accurate placing of the theodolite to ensure the first small triangles closing with sufficient accuracy.

Plum-
mets.

To find the spot vertically underneath a given point, such as a flag on a bamboo, which it is undesirable to move and from which a plummet cannot conveniently be dropped, set up two boat-hook staves slightly out of the perpendicular, with a weight hanging from the end of each, in such positions as to subtend about 90° between them at the spot it is required to find, and at some little distance from it ; the cord from which

ERRATUM

Page 83, line 19 to line 23.—Erase, and substitute the following:

In measuring an angle between an elevated object and one on the horizon, a direct measurement may introduce more error than would result from reflecting to each object in succession a third object also on the horizon, making an angle as nearly as possible 90° with the elevated object, and taking the difference between the two observed angles. It should be noted that whatever may be the elevation of the object, if another object be on the horizon and distant exactly 90° , the true horizontal angular distance is also 90° , and therefore equal to the observed distance.

This follows from the formula given on p. 357.

\cos horizontal angle $= \cos$ angular distance $\times \sec.$ alt.

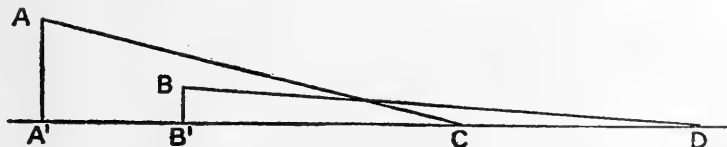
If angular distance $= 90^\circ$,

Then \cos horizontal angle $= \cos 90^\circ \times \sec.$ alt. $= 0$.

Therefore horizontal angle $= 90^\circ =$ obsd. angular distance.

It also follows that if the observed angular distance is greater than 90° , the true horizontal angle is greater than the observed angle, and *vice versa*.

In the event of both objects being elevated, a natural mark on the horizon should be selected as nearly as possible 90° from one of the objects; and a second mark on the horizon, making an angle with the other object, also as nearly as possible 90° . From measurement of the angles between the horizon marks and both elevated objects, the required horizontal angle between the latter may be deduced with approximate accuracy dependent on the degree in which the necessary conditions governing the selection of the horizon marks have been fulfilled.



In the above figure, let A and B be two elevated objects; required A'B' the horizontal angular distance between them.

ERRATUM—*Continued.*

Select an object C on the horizon so that $AC=90^\circ$ as nearly as possible.

Select an object D on the horizon so that $BD=90^\circ$ as nearly as possible.

Then $A'C=AC$ and $B'D=BD$.

Measure CD.

Then $A'C+CD-B'D=A'B'$, the angle required.

Or $AC+CD-BD=A'B'$,, ,,

In practice the ideal conditions required are seldom or ever found, and a few degrees divergence from 90° in selecting the horizon marks will introduce error. The method can therefore only be regarded as an expedient which may be occasionally useful. If the horizon marks shall be on opposite sides of the elevated object, the same reasoning would apply, but the angle between the horizon marks being in that case very large, it would have to be measured by means of an intermediate mark. If forced to use the sextant for triangulation another means may be used.

each plummet is suspended, brought in line with the flag, will indicate the direction of a line passing exactly underneath the flag; the intersection of those lines is the spot required.

Although we are about to speak of triangulation from shore stations, as carried on by means of the theodolite, as this instrument is always available in a surveying ship, it must be understood that, *with care*, an excellent triangulation may be obtained with that invaluable instrument, the sextant. Triangulation by Sextant.

When entering sextant angles in the "Main Angle Book," it is convenient to refer them all to one zero, measuring round to the right as if taken with a theodolite. This facilitates the correction of angles if taken from a false station. Sextant Angles at a Station

The point on which care is principally needed is that the angles measured should be horizontal angles. A practised surveyor will usually be able to note some small natural mark directly above or below the object whose angle is required, and *at his own level*, to which to measure his angle, and in most cases of using the sextant this will give a sufficiently near result. Horizontal Angles with Sextant.

If the angle is small, a direct measurement will introduce more error than would result from reflecting to each object in succession a third object making a very large angle, and taking the difference between the two angles observed. If forced to use the sextant for triangulation, another means may be used.

From the end of a longish pole (boat-hook staff will do), planted at a slight angle from the perpendicular, let a plumb-line fall, and getting the object transit one point in the line, the angle can be taken to any other part of it. The plumb-line must not be too close to the observer, or it will be difficult to keep the transit on, and parallax will creep in.

When possible, a sextant angle between two objects at different levels may be corrected, by observing the angles of elevation (corrected for distance of shore horizon) of the two objects, and calculating the true horizontal angle between them by spherical trigonometry.

It is a question of circumstances as to whether the main triangulation is to be carried on by itself first of all, or in combination with the secondary stations and marks. This in no way affects the principle of the work, but only the detail of what is done when the angles at the main stations are observed.

The main triangles should be as large as possible. The fewer triangles there are, the fewer are the chances of errors of observation.

MAKING A MAIN STATION.

Choice of
Zero.

Observing angles at a station is technically called "making" it. Let us suppose a surveyor making a first station, probably one of the base stations.

He has been previously furnished with a list of the main stations visible from him, and has been told how many times his angles to them are to be repeated. He has also received instructions about the secondary stations and minor marks, if any have been selected and marked.

Having levelled the theodolite, the first thing is the choice of an object from which to measure all the angles, which is called the zero.

A zero should be, if possible, another main station. It must be at some distance, but not so far as to be easily obscured on a hazy day; well defined; so placed that the rays of the sun, when it moves from the position in which it happens to be when the station is commenced, will not obliterate it. It should be a fixed object—*i.e.*, not likely to be removed, or to tumble down, and not so high as to be covered with clouds, as a mountain peak.

A great deal of trouble is given when a zero has to be changed, or when on a subsequent visit to a station the same zero cannot be used. Attention to the above-mentioned points is, therefore, of importance.

The bearing of the zero by the theodolite compass should always be entered in the book.

Observe
Main
Angles
first.

The zero fixed upon, and the theodolite directed upon it, observe the main angles, or those to the main stations, first, repeating them the required number of times, by either of the two methods described under "Theodolite."

These completed, observe the secondary stations a sufficient number of times, as well as all marks and conspicuous objects.

It is important to remember that the position of the sun has a great effect on the visibility of objects, and therefore that those stations and objects on which the sun is shining

should be secured first, because later, when they fall into shadow, they may be wholly invisible.

In most instances a sketch will be also necessary, on which the angles to conspicuous objects, tangents, etc., will then be recorded, instead of in the book.

All angles should be read twice, in order to prevent mistakes; but to ensure accuracy when required, the angles must be repeated on different parts of the circular arch, for the following reasons:

Repetition of Angles.

A theodolite, however well turned out, is seldom exactly centred, hence arises error, as, no matter how uniform the graduation of the circular arc may be, a slight deviation of the axis from true centring will give a difference of reading for an angle on different parts of the arc.

The mean of the readings of the two verniers is supposed to correct errors of centring, but for remarks on this, see "Theodolite."

The reading itself of an angle can never be considered as perfectly correct. Slight parallax frequently exists, especially when an instrument has been some time at work, and is getting worn. In small theodolites the marking of vernier and arc at any given angle will often not coincide exactly, and judgment may assign the wrong reading.

By multiplying readings, then, a mean will be obtained which will to a great extent eliminate these errors, and this must always be done in observing main angles.

Excepting for main angles, forms ruled in the angle book are unnecessary, and in this case the form is simple, consisting of columns to keep the figures separate, as under.

½ July 4th, 1881, at Pagoda \triangle , Theod. 77.

| \triangle^s | \angle Observed. | Difference. |
|-----------------------------|--------------------|-------------|
| | ° ' " | ° ' " |
| \oplus Patero \triangle | 360 00 00 | |
| Mango \triangle (flash) | 25 14 30 | 25 14 30 |
| | 50 29 30 | 25 15 00 |
| | 75 44 00 | 14 30 |
| | 100 59 00 | 15 00 |
| | Mean | 25 14 45 |

h July 4th, 1881, at Pagoda $\triangle \oplus$ Patero \triangle , Theod. 77.

| \oplus | Prince \triangle | Flag \triangle | Snow \triangle | |
|----------|--------------------|------------------|------------------|----------|
| ° | ° ' " | ° ' " | ° ' " | |
| 360 | 24 18 00 | 29 10 30 | 48 26 00 | Z. O. K. |
| 100 | 124 19 00 | 129 11 00 | 148 27 00 | Z. O. K. |
| 200 | 20 00 | 10 30 | 27 00 | Z. O. K. |
| 300 | 19 00 | 11 00 | 27 00 | Z. O. K. |
| Means | 24 19 00 | 29 10 45 | 48 26 45 | |

The first of these forms is adapted for the observation of main angles by repeating round and round singly; which is done when a solitary angle is required to be observed accurately, but to obtain great accuracy the reading should be repeated right round the arc.

The second is for ordinary main angles. This method saves much time when there are a number of angles required, and is as correct as is generally necessary.

Compari-
son of
Methods.

The weak point of the first method is that the zero cannot be referred to, but, as only one angle is taken each time, a theodolite must be very much out of order to introduce error.

If the angle to be observed is small, this method will not answer the purpose, as the theodolite will only be rotated through a small part of the circle, unless an inconveniently large number of repetitions be made.

The weak point of the second method is that any slight error in setting or reading the zero affects every station observed; whereas in the other, the vernier being once *set* at the commencement, is afterwards *read* only.

By either method the observer will see if his different observations of each angle are agreeing together, and can take more if requisite.

Verifying
the Zero.

In all observations of angles with the theodolite (except the case referred to above), the zero must be looked at from time to time, and invariably at the conclusion of the set of angles, to make certain that the direction of the instrument has not changed by any unnoticed touch or shock. On every occasion of doing this it must be noted in the book, so as to know, in

the event of the zero presently being discovered to be wrong, how far back the angles must be recommenced. A common form of notation of this is, Z. O. K., or Z. K., for zero correct.

If the zero is found continually getting displaced without any apparent cause, something is loose, and this must be looked to at once, or nothing will be satisfactory. The parts most liable to go wrong have been mentioned under the head of "Theodolite."

Defects of
Instru-
ment.

Placing a heavy stone against each leg of a theodolite, or one heavy stone triced close up under the apex of the tripod, is very effective in steadying the instrument. The shelter of an umbrella will often enable angles to be accurately obtained which would otherwise be doubtful.

Precau-
tions in a
Strong
Wind.

If using a heliostat, it must be placed in front of the theodolite, in the direction of the station to which you mean to flash. When the stations are distant one from the other, it is desirable to arrange who shall flash first; the receiver of the flash, say at A from B, then takes his angles to it, and does not direct his flash to B until he has got the requisite number of repetitions. When he does flash to B, the latter will know A has done with him, and can direct his flash to some other station, while he observes A. When B in turn has finished with A, he must give the latter another flash to acquaint him with the fact. A's turning off his flash will show B he understands.

Arrange-
ment for
using
Heliostat.

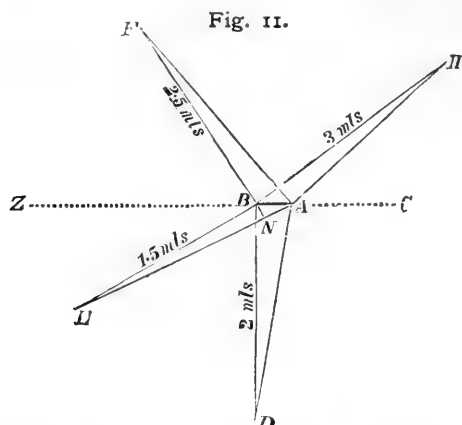
As already remarked, the amount of time saved when the sun is visible, by the use of a heliostat, is incalculable. It is useful for long distances, and short also, and on all sorts of occasions, and is, in fact, one of the surveyor's greatest friends.

Heliostat
Invalu-
able.

FALSE STATION.

It will often happen that a beacon having been erected, the theodolite cannot be placed exactly on the spot, at any rate without a great deal of trouble; or if a building or tree has been selected as a station, that the observer finds on going there that he has to make his station on one side of it in order to see what he wants, or has to make a supplementary station to see a few objects obscured by the building, etc. This is called "False Station," and if the object is already plotted, or

it is desirable to plot it instead of the actual theodolite spot, the angles taken there must be corrected for the distance the theodolite was from such object.



The correction will vary according to the direction of the objects with regard to the true station, as the figure annexed will show.

In Fig. 11 let A be the true station at which angles are required; B the false station; D, E, F, H objects so placed as to illustrate all positions of false and true stations with regard to them. We have observed at B the angles to D, E, F, H, and A, and measured the distance B A.

Firstly. Required the angle E A D.

Produce B A towards C and Z.

$$\text{Now } E A C = E B C + B E A$$

$$\text{and } D A C = D B C + B D A.$$

Subtracting we have

$$E A C - D A C = E B C - D B C + B E A - B D A$$

$$\text{or } E A D = E B D + (B E A - B D A).$$

Secondly. Required the angle E A F.

$$\text{Here } Z A E = Z B E - B E A$$

$$\text{and } Z A F = Z B F - B F A.$$

$$\text{Adding, we get } Z A E + Z A F = Z B E + Z B F - (B E A + B F A)$$

$$\text{or } E A F = E B F - (B E A + B F A).$$

Thirdly. Required the angle D A H.

Here $C A H = C B H + B H A$

and $C A D = C B D + B D A$.

Adding, we get $D A H = D B H + (B H A + B D A)$.

These small angles, B D A, B E A, etc., the angles subtended by the distance between true and false station at each object observed, must be either calculated in each triangle, having two sides and the included angle (for the rough distances B D, B E, etc., will answer the purpose), or else, which is simpler, have a table* made of the angles which are subtended by different lengths at different distances, and take the required angles out, thus :

Let us suppose the theodolite angle in our book corresponding to A is 60° , D 160° , and E 220° . A B is 12 feet. E B is $1\frac{1}{2}$ miles, and B D 2 miles, measured roughly on the sheet.

Calcula-
ting Cor-
rection by
Table.

Required B E A by the table.

It will be evident that the angle B E A is that subtended by a chord drawn across to E A from B. This chord we get near enough by considering B N as at right angles to both E B and E A, and looking out in the traverse table with B A, or 12 feet, as a distance, and B A N or 20° ($180^\circ - 160^\circ$) as a course, and taking the departure for the length of B N, which in this case is 4.1 feet.

We then turn to our table, and see that 4 feet at $1\frac{1}{2}$ miles subtends $1' 31''$, which is the angle B E A. In a similar manner we can deduce any of the required angles, quite near enough for ordinary purposes.

Now, this process becomes far simpler, and much time is saved if, in making a false station, a zero for the theodolite is chosen in a direction exactly opposite to the true station, as, for example, in our figure at Z ; for then each angle taken can easily be corrected separately for the error of the false station, and the true angle entered in the book. Difficulties as to whether the ultimate correction is + or - will be avoided, as in correcting the angles the error is subtracted from all theodolite angles up to 180° , and added to all angles between 180° and 360° .

Arrange-
ments for
Working
by the
Table.

* Appendix, Table O.

Thus, in the case as in the figure above, the angles will stand in the book :

| Object. | | | Observed Angle. | Correction. | Angle at true \triangle |
|---------|----|----|-----------------|-------------|---------------------------|
| | | | $^{\circ}$ | ' " | $^{\circ}$ ' " |
| Zero, Z | .. | | 360 | 0 00 | 360 00 00 |
| F | .. | .. | 50 | 2 08 | 49 57 52 |
| H | .. | .. | 130 | 1 42 | 129 58 48 |
| D | .. | .. | 280 | 3 23 | 280 03 23 |
| E | .. | .. | 340 | 1 31 | 340 01 31 |

In using the traverse table, take for the course—

Up to 90° the observed theodolite angle itself

Between 90° and 180° .. 180° —the observed angle

„ 180° „ 270° .. observed angle— 180°

„ 270° „ 360° .. 360° —observed angle

—and the departure is looked out in each instance.

Table
generally
Useful.

The table of angles subtended by different lengths is useful for other purposes. As when an angle is taken to an object, and it is afterwards decided to plot a station made near that object instead of the object itself, the angles to the station can be corrected by it, in precisely the same manner as described above, the distances and direction of the station from the object being known.

Extension
of Table.

Distances or lengths, greater than those included in the table, can be got by multiplication or division.

Thus, if the angle of 18 feet at 5 miles is required, it is double that of 9 feet.

Again, if the angle of 12 feet at 10 miles is wanted, it is half that at 5 miles.

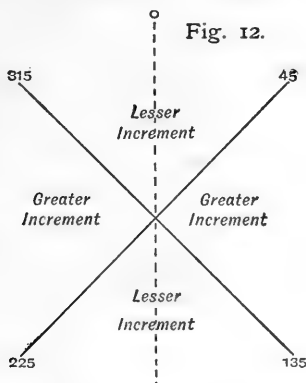
Rule of
Thumb.

The correction for “false station” may readily be found as follows, without having to bestow a moment’s thought beyond applying the rule, which is a matter of no small gain in time when a number of angles have to be corrected.

Rule.—Put down the theodolite reading which it is required to correct (increased if necessary by 360°), and from it subtract the theodolite reading of the centre of the true station. Call this remainder θ .

With θ as a “course,” and the number of feet from the

theodolite to the true station as a "distance," enter the traverse table, and take out the greater increment if θ lies between 45° and 135° , or between 225° and 315° , and the lesser increment for other angles. The accompanying diagram (Fig. 12) will assist the memory. Refer this increment to the



"table of subtended angles by various lengths at different distances" (using the distance of the object observed), and find the corresponding correction in arc, which mark + or - according as θ is under or over 180° . Apply this correction to the observed theodolite angle. A "table of subtended angles" is unnecessary if the formula

$$\text{Angle in seconds} = \frac{\text{number of feet subtended} \times 34}{\text{distance of object in sea miles}}$$

be used instead.

The angle to the zero must be corrected in the same manner as other angles; the correction to zero will be nil if the angle to the true \triangle reads 180° or 360° .

The sign of the correction to the true station will be reversed when correcting angles taken to the false station at one of the other stations, but will be the same in amount.

The use of the foregoing method is facilitated by placing the false \triangle in such a position that the theodolite reading of the true \triangle is not large.

A simple and effective way of reducing the angles to the true \triangle is to measure the angles taken to the same object from two positions at equal distances on either side of the true \triangle , the line joining the two positions passing through the true \triangle . Two False Stations on Opposite Sides of the True \triangle

Con-
nec-
tion
of
a
Con-
spic-
uous
Natural
Object
with a \triangle
adjacent
to, but
invisible
from, it.

It sometimes happens that a \triangle is made near the top of a wooded hill, on which stands a tree which overtops all others, but is difficult or impossible to identify when on the hill itself. This tree is the natural object which will be observed from all other stations, and it is therefore often desirable to connect it by angle and distance with the \triangle itself. This may be effected thus :

When at a convenient distance from the hill, fix the ship, shoot up the \triangle , and take a careful angle between it and the tree. When the bearing of the hill has changed by a fairly large angle— 70° to 90° —fix as before, and again measure the

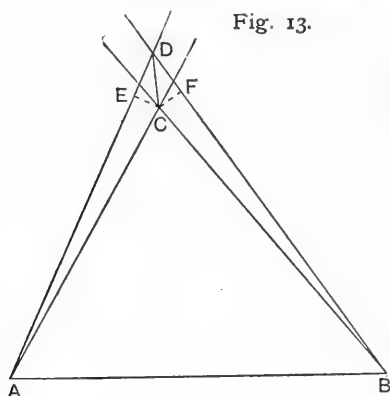


Fig. 13.

angle subtended by the tree and the \triangle . The problem is then quite a simple one, and as it is working down from two long bases (the distance of the hill at each of the ship's fixes) to a small side, it is very accurate. The easiest way to solve it is by the aid of the traverse table and protraction, as follows :

In Fig. 13, let C be the \triangle ; D, the tree ; A and B, the two ships' positions.

C bears from A, N. 30° E. (true), taken from the plotting-sheet.

C A = 6,000 feet. $\angle D A C = 1^\circ 00'$.

C bears from B, N. 40° W. (true).

C B = 8,000 feet. $\angle D B C = 1^\circ 30'$.

Required True Bearing and distance of D from C.

Draw C E and C F \perp A D and B D.

Then from traverse table :

C E = 105 feet in direction N. 61° W. from C.

C F = 209 feet in direction N. $51\frac{1}{2}^{\circ}$ E. from C.

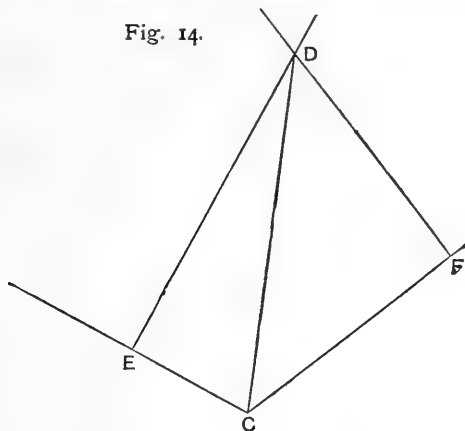
Bearing of D from A = N. 29° E.

Bearing of D from B = N. $38\frac{1}{2}^{\circ}$ W.

From the above data plot the positions of C, E, D, F, on
as large a scale as convenient (say 1 inch = 100 feet),
as in Fig. 14.

Result (by protraction) : D bears from C, N. 8° E. 285 feet.

Fig. 14.



SKETCH.

A sketch taken from a station is made with the object of more easily identifying details to which it is necessary to take angles. By having a view of hills, islands, houses, trees, etc., from two or three stations they can, if fairly placed in their proper positions, be easily recognised in the different sketches when plotting. No description in the angle book will do this so well, unless, of course, there is something very remarkable in the object, but even then the position of it as shown in the sketch will assist materially to prevent mistakes, and a curt description is also written against it on the sketch.

Sketching to this extent is within anybody's reach. A fairly correct outline is all that is absolutely necessary, and a very little practice will enable the least likely draughtsman to make a sufficient sketch for practical purposes.

**Checking
Scale by
Angles.**

It is well for a beginner to commence by taking some rough angles to check his scale, or, until he is used to it, he will probably have one part of his view two or three times as big as the other, which is confusing afterwards, although the proper angles will be written against the prominent objects when the sketch is finished.

**Preserva-
tion of
Scale.**

Always put the most distant outline on the paper first, as it is far easier to keep the scale uniform if this is done.

Begin on the extreme left of your view, or if it is an all-round view, choose a point, in the direction least required, to be the left, and always work to the right.

**Useful
Hints for
Sketch-
ing.**

If the sketch is too long for one double page of the sketch-book, when the right-hand end of it is reached, turn over, and turn 1 or 2 inches of the last page down, so as to show on the fresh page; this will give a commencement for the part to follow, and the sketch will be continuous.

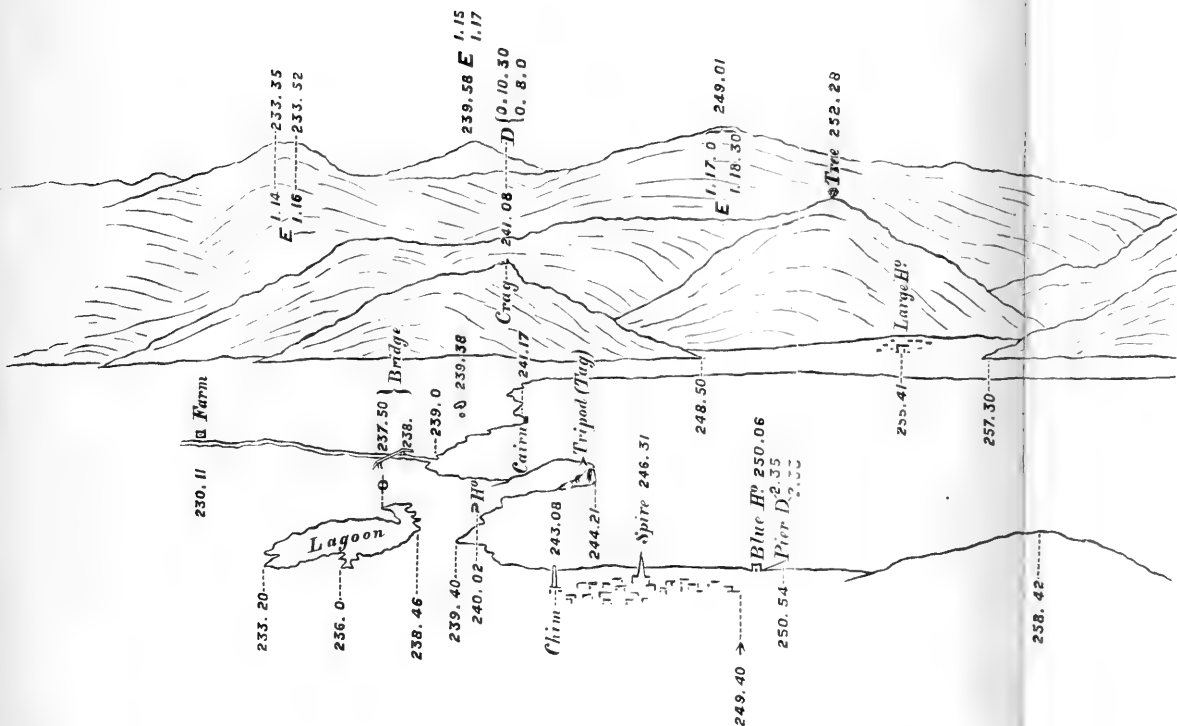
Commence by settling whereabouts on the paper some two well-defined points of the distance are to be, and use these after as a scale from which to measure by eye the proper position of everything else.

If taking angles to assist correct drawing, as suggested above, a scale for the sketch must be decided on, say about $\frac{1}{3}$ inch to a degree, but this will vary according to the complication of the sketch. If no divided scale is at hand, mark the edge of a strip of paper by eye, which will answer the purpose perfectly.

Take an angle from some definite point of the distance on the extreme left to some other, say about 20° to its right. Make a dot for the first object, lay the scale or strip of paper on the sketch, and dot again at the proper number of degrees, and at the proper height, with regard to difference of altitude, for the second object. Other angles can be taken to other objects between these, and the view sketched in between these dots, commencing as already said with the outline most distant, and therefore highest in the sketch.

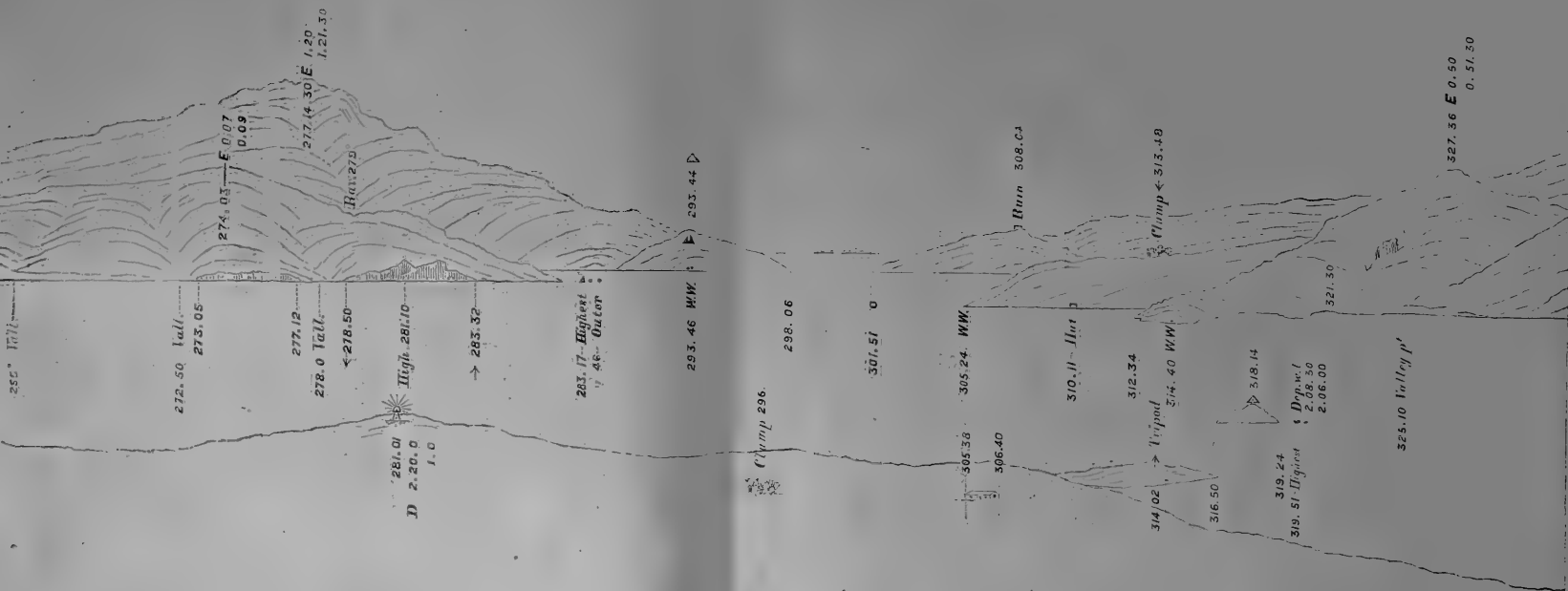
In sketching for this purpose, it is well to rather exaggerate the height of objects, as, where there are hills, range upon range, or many objects, as houses, trees, etc., at different altitudes, they will get so crowded up as to make the sketch difficult to decipher, unless this course is adopted.

F R O M



Rough Sketch fro

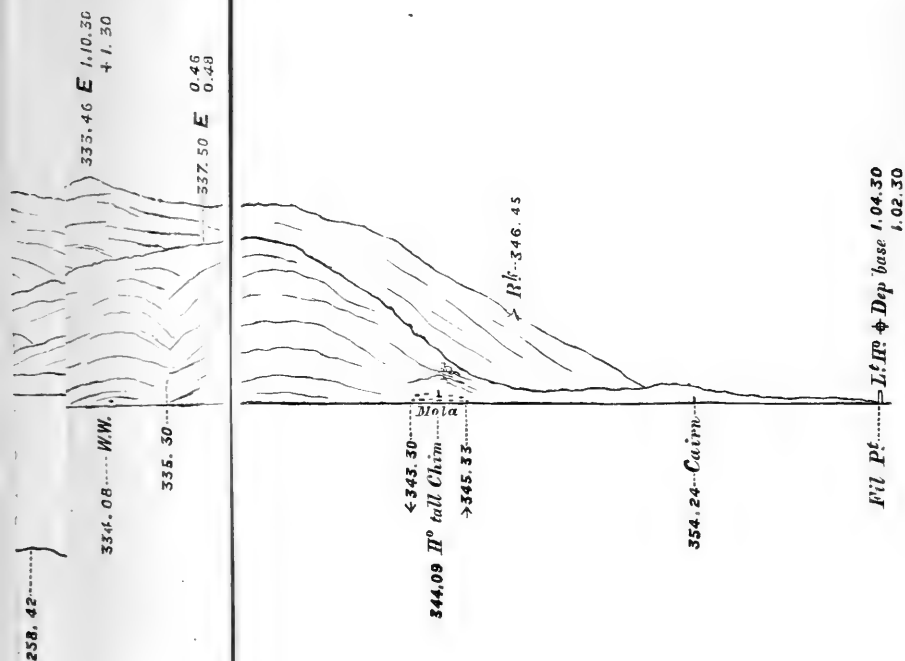
To face p. 94.



m a Δ showing method of marking angles, elevations, depressions, &c., and also mode of tur

24TH,

1 8 8 1.



tech n ing dow n paper for continuous panorama.

The great thing in a sketch is to place objects fairly correctly with regard to each other horizontally considered; *e.g.*, if there is a hill with a point nearly underneath it, take care that the latter is drawn on the correct side of the hill, right or left. Nothing is more calculated to confuse anybody plotting angles from a sketch, than to find that an object drawn apparently to one side of another object has an angle which shows it should have been on the other side. Doubt is at once thrown on the angle, when it is probably the drawing of the sketch which is incorrect.

Important
Point to
observe.

When the sketch is finished resume the theodolite, using the same zero, and mark the angles on the sketch itself, noting what the object is, when it may be doubtful, as for instance—Chimney of red house, Right of two fir-trees, Big white boulder, etc. See example of sketch attached.

Descrip-
tion of
Objects in
Sketch.

Views of distant land intended for reproduction on the published chart should be drawn very accurately to scale, both vertically and horizontally, from sextant angles. A convenient scale for horizontal angles in a long sketch is 0.2 inch to a degree, and for vertical angles 0.3 inch, or even larger. In sketches for leading marks, and where the portion of coast embraced is not large, the scale should be larger. The position from which a sketch is taken should always be noted upon it.

Views.

Photography may often usefully be employed if circumstances are favourable.

Views are very valuable for picking up the land and identifying landmarks shown on the chart, but the positions from which they are taken should be carefully considered; if not taken from a sufficient distance seaward, they may be not worth engraving. The requirements of the navigator must be remembered.

PREPARATION FOR CALCULATING THE TRIANGULATION.

It is well that a true bearing be obtained between two distant stations before plotting; but the method of doing this will be described under observations, and as far as absolute necessity goes, a good compass bearing from a shore station is

A Bearing
required
for Ori-
entation of
Chart.

quite sufficient to begin on. The bearing is only wanted to plant the meridian fairly square on the paper, and the compass bearing will give us this near enough to be able to lay off any bearings which may be taken in the course of mapping the detail. The compass will never be used in any of the important parts of the chart, unless our survey partakes of the nature of a sketch or running survey.

If, however, regular triangulation is likely to fail, true bearings in the course of the work may be necessary to carry it on, and in this case we *must* begin with a careful true bearing.

Prepara-
tion of
Triangles.

In preparing the triangles for working, they will of course never be found exactly correct, *i.e.*, the three observed angles will be either more or less than 180° .

Spherical
Excess.

In dealing with this theoretically, the sum of the three theodolite angles taken at the corners of any triangle will be greater than 180° , in consequence of each angle observed being in a different plane. This is known as the spherical excess, and in extended triangulations for topographical purposes, as the survey of India, etc., must be taken into account. For practical nautical work we need not regard it, as our instruments are not large enough to measure angles so exactly, nor is our work of sufficient extent

Correct-
ing the
Triangles.

In dealing with the amount the triangle is in error, for the three angles of the triangle must be corrected to make the precise 180° , before using them for calculation, circumstances must guide its distribution among the angles.

An angle observed with a large theodolite should have more value given to it than others. One station may have been more exposed to the wind than others, which would depreciate the value of the angles observed there.

Without any indications of this kind to guide, it is as well to divide the error equally among the angles; but it must be remembered, that an alteration in the small angle will make more difference in the resulting position than in either of the other two, so that if this angle at all approaches the limit which should be used for a receiving angle (30°), it is perhaps well to put the smallest amount of change into it, but it is of course impossible to *guess* where the error is. If the angles have been repeated often enough, the resulting error any way will be very small.

No rule can be laid down with regard to the amount of deviation from the 180° that can be admitted, it so much depends on the degree of accuracy required ; but in an ordinary theodolite survey the error should not be more than two minutes, and ought to be under one, working with 5-inch theodolites, and repeating the angles three times if satisfactory, or more if they vary much.

In the first few triangles, the error should not be more than one minute.

Having corrected the triangles, we come to the calculation.

The working out of the triangulation is the very simple affair of plane triangles which every naval officer understands. The rule of sines, and the rule to find the third side,* when two sides and the included angle are given, are all that are required.

Logarithms of all angles must be taken out to seconds, so that the possession of tables giving these for every second of arc will save much time and chance of mistake.

Into the final calculation of an extended calculated triangulation some other considerations enter.

The actual working of the triangles will be the same ; but here we want the bearing of every side, as well as the distance, and the "convergency of the meridians" must be considered. This convergency will be explained before proceeding further.

CONVERGENCY OF THE MERIDIANS.

The true bearing of any two points on the earth, taken one from the other, in both directions, will be found to differ by a quantity which is called the convergency, and varies with the latitude, distance apart, and position of the points in bearing, or, in other words, with latitude and departure.

Thus, if R and L are two stations lying roughly N.E. and S.W. of one another, R being nearest the pole, in this case the North Pole, the true bearing of L from R will be found to be a greater number of degrees and minutes as measured from the meridian than the reverse bearing of R from L.

This results from the form of the earth. The true bearing of one position from another is the angle which the arc of a

* The rule where sines only are involved must be used.

great circle drawn between the two positions makes with the meridian of the observing position. As meridians are not parallel, but converge at the poles, the great circle will cut each meridian it passes at a different angle, the amount of difference, for equal meridians, depending on the latitude.

To further the comprehension of this, let us consider the method of projection of the sphere used when graduating a map, made from the original data of angles and measurements.

Projections
of the
Sphere.

It will be evident to anyone who considers the subject that, as our globe is a sphere (speaking roughly), a portion of its surface cannot be shown on a flat piece of paper without distortion, more or less, according to the extent so shown.

There are a variety of methods used to delineate a portion of the earth's surface on a map, which are called "projections." Into this variety it is not proposed here to enter, as but one can be used when actually making a survey, which is the "Gnomonic Projection."

Gnomonic
Projection.

This projection is the only one on which great circles of the earth are shown as straight lines. As it is on the chord of a great circle that we see one object from another, it is evident that in graduating a map on which we have laid down, or are going to lay down, one position from another by drawing straight lines, we *must* use this projection.

A chart on the Gnomonic Projection is supposed to be drawn on a flat surface laid against the earth, touching it at the central point of the flat surface, and there only. From the centre of the earth lines are supposed to be drawn, passing through the different points to be shown on the map, until they pierce the flat surface.

The positions so indicated on the upper side of the flat surface are those corresponding to the points required.

Here, in Fig. 15, P Q S is the globe, and A B C D a flat surface laid against it, touching at the point J, the centre of the flat surface, the *under* side of which is shown. P is the pole. M, F, are points taken on the same meridian as J. Imaginary lines drawn from the centre of the earth through these points will touch the flat surface in N and G, and the line joining them, the central meridian of the chart, will be a straight one. K, another point on the globe east of the central meridian, will be projected at L, by the same method of

earth. L is the position on our chart of K (see other figure). R is the position of a similar point, invisible in first figure, being on the other side of the earth. Meridians passing through L and R are projected on the chart by the same method as before, *i.e.*, by drawing imaginary lines from the centre of the earth through different points in the required meridian; they will be found to lie as T L, R O, and their extension will also pass through X, making an angle R X L,

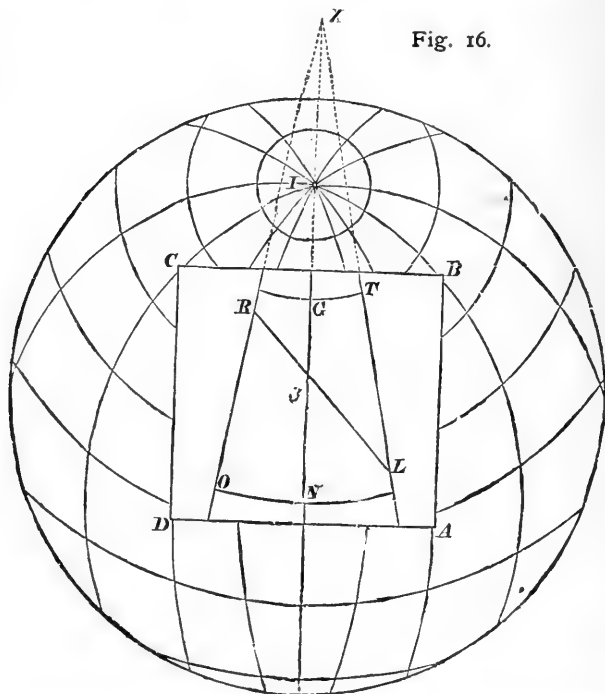


Fig. 16.

which is the Convergency of the meridians; and this will be seen at once to be equal to the difference of the reverse bearings of R and L, for—

$$\angle ORL = OXL + R LX,$$

or

$$OXL = ORL - R LX,$$

i.e., Convergency = Bearing of L from R — Bearing of R from L.

A little consideration of this last figure will show that the further towards the pole the central point J is, the greater

will be the convergency of two meridians a fixed number of degrees apart; that when the pole P and J coincide, the meridians will radiate over the chart from that centre, and the convergency will equal the distance between the meridians; and that when J is on the Equator, the meridians will be parallel, and convergency will be nothing. Convergency nil at Equator, and equal to diff. long. at Poles.

Parallels of latitude will appear on the chart as curves, concave towards the poles, and cutting each meridian at right angles. Parallels.

The Equator being a great circle will be a straight line, and, generally, the further from the Equator, *i.e.*, the higher the latitude, the greater will be the degree of curvature in the parallels.

More consideration will show that, the farther a part of the flat surface is from the surface of the earth, the greater will be the distortion of the positions resulting from this method of delineating the globe; or, in other words, that the distortion increases from the centre of a gnomonic chart, and will become very considerable towards the edges, if a large area of the earth is attempted to be shown on a flat surface. But in practice, a marine survey does not extend over a sufficient area to make this distortion in any way apparent. Our diagrams are of course much exaggerated in this respect. Distortion.

It will be understood that the convergency is an actual fact, and does not result merely from the method employed in this projection. We have only considered it in connection with the projection, as it is thought that by so doing the nature of the convergency becomes more plainly apparent.

The mean of the two reverse bearings, or either one of them, plus or minus half the convergency, will give the Mercatorial Bearing, so called from being the bearing which each station will be from the other in a Mercator's chart, where, the meridians being all parallel, the line joining the stations will cut them at the same angle, this angle being also the one at which the line on our gnomonic chart will cut a meridian midway between the stations. Mercatorial Bearing.

The actual observed bearing of a distant object, if protracted on a Mercator's Chart, will not pass through its

position, in consequence of the existence of convergency. Mercator's charts are generally on such a small scale that, for navigating purposes, the error of taking the bearing swallows up the error introduced by not allowing for convergency.

The formula for Convergency is—

Convergency
Formulae.

Tangent Convergency = Tan. departure \times Tan. Mid. lat. (1)

Or in anything but high latitude, or when the departure is great, it is correct enough to say—

Convergency (in mins.) = dep. (in mins.) \times Tan. Mid. lat. (2), which can be converted into—

Convergency = d. long. \times Sin Mid. lat. (3)

Convergency = dist. \times Sin Merc. Bearing \times Tan. Mid. lat. (4)

any of which can be used as convenient.

The proof of the formula is given in the Appendix A.

Convergency by
Spherical
Triangle.

The convergency can also be found when the latitudes of and difference of longitude between the two stations is known, by working out the spherical triangle, with the pole, and the two stations, as the three points. Here we have the two colatitudes as the sides, containing the difference of longitude as the polar angle, to find the other two angles, which will be the bearings of each station from the other. The difference of these will be, as before, the convergency.*

CALCULATING THE TRIANGULATION.

We now resume our remarks on working out a calculated main triangulation. All sides being calculated by the ordinary method of plane triangles, we now want the bearing, the mercatorial bearing, of each side, or, at any rate, a considerable number of them, in order that we can take any triangles or sides to work up details on, on a separate sheet, and that such sheet may be complete in itself as to bearing, distance, and position, with regard to other portions of the main triangulation.

* See following article for application of Convergency.

We will take as an example the following :

Lat. A, 49° 30' 24" N.

True bearing observed B from A. N. 69° 05' 00" W.

Angles Observed and as Corrected.

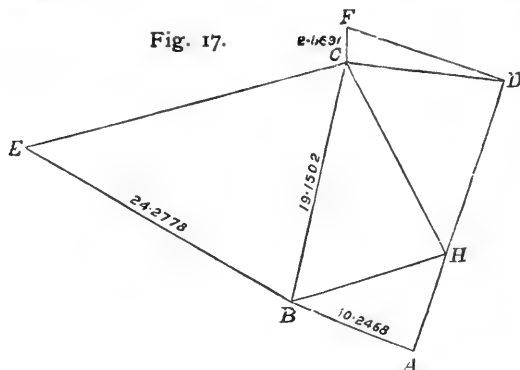
| | | Observed. | | | | Corrected. | | |
|---|----|-----------|----|----|--|------------|----|----|
| | | ° | ' | " | | ° | ' | " |
| A | .. | 86 | 06 | 35 | | 86 | 06 | 19 |
| B | .. | 38 | 52 | 02 | | 38 | 51 | 47 |
| H | .. | 55 | 02 | 04 | | 55 | 01 | 54 |
| | | <hr/> | | | | <hr/> | | |
| | | 180 | 00 | 41 | | 180 | 00 | 00 |
| | | | | | | | | |
| B | .. | 59 | 33 | 10 | | 59 | 33 | 27 |
| H | .. | 80 | 27 | 51 | | 80 | 28 | 09 |
| C | .. | 39 | 58 | 14 | | 39 | 58 | 24 |
| | | <hr/> | | | | <hr/> | | |
| | | 179 | 59 | 15 | | 180 | 00 | 00 |
| | | | | | | | | |
| C | .. | 56 | 58 | 08 | | 56 | 58 | 44 |
| H | .. | 46 | 26 | 22 | | 46 | 26 | 58 |
| D | .. | 76 | 33 | 43 | | 76 | 34 | 18 |
| | | <hr/> | | | | <hr/> | | |
| | | 179 | 58 | 13 | | 180 | 00 | 00 |
| | | | | | | | | |
| C | .. | 96 | 50 | 21 | | 96 | 50 | 27 |
| D | .. | 11 | 17 | 06 | | 11 | 17 | 13 |
| F | .. | 71 | 52 | 13 | | 71 | 52 | 20 |
| | | <hr/> | | | | <hr/> | | |
| | | 179 | 59 | 40 | | 180 | 00 | 00 |
| | | | | | | | | |
| B | .. | 72 | 40 | 17 | | 72 | 40 | 31 |
| C | .. | 62 | 46 | 39 | | 62 | 46 | 53 |
| E | .. | 44 | 32 | 22 | | 44 | 32 | 36 |
| | | <hr/> | | | | <hr/> | | |
| | | 179 | 59 | 18 | | 180 | 00 | 00 |

We have in the annexed figure (17) a portion of a triangulation, where all the angles have been observed at each station. The latitude of A is known, A B is the original long side obtained by extending the base, and the true bearing of B and A has been taken from one another, from which we have deduced a mean bearing of B from A with which we intend to work. The length of each side has been calculated by ordinary trigonometry. We now want to calculate the bearings of the different sides, so as to be able to break up the triangulation into different sheets. We shall want also

Example of Calculated Triangulation.

the latitude, and difference of longitude from A, of F, which is a station in a plan on a large scale we have made.

For the purposes of this plan we have obtained the side FC in the triangulation, which will serve as our base instead of measuring another.



We shall commence by calculating the convergency for ten miles of departure at the average latitude of the chart, as we shall want it directly.

In this case we find that—

Convergency for 10' of departure = 11.92'.

Or for each mile of departure = 1.2'.

We then find approximate latitude of B by the formula—

Diff. lat. = A B \times Cos rough mercatorial bearing.

We obtain the bearing, near enough for this purpose, by finding the rough convergency and applying half of it to the observed bearing of B from A, thus :

Take departure from the traverse table, in this instance 9.5. Multiply it by the convergency for a mile, just found to be 1.2, which gives us 11.4 as the rough convergency. Adding half of this to the bearing of B from A, we get rough mercatorial bearing N. 69° 11' W., and working out the difference of latitude, we find it to be 3' 38", which gives for the latitude of B, 49° 34' 02", and for middle latitude 49° 32' 13".

Then convergency = dist. \times Sin merc. bearing \times tan. mid. lat.

Using the rough bearing just found, we get—

Convergency for A B = 11' 13.8".

This convergency, and half of it, added respectively to the bearing of B from A, will give the reverse bearing of A from B, and the mercatorial bearing, thus :

B from A = N. $69^{\circ} 05' 00''$ W.

A from B = S. $69^{\circ} 16' 14''$ E.

Mercatorial bearing $\frac{\text{N.}}{\text{S.}} 69^{\circ} 10' 37'' \frac{\text{W.}}{\text{E.}}$

If this differs much from the rough mercatorial bearing, we must recalculate the latitude of B before proceeding further, but this should not be necessary.

Then to calculate bearing of B E, we have the bearing of A from B, just found, to start from. Adding the three angles, A B H, H B C, C B E, to it, we shall get the bearing of E from B. The convergency for B E is calculated in the same manner as above, and we shall then have mercatorial bearing of B E. Thus :

| | | | | | | | | | | |
|---------------------------|---|---|---|---|---|-------------------------------|--------------|-------|--------|-------------------------------|
| A from B | . | . | . | . | . | S. | 69° | $16'$ | $14''$ | E. |
| A B H | . | . | . | . | . | | 38 | 51 | 47 | |
| H B C | . | . | . | . | . | | 59 | 33 | 27 | |
| C B E | . | . | . | . | . | | 72 | 40 | 31 | |
| <hr/> | | | | | | | | | | |
| E from B | . | . | . | . | . | S. | 240 | 21 | 59 | E. |
| Or | . | . | . | . | . | N. | 60 | 21 | 59 | W. |
| $\frac{1}{2}$ convergency | . | . | . | . | . | | | 12 | 26 | |
| <hr/> | | | | | | | | | | |
| Mercatorial bearing of | | | | | | $\frac{\text{N.}}{\text{S.}}$ | 60° | $34'$ | $26''$ | $\frac{\text{W.}}{\text{E.}}$ |
| B E | . | . | . | . | . | | | | | |

In like manner we must calculate the mercatorial bearing of all the sides we require, remembering that of the reverse bearings, the bearing of the station nearest the pole from the one farthest from the pole is the smallest. In this case, then, being in the northern hemisphere, where a bearing is measured from the north point, the convergency is added to obtain the reverse bearing.

Having obtained the bearing of each side, we can calculate the relative position of any two stations by working out the traverse between them.

Applica-
tion of
Conver-
gency.

Thus, to get position of F, we have—

| | | |
|-------------|---------------------------|----------------|
| A B | N. 69° 10' 37" W. | 10·2468 miles. |
| B C | N. 12 20 56 E. | 19·1502 miles. |
| C F | N. 1 24 17 W. | 2·5691 miles. |

From which we calculate difference of latitude and departure in the ordinary manner.

We thus get the mercatorial bearing of A F, N. 12° 32' 45" W., and distance 25·5269 miles.

Calcula-
tion of
Triangu-
lation not
generally
necessary
early in
Survey.

It will be understood that it is by no means necessary to work out all the triangulation as just described when commencing the plotting. All that is then required is as long a side as we can get on which to begin. The main triangulation can be calculated afterwards, and in many instances must be, as the whole of the angles will not be obtained till later on. In some nautical surveys it will not be necessary to calculate any triangulation at all.

Triangles
contain-
ing Small
Angles not
always
ill-condi-
tioned.

It will be observed that we have a triangle C D E with a very small angle. This not being a receiving angle does not matter in the least. We are obtaining the position of F from C and D, which are already fixed, and the angle of intersection at F being nearly a right angle, the change of position in F, resulting from a small error in the angle at either C or D, will be as small as is possible, and much less than if, the angle at C being the same, that at D was 60°, which would result in the intersection at F being more acute, and any error would consequently change the position of F to a greater degree.

If we were obtaining D from C and E, such a small angle would not be admissible for a moment, as it is evident that any small error at C or F would result in a great change of position in D.

It would be awkward and inconvenient to have many such triangles in the main framework of the triangulation, as the small side is of no use in carrying on the chain, and we should be forced to multiply triangles in consequence; but we are, notwithstanding, sometimes obliged to include some such in our work, from the lie of the land and other causes, and as long as we use them as in the example they will not affect the result, as far as chance of accuracy goes, and should not be under these circumstances considered as "ill-conditioned."

In working out the diff. lat. and diff. long. of two positions from the triangulation geodetically, we have been treating the earth as a sphere. This is not strictly the case, as the form of our globe is that of an oblate spheroid; but the error introduced by assuming it to be a sphere is small, and can often be disregarded in hydrographical work, as being swallowed up in the larger errors incident on imperfect triangulation. Correc-
tion for
the
Spheroid.

When, however, a triangulation has been carefully done, and we wish to get the difference of longitude as near as we can, either for the scale of the chart, or for purposes of comparison with that deduced from the astronomical positions, or in latitudes far from the Equator, the necessary correction for the spheroid should be applied.

This correction is $2 \cos^2 \text{Mid. lat.} \times \text{compression}$.

The compression of the earth is the proportion that the difference of the equatorial and polar diameters bears to the diameter, and can be taken as $\frac{1}{300}$.

The formula for correction for a given difference of longitude will then stand :

$$\text{Correction} = \text{diff. long.} \frac{\cos^2 \text{Mid. lat.}}{150}.$$

This is subtractive from the calculated difference of longitude by the triangulation.

In the latitude of 20° , this correction for a difference of longitude of $100'$, amounts to $35''$. as will be seen by the following example :

In latitude 20° the departure deduced from a triangulation was found to be $94'$, required true difference of longitude. Example.

| | | | |
|-----------------------------|-----------|------------------|---------------|
| Dep. | 1.973128 | | |
| Sec. lat. | 0.027014 | | |
| <hr/> | | | |
| Spherical d. long. | 2.000142 | | 100.0327' |
| Cos ² lat. | 9.945972 | | |
| <hr/> | | | |
| | 11.946114 | | |
| 150 | 2.176091 | | |
| <hr/> | | | |
| Reduction | 1.770023 | | -0.5889 |
| <hr/> | | | |
| | | True diff. long. | 99.4438' |
| | | or | 1° 39' 26.6'' |

The true difference of longitude can also be calculated from the tables of lengths of a minute of latitude and longitude in the Appendix M as follows :

$$\text{True diff. long.} = \text{dep.} \frac{\text{No. of feet in minute of Lat.}}{\text{No. of feet in minute of Long.}}$$

Working out the above example this way, we have :

| | | |
|-----------------|----------|--------------|
| Dep. | 1·973128 | |
| 6053 | 3·781971 | |
| | <hr/> | |
| | 5·755099 | |
| 5722 | 3·757548 | |
| | <hr/> | |
| True d. long .. | 1·997551 | 99·44' |
| | <hr/> | |
| | or | 1° 39' 26·4" |

which gives the same result as the other method.

Calcula-
tion of
True Bear-
ing be-
tween Two
Astrono-
mical
Positions.

When calculating the true bearing between two points whose astronomical positions are known, it is necessary in the first place to calculate the spheroidal correction, and *add* the correction to the true (or chronometric) longitude to obtain the spherical diff. long.

With the spherical diff. long. and diff. lat. the mercatorial bearing and distance is found by middle latitude sailing, which is an equally correct but shorter method than that by spherical trigonometry, and may be safely used when dealing with distances not exceeding 120 miles.

Correct-
ing Tri-
angula-
tion for
Error of
Tempo-
rary Base
and Bear-
ing.

In the example of triangulation we have given we have supposed ourselves to be working from a measured base. If the survey is extensive, the ultimate scale of the chart will depend upon the astronomical positions. It is very unlikely that when these are obtained, the distance between the extreme points depending upon them will agree exactly with that deduced from the short side, and therefore all the sides will want correction in probably both bearing and distance.

The readiest way of doing this is to get a proportion between the two total distances, as found by the triangulation and by the astronomical positions respectively, in the shape of a logarithm, and multiply each side found by it, which will give the true value as dependent on observations. The bearing

of every side will have to be corrected by the difference of the bearings of the extreme points.

Thus, referring again to our example (which, for the sake of brevity, we have confined to only a few sides), let us suppose we find by observations that A F is N. $12^{\circ} 36'$ W. 26.248 miles.

Dividing this distance by the former one, we get a proportion whose logarithm is 0.012097. Adding this to the log of each side required to be corrected will give us the true value.

The difference of bearing is $3' 15''$ more to the westward. The bearing of each side will then have to be corrected by this amount. Thus, the bearing of A B will stand N. $69^{\circ} 13' 52''$ W.

This difference is somewhat exaggerated. It should seldom, when true bearings have been well observed, amount to so much, but in some climates it may be unavoidable.

In a case of this kind the result of both triangulation and astronomical observations would be transmitted home, as their concurrence or otherwise will form a good test of the value of the work generally.

In stating as we have that the ultimate scale of the chart of an extended piece of coast will depend upon the astronomical positions at either end, it is not intended to lay down a too hard-and-fast rule. The conditions of each element, triangulation and positions by astronomical observations, must be considered. Both, under the ordinary circumstances of a marine survey, are liable to error. In a rigorous trigonometrical survey, the triangulation is more likely to be correct owing to the unknown error in the astronomical positions due to local attraction of the pendulum, or in other words of the mercury in the artificial horizon; but, in an ordinary marine survey, the triangulation is carried on under conditions which prevent the possibility of ensuring freedom from errors in it. Nevertheless, should the discrepancy in bearings be large, when we know our true bearings have been well observed and our triangulation to have carried it on within the limits of the discrepancy, it is desirable to adjust the astronomical positions so as to reduce the discrepancy in bearings.

In such circumstances we must examine critically the various

causes contributing to this discrepancy, and assign to each its due weight.

The difference between the results by triangulation and by astronomical observations is attributable on the one hand to errors in triangulation, viz. :

- (1) Errors in the base as measured ;
- (2) Accumulated effect of errors in the triangles involved ;
- (3) Errors in the true bearings ;

and on the other hand to errors in the astronomical observations.

If proper care has been exercised throughout all the operations, each should bear its fair share in the final adjustment within the limits of its probable error.

This is more particularly the case when it becomes a question of altering the true bearings to any material extent, especially if the bearings obtained at one end of the survey, on being transferred to the other end, throughout the whole length of the triangulation, are found to agree fairly closely with the bearings observed at that end.

We should also be reluctant to throw over entirely the results by triangulation if a check base has been measured and found to corroborate that part of the work.

These considerations have greater weight in the case of a survey where the limits are not very extended. An error of a fraction of a second of arc in the astronomical position of one or both of the terminal points of a survey of small extent might produce a somewhat startling discrepancy in bearing.

Although the observed latitudes will not probably be much in error, the sextant being capable of very great accuracy, yet an error of 1" or even 2" may easily occur in either latitude, involving a possible error of 2" or 4" in the resulting diff. lat.

In the case of a chronometric meridian distance, the possible error in diff. long. may be even greater, and, as stated above, astronomical positions are liable to be seriously affected by errors due to the deflection of the plumb-line.

We must therefore endeavour to obtain final results without placing undue reliance on any particular element of the problem to the exclusion of others when circumstances permit.

CO-ORDINATING RESULTS OBTAINED ASTRONOMICALLY AND BY TRIANGULATION.

Suppose A and B to be the two terminal points of the survey ; that we have carefully triangulated between the two, have observed the latitude at both, and have measured the meridian distance between them.

We can now calculate the bearing and distance between A and B both astronomically and by triangulation. As has been said before, we may expect to obtain different results. How to harmonize the two ?

I. *Consider first the Astronomical Result.*

The observed latitudes will not probably be much in error, yet, as before stated, an error of 1" or even 2" may easily occur in either latitude.

The actual amount of probable error of either latitude can be very fairly estimated after the sights have been worked out ; the *sum* of the probable errors of the two latitudes will be the " maximum probable error " of the resulting diff. lat.

The probable error of the meridian distance can also be estimated, and this will be the same thing as the " maximum probable error " of the resulting diff. long.

II. *Consider next the Triangulation Results.*

Errors will occur both in distance and bearing.

(a) *Error in distance* will be due to errors in the measured base, and to errors in the angles of the triangles used in calculating the triangulation.

If a base has been measured near A at one end of the survey, and a check base near B at the other end, and the distance A B calculated separately from both bases, it will be as well to adopt the mean of the two values as the accepted length A B (by triangulation), and consider half the difference of the two as the probable error in distance A B.

Knowing the bearing of the line A B, find the diff. lat. and diff. long. corresponding to this probable error, in seconds, by means of Traverse and Carrington's Tables.

This is the " maximum probable error " in distance A B (by triangulation) expressed in terms of diff. lat. and diff. long.

If no check base has been measured, we can still form a good idea of the probable error of—

(1) The measured base in “feet per mile.”

(2) The sides of the various triangles, if, as should be the case in a properly arranged triangulation, we can occasionally calculate the individual side of a triangle through two or more different lots of triangles, getting two or more results from the same side, and adopting the mean, and so obtaining a probable error in terms of “feet per mile.”

Then $(1) + (2) =$ “probable error” of the distance A B expressed in “feet per mile”; and $[(1) + (2)] \times \text{length of A B in miles} =$ probable error of A B in feet.

Knowing the bearing of A B, find the diff. lat. and diff. long. corresponding to this probable error, in seconds, as before.

This is the “probable error” of the length of A B (by triangulation) expressed in terms of diff. lat. and diff. long.

(b) *Error in bearing* can be estimated if true bearings have been observed at or near both A and B.

Let the true bearing observed at or near A, say, be worked through the triangulation until it can be compared with the true bearing observed at or near B by means of a common side.

The mean of the two will be the “Mean True Bearing” of that particular side, and should be adopted.

From this mean true bearing calculate the mercatorial bearing of A B (by triangulation).

Half the difference of the aforesaid true bearings will be the probable error of the mercatorial bearing of A B (by triangulation) expressed in arc.

Calculate the number of feet subtended by this angle at the distance A B in a direction at right angles to A B by the formula—

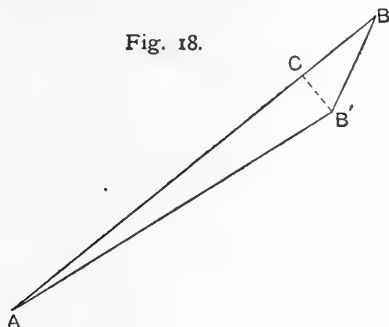
$$\text{Angle in seconds} = \frac{\text{feet subtended} \times 34}{\text{distance in miles}},$$

and thence the corresponding diff. lat. and diff. long. in seconds.

The “maximum probable error” in the triangulation will thus be the probable error due to distance as found in (a) + the

probable error due to bearing as found in (b), expressed in terms of seconds of diff. lat. and diff. long.

III. Now calculate the mercatorial bearing and distance of A B from the astronomical positions of A and B, not for-



getting to apply the spheroidal correction to the diff. long. by meridian distance before calculating.

We have already in II. (b) calculated the mean mercatorial bearing and distance of A B from the triangulation.

Project the two bearings and distances from a common point A (Fig. 18), terminating in B and B' respectively; then B B' represents the difference between the two positions of B with reference to A, as found astronomically and by triangulation.

From B' drop perpendicular B' C on A B.

Since B A B', the difference between the two mercatorial bearings, is a small angle, B C may be considered as the difference of the two determinations of A B, or A B - A B'.

B' C may be calculated from the formula—

$$\text{Angle in seconds} = \frac{\text{feet subtended} \times 34}{\text{distance in miles}}, \text{ or } B A B' = \frac{B' C \times 34}{A B}.$$

With B C and B' C in the right-angled triangle B C B' calculate bearing and distance B B', and find the corresponding diff. lat. and diff. long. between B and B'.

Divide each of these in the proportion of the "maximum probable error" of diff. lat. and diff. long. due to astronomical positions and triangulation respectively, and the result will be the amount required to apply to the astronomical positions and to the triangulation positions to make them agree.

EXAMPLES.

(1) *By Astronomical Observation.*

| | | | | |
|---|-------------------------------------|----|-----------------|---------------|
| Let A (by observation) | 26° 57' 51.6" N. | .. | .. | ± 0.8" |
| Let B (by observation) | 28° 02' 08.4" N. | .. | .. | ± 1.6" |
| Maximum probable error of diff. lat. | | .. | .. | <u>= 2.4"</u> |
| Mercatorial distance A B (by observation) | 1° 25' 54" | .. | | ± 5.0" |
| Maximum probable error of diff. long. | | .. | .. | = 5.0" |
| Mercatorial bearing and distance A B | $\frac{N.}{S.} 50^{\circ} 00' 00''$ | | $\frac{E.}{W.}$ | 100.00' |

(2) *By Triangulation.*

(a) A B calculated from base measured near A = 99.88'.

A B calculated from base measured near B = 99.72'.

Mean value of A B = 99.80'.

Probable error in A B ± 0.08'.

Or suppose no check base has been measured :

(i.) Probable error in the measured base owing to irregularities of ground, etc., is estimated at 1 foot per mile.

(ii.) Mean probable error in length of any side is estimated from the results obtained whilst working out the triangulation to be 3.82 feet per mile.

Thus :

$$(i.) + (ii.) = 1 + 3.8 = 4.8 \text{ feet per mile.}$$

$$4.8 \times A B = 4.8 \times 100 = 480 \text{ feet} = 0.08'.$$

Or probable error in A B = ± 0.08'.

With 0.08' distance, and bearing 50°—

Probable error of A B expressed in terms of diff. lat. = ± 3.1".

Probable error of A B expressed in terms of diff. long. = ± 4.1".

(b) The true bearing, as observed at or near A, when worked through the triangulation and compared with the true bearing observed at or near B, by means of a common side, is found to differ 3' 00".

The mean of these two true bearings is now adopted as the

"mean true bearing" of this common side; using this, the triangulation is reworked, giving

$$\text{Mercatorial bearing A B} = \frac{\text{N.}}{\text{S.}} 50^\circ 06' 00'' \frac{\text{E.}}{\text{W.}}$$

Probable error = half the difference of the bearings = $1' 30''$.

$$1' 30'' = \frac{\text{feet subtended} \times 34}{100}.$$

\therefore Feet subtended = 265 feet.

With distance 265, and bearing 50° , from Traverse table diff. lat. = $1.7''$ and diff. long. = $2.3''$, which is the probable error of mercatorial bearing A B expressed in terms of diff. lat. and diff. long.

Whence, from (a) and (b), maximum probable error in the triangulation, due to errors of both distance and bearing, expressed in terms of diff. lat. and diff. long.

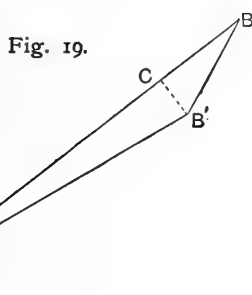
$$= \text{Diff. lat. } (3.1'' + 1.7'') = \pm 4.8''.$$

$$\text{and Diff. long. } (4.1'' + 2.3'') = \pm 6.4''.$$

$$\text{A B (from astronomical observation)} = \text{N. } 50^\circ 00' 00'' \text{ E. } 100.00'.$$

$$\text{A B (from triangulation)} = \text{N. } 50^\circ 06' 00'' \text{ E. } 99.80'.$$

$$\text{Difference} = \frac{6' 00''}{\underline{\underline{\hspace{1cm}}}} \quad \frac{0.20'}{\underline{\underline{\hspace{1cm}}}}.$$



$$\text{B A B}' = 6' 0'', \text{ B C} = 0.20'.$$

$$6' 0'' = \frac{\text{B}' \text{ C} \times 34}{100}.$$

Whence—

$$\begin{aligned} \text{B}' \text{ C} &= 1,059 \text{ feet, } \text{B C} = 1,212 \text{ feet.} \\ \text{C B B}' &= 41^\circ, \quad \text{B B}' = 1,610 \text{ feet.} \end{aligned}$$

With 1,610 as distance, and $(50^\circ - 41^\circ) = 9^\circ$ as "course,"
diff. lat. = $15.8''$, diff. long. = $2.8''$.

Now, since from—

(1) Maximum probable error (astronomically) = $2.4''$ diff. lat.,
 $5.0''$ diff. long., and from

(2) Maximum probable error (triangulation) = $4.8''$ diff. lat.,
 $6.4''$ diff. long.

we must divide diff. lat. $15.8''$ in proportion of $\frac{2.4}{5.0} = \frac{5.3}{10.5}$,

and diff. long. $2.8''$ in proportion of $\frac{5.0}{6.4} = \frac{1.3}{1.5}$

Then, since—

Astronomical diff. lat. = $1^\circ 04' 16.8''$, diff. long. = $1^\circ 25' 54''$

Correction = $-5.3''$ $-1.3''$

Accepted diff. lat. = $1^\circ 04' 10.5''$, diff. long. = $1^\circ 25' 52.7''$.

Or—

Triangulation diff. lat. = $1^\circ 04' 01.0''$, diff. long. = $1^\circ 25' 51.2''$.

Correction = $+10.5''$ $+1.5''$

Accepted diff. lat. = $1^\circ 04' 11.5''$, diff. long. = $1^\circ 25' 52.7''$.

These results, being identical, show that the working has been correct.

The latitude of A and B (by observation) must now be corrected for the total error of $8.1''$ in the diff. lat., in the proportion of their individual probable errors, viz., $0.8''$ and $1.6''$.

Thus:—

$$\text{Correction to lat. of A} = \frac{8}{24} \times 5.3 = 1.8''.$$

$$\text{Correction of lat. of B} = \frac{16}{24} \times 5.3 = 3.6''.$$

Whence:—

Lat. A (by obs.) = $26^\circ 57' 51.6''$ N., Lat. B = $28^\circ 02' 08.4''$ N.

Correction = $+1.8''$ $-3.6''$

Accepted Lat. A = $26^\circ 57' 53.4''$ N., Lat. B = $28^\circ 02' 04.8''$ N.

As regards the longitude we must accept that of either A or B, and apply the accepted diff. long., viz., $1^\circ 25' 52.7''$.

The diff. lat. and the diff. long. between A and B, both astro-

nominally and by triangulation, being now identically the same, we calculate the final accepted mercatorial bearing and distance $AB = \frac{N.}{S.} 50^{\circ} 01' 50'' \frac{E.}{W.} 99.929'.$

The difference between this bearing and the mercatorial bearing $\left(\frac{N.}{S.} 50^{\circ} 06' 00'' \frac{E.}{W.} \right)$ previously calculated from the triangulation, is $4' 10''.$

This is the correction to be applied to the mean true bearing of each side, applying it to the *left* in all cases since the accepted bearing is to the left of the previously calculated bearing.

The difference between the log. of the accepted distance 99.929' found above, and the log. of the distance 99.80' previously calculated from the triangulation, is the log. to be *added* to the log. of each side, since the accepted distance is greater than the calculated distance.

The foregoing example illustrates an extreme case. The difference in bearing between the results by triangulation and astronomical observations would seldom be so large as $6'$; and since the amount by which the observed astronomical positions require to be altered exceeds the probable error of the observations, it points to the existence of other errors due to deflection of plumb-line.

Under such circumstances it might be desirable to take this possibility into consideration by assigning larger probable errors to the astronomical positions, and dividing the diff. lat. and diff. long. in corresponding proportions, thus giving relatively greater weight to the result by triangulation if it is sufficiently reliable.

The effect of any such readjustments in connection with the bearings of the lines AB and AB' may be studied by plotting the triangle BCB' on a large scale.

In the memoir of the chart, the latitudes, meridian distance, and true bearings as actually observed, should be stated, together with the corrections that have been applied to them to obtain the finally accepted results. The number of feet per mile by which the distances by triangulation have been corrected to agree with the accepted positions should also be stated.

CHAPTER V

PLOTTING

Subject
com-
prised in
Chapter.

THIS chapter will comprise, besides a description of the method of placing the points on the paper, which is more generally understood by the term "plotting," an account of the different manners in which those points may be obtained, other than by a regular chain of triangles. This is, perhaps, more correctly a part of triangulation, and for some reasons should be described under that article, but it is thought that it will tend to clearness of comprehension, if it is taken in connection with the mode of laying down the points as obtained, as it is not easy to separate the two steps in many instances.

In discussing the general question of Plotting, therefore, we will first take the placing of the points of an ordinary triangulated survey on paper, and then consider some other systems to be adopted when regular triangulation fails us.

Great
Care
requisite
in Plot-
ting.

Plotting the points is a most important operation, and one requiring great care.

No matter on what scale or on what system a survey is being made, equal pains must be bestowed on plotting the points. Indeed, it may almost be said that in proportion as the elements of a survey approach to the least accurate form, viz., a sketch survey, so does the necessity for careful plotting increase, as the numerous checks, which in a detailed triangulation will instantly make any error in plotting apparent, will be more or less absent in proportion to the departure from such regular triangulation; and not only will the minor details of such a chart be inaccurate, which we expect, but the main and prominent points may be unnecessarily out of place unless care is bestowed on the plotting.

Before describing in detail the different methods in plotting, it is necessary to understand the system of laying down angles by chords, and why this is done. Plotting
by Cords.

It will easily be seen that, where lines are to be drawn of considerable length, a protractor whose radius will be much shorter than the desired line, can hardly give the angle exact enough to ensure the extremity of the line being precisely placed; for the straight-edge, perhaps 6 feet in length, by which the required line is to be drawn, will only be directed by two pricks in the paper, which, with the largest protractor, will not be more than 18 inches apart. However exactly the protractor has been placed, and the pricks made, the mere laying of the straight-edge so that the line drawn will pass *precisely* through the centre of the two pricks near together, is almost an impossibility, and an error, quite imperceptible at the pricks, will be very appreciable at the end of the straight-edge.

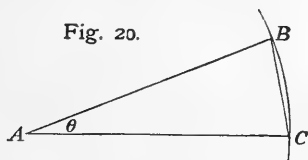
For this reason, we want our directing prick as far along the straight-edge as we can get it.

We accomplish this by using chords.

If two radii of a circle of given length of radius, containing between them a given angle θ , be drawn to cut the circumference of the circle, the chord to the arc of the circumference

thus cut off is $2 \text{ radius } \sin \frac{\theta}{2}$.*

Thus, by reversing this and describing from the centre A, Fig. 20, an arc of a circle of any radius, drawing the line A C, and measuring the chord C B (which will be done in practice



by describing a short arc of a circle with the required chord as radius, from the centre C), the point B, where the chord cuts the circumference (or the two arcs intersect) joined to A, will give the required angle θ .

* *Vide* proof of this rule in Appendix C.

**Table of
Chords**

A table of chords for a radius of 10 inches is given in Appendix,* which saves much time and chance of errors, as the chord to the angle required can be taken from the table, and multiplied by the radius with which it is meant to lay off the angle, divided by ten; but in case this is not at hand, we must calculate our own chords.

**Calculat-
ing
Chords.**

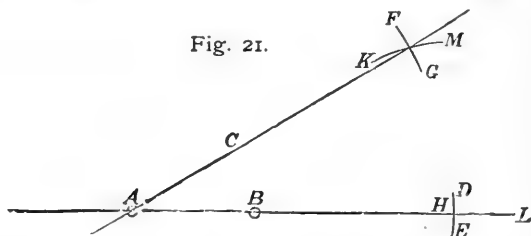
Tables of natural sines are not included in Inman's, the tables generally in use at sea, and logarithms of sines are in that work only given for every fifteen seconds, and we may want to take the angles out exactly. Moreover, by using the logsine, three logarithms will have to be taken out, and the process is somewhat longer. It is simpler, therefore, to use the table of natural versines, which are given in Inman to seconds.

As $\sin \frac{\theta}{2} = \text{versine } (90 + \frac{\theta}{2}) - 1$, our required chord will be
 $2 \text{ radius (vers. } (90 + \frac{\theta}{2}) - 1)$.

Versines are given for a radius of 1,000,000, so we have to divide the versine taken out by that number. This reduces the rule in practice to this.

Look out the natural versine of $90^\circ + \text{half the required angle}$, leaving out the left-hand figure 1, and putting a decimal

Fig. 21.



point before the remaining six figures. Multiply this number by twice the radius, and the result will be the chord required.

Example. Let us take now an example in practice.

At A, Fig. 21, the angle between B and C is $35^\circ 14' 30''$. The line A L from A passing through B is already drawn. We want to lay off this angle, and requiring accuracy, we take a long radius, *i.e.*, 45 inches.

* Appendix J.

Forty-five inches must be carefully measured, by the brass diagonal scale, on to a pair of beam compasses, with the two steel points shipped. Flattening the paper down by placing the straight-edge close to the line A B, and putting weights on it, with the centre A describe a short arc of circle D E, scratching lightly the surface of the paper. Then moving the straight-edge into the direction of C (which can be ascertained roughly by a protractor), and again weighting it, make another small scratch F G. With the assistance of a reading-glass, and by means of a needle mounted in a handle, and spoken of as the "Pricker," make a fine prick at the intersection of the lines A B, D E, *i.e.*, at H.

Look out the versine of $107^{\circ} 37' 15''$ ($90 +$ half the required angle), which is 1,302,717. This becomes .302717, which, multiplied by 90, gives 27.244 inches as the chord.

Measure this distance on the beam compass, and flattening the paper as before, draw, with H as a centre, a short arc K M crossing F G. The point of intersection is to be pricked carefully as before, and the straight-edge can now be laid on A and it, and the line ruled will be at exactly the angle required. This seems a tedious operation, but it is the only way in which points can be got to go down satisfactorily, and in the end much time will be saved.

It may be noted here, that it is preferable to make a mark with a steel point instead of a pencil, from the practical difficulty of measuring accurately the required distance on the beam compass when the pencil-point is used, as, when the pencil-point is cut sharp enough to make a fine line, it is almost impossible to prevent breakage in applying it to the brass scale divisions. It is also cleaner. In marking, the point must be held sloping, so as only to impress, and not actually to scratch the surface of the paper, which it will do if held perfectly upright.

Of course, if the paper is stretched on a board instead of being loose on the table, the time and trouble of seeing the paper flat is saved, but this is seldom used in our work.

If the table of chords is available, look out the chord for $37^{\circ} 14' 30''$ and multiply it by 4.5, as the table is made out for a radius of 10 inches. This will give the same quantity of 27.244 inches as found above.

Steel
Points to
be Used.

C may be of course anywhere on the line A B, and supposing ourselves to be plotting from an original base A B will probably be much nearer to A than to F G, but by taking such a long radius we get a straight line in the true direction of the angle laid off, and when we want to measure another angle on to another object, perhaps three times the distance of C from A, we have a long line we are certain of, to do it from.

Always
draw Long
Lines.

Here let it be impressed upon the surveyor that all lines drawn for plotting the main points, and indeed all points (except very minor ones, on which the position of nothing else will depend), must be drawn as long as possible, and with more or less long chords, if we desire correctness. If we have a line drawn between two stations which lie say 6 inches apart on the paper, and it only projects a few inches beyond each, and we hereafter require to lay off an angle from one, having the other as zero, to a station which will be say 2 feet or more distant, we cannot do it correctly, as this longer line will have to be directed by a prick which cannot be farther off than the length of the zero line; but by drawing long lines with long chords, we are ready for anything, and it will not matter whether the station we take for zero be near or far, as we use, not it, but the long line ruled through it.

Length of
Radius.

In no case should a line to a station be laid off with a protractor or chord whose radius is less than the distance of the station, excepting in a rough plan which we want to do rapidly, or in most parts of a running survey, where, pretensions to accuracy being thrown to the winds, we get points near enough for our purpose down with a protractor.

Zero
Lines.

It is important to remember in the selection of zero lines that the one should be preferred which makes the smallest angle with the line to be projected from it, provided the object selected for zero is farther off than the point to be plotted.

Lengthen-
ing a Line.

It is difficult to extend correctly a short line once drawn, by simply ruling on with the straight-edge. If a longer line is wanted, it is better to lay off the angle to it again from some other long line, with a sufficient radius.

Ruling a
Straight
Line.

To rule a true straight line which will pass exactly over the centre of the pricks is by no means an easy thing. The ruling pencil, which should be of the hardest lead manu-

factured, should be cut to an edge, not a point, and the straight-edge being placed in position, and weighted to keep it in contact with the paper throughout its length, the flat side of the pencil is placed against it, and tried at both points, to see whether the line will pass truly over them. Care must then be taken to hold the pencil in the same position while drawing the whole line.

In laying off by chords an angle over 60° , or a little under 60° , it will be found best to mark off 60° first, and measure the remainder of the angle from the 60° prick. This is done by drawing short arcs with the radius used, from the station from which it is desired to lay off the angle, and from the

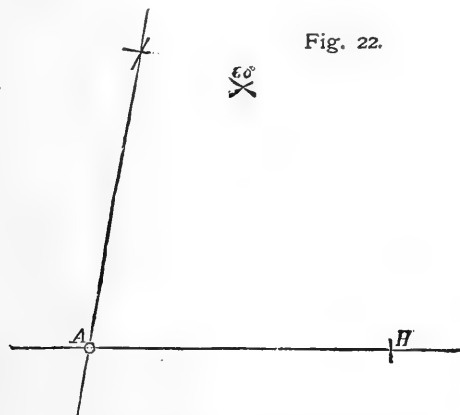


Fig. 22.

radius prick (H in last figure), the intersection of these must be pricked off as 60° , and another short arc being drawn with the originating station as centre, the chord of the difference of the angle from 60° is measured from the 60° prick to the last short arc, as in Fig. 22 (above).

This is done not from any incorrectness of the principle if the angle were laid off at once, but because it is inconvenient to be measuring long distances as chords, as there is a greater chance of some little inequality of the paper causing error, and also, the longer the chord measured, the more acute will be the angle between the two intersecting arcs, and consequently the greater the difficulty of pricking in accurately at the intersection.

Com-
mence-
ment of
Plotting.

Understanding then how to lay off angles by chords, and having obtained by calculation as long a side as we can for a plotting base line, so as to plot as much as possible *inwards*, or with decreasing distances, and not *outwards* to stations farther distant than the original two, and having settled whereabouts on the sheet this base line shall be placed, draw a meridian line, parallel with the side of the paper, and passing at one end of where the base is to be. Make a prick on this line for one end of the base, using, as always for pricking, a reading-glass, to ensure getting the prick exactly on the line. Let us call this A.

From A, lay off, with as long a chord as can be commanded, the true bearing of the base, and having ruled this line of bearing as long as possible, make another prick on it at the required distance from A, for the other end of the base. From the two base stations lay off angles to two other main positions, and choose the one of these where the intersection of the lines makes the nearest angle to 90° as the third station to prick in, doing so with great care on the intersection of the two lines. Then from this third station lay off an angle to the fourth, and if this, when ruled, passes exactly over the intersection of the two lines from the base stations, it can be pricked in. All four stations are correct, and the groundwork of the chart is laid; but if there is any little triangle visible with the reading-glass, all must be plotted over again, for unless these first four stations are exactly right, nothing will ever go right afterwards.

These four stations settled, proceed in like manner with other main stations; but now we shall of course have three intersecting lines for each station, and care must be taken that these lines do truly intersect, and no station must be pricked in that has not got three such converging lines through it.

The main stations down, smaller chords may be used for secondary theodolite stations, and the protractors will come in in plotting the marks and other minor points, the necessary angles for which we may suppose some of the party are getting, whilst the first main points are being carefully plotted.

As the chart fills, there will be many lines from which the angle to a new point can be measured, and it is well to

remember that as a standing rule the smallest angles both give less trouble and present least chance of error.

The ordinary way of marking the points is to ring a small circle of carmine round them. Larger circles can conveniently be used to distinguish the main stations.

Marking
"Points."

It will be found in the course of plotting that the paper will vary so much, expanding at one time and contracting at another, that the arcs of radii once measured and scratched on the paper, cannot be considered as so done once for all. If some hours have elapsed since marking any radius, it must be remeasured, to ascertain if it has altered.

Stretch-
ing of the
Paper.

In getting angles for plotting stations of all kinds, it must be remembered that two angles of a triangle will always give the third, and that as far as mere plotting goes, it is not necessary to waste time in observing the third angle. If the two observed angles have been got fairly accurately, the double error which will be thrown into the third angle deduced from them should not be enough to show in plotting, and if it does, it will soon make itself apparent by not intersecting. An angle from a fourth station will show which of the other three angles is wrong.

Calculat-
ing Third
Angle.

Thus if we have observed at a station C, which we want to plot, the angle between A and B, and also the angle at A between B and C, the angle at B which is wanted to draw a line to C can be calculated without the trouble of visiting B. It is indeed a blessed circumstance for the marine surveyor that the three angles of any triangle equal 180° .

It is a great assistance when plotting, to note in red ink in the "Main Angle Book," each "calculated angle" as it is obtained placing it under the heading of the particular \triangle from which the angle is calculated. The angle should be referred whenever possible to the same zero that has been used at that \triangle , and then it can be treated as if it had been actually observed. The \triangle through which the angle has been obtained being also stated in brackets, the origin of a calculated angle can be traced. Such angles must, of course, be used with great caution, but when an angle can be calculated through two or three different stations, and the results are in agreement when reduced to a common zero, it becomes assured within much smaller limits than would otherwise be possible.

Entering
"Calcu-
lated
Angles"
in Main
Angle
Book.

It is advisable to pursue this system even in the case of peaks that are not visited, if they are of importance in the survey, and are likely to be much used for purposes of "shooting back."

Double
Calcu-
lated
Angles.

Calculated angles being entered in this manner, they may be used in their turn to obtain a second calculated angle, with the further advantage of giving a choice of zeros from which to lay down the calculated angle, and thus to select a zero making a smaller angle for plotting than if obliged to use the original \triangle as a zero from which the angle was calculated.

A thorough grasp of the foregoing greatly facilitates plotting in certain cases, and provides resources to fall back upon when, from any cause, observed angles become scanty.

Station
Pointers
for Chart
Room Use.

A large pair of station pointers carefully tested for errors is very useful for laying down small angles (under 10°) for topography, etc. A single peak on a distant range of hills being fixed by chords, it can be used as a zero, and shots to the other peaks in the vicinity on the range can be laid down by the station pointer with great precision, for the reason that when the angles are small any slight error in placing the centre of the station pointer over the centre of the \triangle has a minimum effect.

Small
Angles de-
sirable for
Plotting.

The fact stated above should be borne in mind when plotting, since it follows that a small error in the actual plotted position of a \triangle shows itself but very slightly when the line to be laid off from that \triangle makes only a small angle with the zero used. Hence the desirability of choosing a zero for plotting which makes a small angle with the line to be laid off from any .

Plotting
by Dis-
tances.

If the triangulation has been calculated beforehand throughout, and the lengths of all the different sides have been found, it is more advantageous to begin plotting the main triangulation by distances rather than by chords. The main stations are thus got down in less time and with less trouble; but these are only a small proportion of the points to be plotted, and long lines must be ruled between the stations as zeros for plotting the other points by chords. In ruling these lines care must be taken to draw them exactly through the centre of the pricks denoting the stations; but however carefully drawn, there is liability to slight error in any line projected to a point lying beyond the distance of the stations between which the zero line is drawn. In plotting by distances, therefore, points

that will subsequently have to be plotted by chords should, if possible, lie well within the area covered by the main triangulation. After laying down the first three stations, three distances must be measured to plot each point on an intersection of the arcs cutting each other at a sufficiently broad angle; the plotting of the main stations once begun must be completed before distortion of the paper can occur from change of humidity of the atmosphere. Plotting, whether by distances or by chords, must be begun on as long a side as possible, so as to plot with decreasing distances.

IRREGULAR METHODS OF PLOTTING.

We have up to the present been considering the plotting of stations for a regularly triangulated survey. Let us now look at some other methods.

In surveys for the ordinary purposes of navigation, it occasionally happens that a regular system of triangulation cannot be carried out, and recourse must be had to a variety of devices; the judicious use of the ship in such cases is sometimes essential, and with proper care excellent results may be obtained. A few examples will best illustrate some of the methods used, but circumstances vary so much in every survey, that it is only possible to meet them properly by studying each case as it arises, and improvising methods.

In plotting the points of a chart which is being constructed on the principle of do-with-what-you-can-get, which is very often what has to be done in marine surveys, it is frequently found necessary to plot a position by its own angles, as, for instance, where the ship, anchored or moored off a low coast, has to be a main station, and only angles from aloft can be obtained to objects inland, such as hills, conspicuous trees, etc., already fixed.

A Position
by its own
Angles.

The use of "angles back," or "calculated angles," is one of the most ordinary expedients for fixing a station that has been shot up from another fixed point. A necessary condition is that the "receiving angle" at the position (A) to be plotted, between any two lines (direct or calculated), must be sufficiently broad to give a good cut; also that the points from which the

“angles back” are calculated should not be situated at too great distances from A, considered absolutely and relatively to the distance between A and the station shooting it up.

Use of
Tracing-
paper.

A station pointer, generally, has some small errors of centring, etc., that prevent it being used where exactness is required, and, moreover, only two angles can be laid off at a time by this instrument. In this case, then, it is better to plot all the angles obtainable on to tracing-paper, using chords for the purpose, and being very careful to make a very minute hole at the centre from which they radiate. If the objects are fairly well placed, a very exact position will be obtained, by laying this tracing on the sheet, and pricking through for the position. This will be much assisted if but one line can be got from a fixed station, as the angles can then be plotted on this line, supposing that in this case back angles cannot be calculated.

The line from the fixed station forms a good zero for laying off angles to other objects.

Only Two
Angles
available.

Again, it may sometimes be found necessary to carry on the main stations with a point plotted by only two angles; but if this happens, efforts must be made to check this, by getting an angle back from stations plotted on by means of this doubtful position, to some old well-fixed station, as a distant mountain; or if this is not to be had, a regular beginning must be made again by plotting two stations with two angles, pricking one, and then laying the angle from that to the fourth, as practised at the commencement of the chart, which will give a certain amount of check.

Moun-
tains In-
valuable.

A well-defined mountain, though miles inland and never visited by the surveyors, will often prove the very keystone of a chart that cannot be regularly and theoretically triangulated. When once well fixed, it will remain to get angles to, long after all the other first points of the survey have sunk below the horizon as the work progresses.

Use of
True
Bearings.

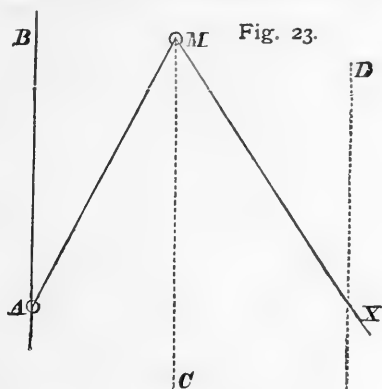
The bearings of this will often be useful, and these can be laid off from the mountain by applying the convergency.

Let us take an example, which will perhaps explain what is required easier by means of a diagram.

We hope that we have made it plain, by what has gone before, that if a distant object bears, say, N. $47^{\circ} 20'$ W., we

do not bear from such object S. $47^{\circ} 20'$ E., but so much less or more by the convergency; and that in all cases of fixing ourselves by means of true bearings observed from our own position, the amount of convergency, due to the bearing and distance of the object, must be calculated and applied to our bearing, before we can use it as a bearing from the object.

Here, Fig. 23, let B A be the original meridian drawn at the commencement of plotting through any station A. M is the distant mountain. At X our main points are falling short from some reason or another, and we are obliged to have recourse to a true bearing of M, which we accordingly obtain. Required to draw this true bearing from the fixed point M.



If we have the sheet graduated, it will not much simplify matters, as it is a great chance if a meridian passes close enough to M to use it without further correction; but let us suppose that we have no other meridian on the chart but A B. We must lay off the true bearing from M, with A as the zero, so we require the angle A M X. If M has been observed from A, whence we had a true bearing by which the meridian A B is directed, we have the bearing or angle B A M. If not, we must measure it from the sheet by reversing the chord method; drawing a line from A to M, and measuring the chord to the line A B at a given radius with beam compasses, and calculating the angle which corresponds to it, or B A M.

Now consider the figure again, M C, X D, being imaginary meridians to assist conception.

The bearing of A from M = bearing of M from A + the convergency, as M is nearer the pole than A, or

$C M A = B A M + \text{convergency for difference of departure of A and M.}$

In like manner :

$C M X = M X D$ (the observed bearing from X) + convergency for difference of departure of M X.

Adding, we have $C M A + C M X = B A M + M X D + \text{convergency for A X.}$

Or $A M X = \text{bearing M from A} + \text{bearing M from X} + \text{convergency for A X.}$

To get convergency in this case, we must assume a position for X, which we can roughly plot for the purpose, and measure the distance A X and bearing B A X.

We can then from this calculate the convergency required, knowing roughly the latitude of A, for

$\text{Convergency} = \text{distance} \times \text{Sin merc. bearing} \times \text{Tan Mid. lat.}$

Drawing
True
Meridian.

If M is likely to be used much in this way, it will be worth while to lay a meridian off through M by plotting the bearing A M C or B A M + the convergency for A M ; from which meridian subsequent bearings can then be laid off, duly corrected for convergency, for the distance between M and the station from which the bearing is observed.

Any line passing through a station, the true bearing of which can be calculated, will serve as a zero for laying down any other true bearing from the same station. This is generally preferable to actually drawing a meridian through the station.

Neglect of
Conver-
gency.

Of course, it will depend on the latitude how much error will be introduced by neglecting the convergency ; but when it is considered that in latitude 45° the convergency is equal to the departure, it will be seen that a large error will result by not applying it ; for in this latitude, supposing A and X are 30 miles apart, an error of half a degree would be made by drawing a meridian parallel to A B, and laying off the bearing observed at X from M.

As a rule, therefore, it can never be safely neglected except very near the Equator.

If it is intended to lay off the true bearing of an object from a station plotted on the chart, the convergency must likewise be borne in mind, and the meridian to be ruled through X

(in this case considered as fixed) from which to measure the bearing, must be, in transferring it from A B, corrected for the convergency due to the distance A X, by, after ruling a line through X parallel to A B, laying off at X, from the parallel just ruled, towards the pole, and on the side of A, an angle equal to the convergency required, which will give the direction of the true meridian.

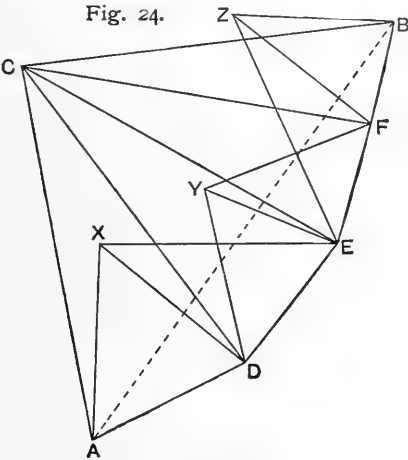
The system of true bearings may be used in many ways whilst carrying on an irregular triangulation. It is impossible to give instances of all the difficulties which may be surmounted by this means, but examples taken from actual practice will show the style of use to which true bearings may be put.

Further
Use of
True Bear-
ings.

EXAMPLES OF IRREGULAR PLOTTING.

EXAMPLE I.—*One inaccessible object visible from both terminal astronomical positions and from two or more intermediate stations.*

Fig. 24 is an illustration of irregular plotting. A and B are astronomical observation spots at the extremes of a



survey, from both of which the high, inaccessible peak C is visible. D, E, F constitute a chain of intermediate stations; A and D, D and E, E and F, F and B being respectively

visible from each other, and C is visible from all. Z is a peak visible from E, F, B. The peaks X and Y are visible from A, D, E, F. The latitudes of A and B and meridian distance between them being determined, and the true bearing of C being observed from both observation spots, angles are observed at all the stations.

(1) Calculating the spheroidal correction (from the formula $\text{correction} = \text{diff. long.} \times \frac{\cos^2 \text{mid. lat.}}{150}$) and adding it to the

true (or chronometric) difference of longitude between A and B, obtain the spherical diff. long. ; with this spherical diff. long. and the diff. lat. the mercatorial bearing and distance A B is found by middle latitude sailing. A B is the long side of the plot.

(2) Assuming A C = 10,000 units, calculate C D, C E, C F, C B in successive triangles.

(3) Calculate the convergences between A and C, and C and B, and apply them to the observed true bearings of C from A and B, to obtain the reversed true bearings. Compare the difference of these reversed true bearings with the sum of the calculated angles A C D, D C E, E C F, F C B ; they should agree.

(4) In $\triangle A C B$, given $\angle A C B$, and A C and B C in units, find $\angle^s C A B, C B A$.

We have now sufficient data to plot on the line A B.

The direct shots to X, Y, Z from the intermediate stations will afford the desired check upon the accuracy of the work.

If some of the stations between A and B are placed somewhat closely to one another, it may be desirable to rely on true bearings of C taken at the different \triangle^s instead of carrying on the original true bearing by means of the calculated angles.

It is very unlikely that the true bearing of B from A, as calculated through the triangulation, will agree with that calculated from their observed astronomical positions ; but this is a discrepancy which must always be expected and must be accepted.

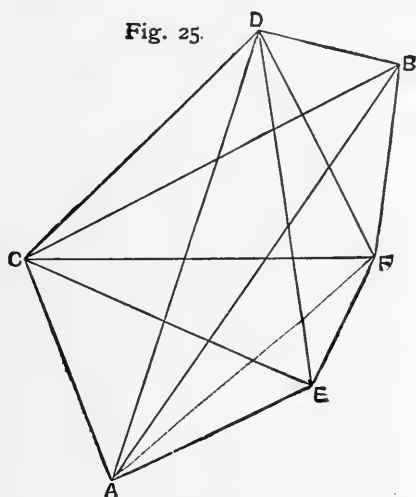
The final astronomical positions being adjusted on the principle already stated for harmonizing such discordances, the sheet is graduated on the finally accepted astronomical positions of A and B.

The point to be noted is that if the observation spots A and B are plotted on their observed astronomical positions, C laid down from its true bearings from A and B, and the intermediate stations D, E, F from the direct shots to them and their true bearings of C (laid down from C as reversed bearings with convergency applied), we shall get an incorrect plot, which will be at once evident on laying down lines to such a point as Z visible from E, F, and B.

The less the distance between A and B, the greater will be this discrepancy, and the more hopeless will be the situation, owing to the fact that probable errors in the astronomical positions are quantities peculiar to themselves, and are independent of distance.

EXAMPLE II.—*Two inaccessible objects visible from both terminal astronomical positions linked by a chain of intermediate stations visible from each other.*

In Fig. 25, A and B are two astronomical positions at the extremes of a survey invisible from each other. C and D



are two conspicuous inaccessible sharp peaks visible from A and B, and from the intermediate stations E and F, which latter are visible from each other and from A and B respectively.

The necessary angles and true bearings having been obtained at the different stations :

(1) From astronomical positions of A and B calculate mercatorial bearing and distance A B [see (1) in Example I.].

(2) Assume A E = 10,000 units.

(3) In $\triangle A E C$, find E C.

(4) In $\triangle E F C$, find E F.

(5) In $\triangle A F E$, given A E, E F, and $\angle F E A$, find $\angle^s A F E$, E A F, and A F.

(6) Knowing the observed angles C A E, D A E, D F E ;

(7) \therefore from (5) and (6) we have $\angle^s C A F$, D A F, D F A.

(8) In $\triangle A D F$, given $\angle^s D A F$, D F A, and A F, find D F.

(9) In $\triangle F B D$, given $\angle^s D F B$, D B F, and D F, find F B.

(10) In $\triangle A F B$, given $\angle A F B$, A F, and F B, find $\angle^s F A B$, F B A.

(11) From (7) and (10) given $\angle^s C A F$, F A B, we have $\angle C A B$.

(12) Given the observed $\angle C B F$ and $\angle F B A$ [from (10)], we have $\angle C B A$.

We have now sufficient data with which to plot on the distance A B, as found in (1).

If A and B lie in a meridional direction, a meridian distance between them is unnecessary, and the distance A B will be obtained from their observed diff. lat. and mercatorial bearing found in the triangulation. This applies to all examples.

EXAMPLE III.—*One inaccessible object visible from both astronomical positions, invisible from each other ; and an intermediate station visible from them, and from which the inaccessible object is also visible.*

With a single intermediate station it is obvious that the work is much simplified, and the chances of error will be correspondingly diminished.

In Fig. 26, A and B are two astronomical positions at the extremes of a survey, and invisible from each other.

C is a conspicuous inaccessible sharp peak, visible from A and B, and also from the intermediate station D, which is also visible from A and B.

The necessary angles and true bearings having been obtained at A, B and D,

(1) Calculate true bearing and distance of B from A and A from B [see (1) in Example I.].

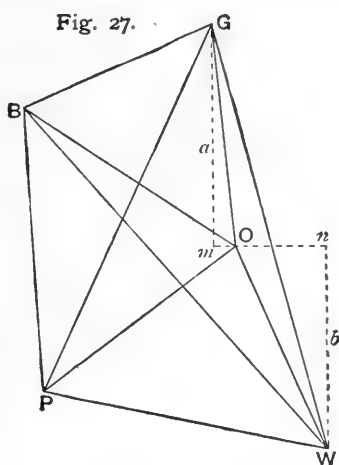
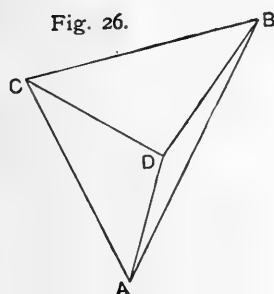
(2) Assume $AD = 10,000$ units.

(3) In $\triangle ADC$, given $\angle CAD$, $\angle ACD$, and AD , find DC .

(4) In $\triangle CDB$, given $\angle CDB$, $\angle DCB$, and DC , find DB .

(5) In $\triangle ADB$, given AD , DB , and $\angle ADB$, find $\angle DAB$, $\angle DBA$.

We have now sufficient data for plotting the points A, B, C and D.



EXAMPLE IV.—*Two inaccessible objects visible from three positions which are invisible from each other.*

In Fig. 27, G, O, W are three stations lying more or less in a meridional direction, but invisible from each other, the latitudes of which have been accurately determined, and from each of which true bearings of the high inaccessible peaks B and P have been observed.

Gm and Wn are true meridians passing through G and W respectively.

Let the observed diff. lat. G m , between G and O = a .

Let the observed diff. lat. W n , between W and O = b .

Let the observed true bearing of B from G, $\angle B G m = G$.

Let the observed angle $BGP = A$.

Let the observed true bearing of B from W, $\angle BWn = H$.

Let the observed angle $BWP = F$.

Let the difference of the reversed true bearings of B from O and G (convergence being applied to the direct bearings), $\angle GBO = B$.

Let the difference of the reversed true bearings of B from O and W (convergence being applied to the direct bearings), $\angle OBW = C$.

Let the difference of the reversed true bearings of P from O and G (convergence being applied to the direct bearings), $\angle GPO = D$.

Let the difference of the reversed true bearings of P from O and W (convergence being applied to the direct bearings), $\angle OPW = E$.

Let the unknown angle $BGO = \phi$

Let the unknown angle $BWO = \theta$

$$\begin{aligned} GO &= Gm \cdot \sec mGO & WO &= Wn \cdot \sec nWO \\ &= a \cdot \sec \overline{\phi - G} & &= b \cdot \sec \overline{H - \theta} \end{aligned}$$

$$\begin{aligned} (1) \therefore \frac{GO}{WO} &= \frac{a \cdot \sec \overline{\phi - G}}{b \cdot \sec \overline{H - \theta}} = \frac{a \cdot \cos \overline{H - \theta}}{b \cdot \cos \overline{\phi - G}} \\ \frac{GO}{OP} &= \frac{\sin D}{\sin \overline{\phi - A}} & \frac{WO}{OP} &= \frac{\sin F}{\sin \overline{\theta + F}} \end{aligned}$$

$$\begin{aligned} (2) \therefore \frac{GO}{WO} &= \frac{\sin D \cdot \sin \overline{\theta + F}}{\sin \overline{E} \cdot \sin \overline{\phi - A}} \\ \frac{GO}{OB} &= \frac{\sin B}{\sin \phi} & \frac{WO}{OB} &= \frac{\sin C}{\sin \theta} \end{aligned}$$

$$(3) \therefore \frac{GO}{WO} = \frac{\sin B \cdot \sin \theta}{\sin C \cdot \sin \phi}$$

$$\text{From (1) and (3)} \therefore \frac{a \cdot \cos \overline{H - \theta}}{b \cdot \cos \overline{\phi - G}} = \frac{\sin B \cdot \sin \theta}{\sin C \cdot \sin \phi}$$

$$\frac{a \cdot (\cos H \cdot \cos \theta + \sin H \cdot \sin \theta)}{b \cdot (\cos \phi \cdot \cos G + \sin \phi \cdot \sin G)} = \frac{\sin B \cdot \sin \theta}{\sin C \cdot \sin \phi}$$

$$(4) \therefore \frac{a \cdot \cos H}{\sin B} \cot \theta + \frac{a \cdot \sin H}{\sin B} = \frac{b \cdot \cos G}{\sin C} \cot \phi + \frac{b \cdot \sin G}{\sin C}.$$

Again, from (2) and (3)

$$\frac{\sin D \cdot \sin \overline{\theta + F}}{\sin E \cdot \sin \overline{\phi - A}} = \frac{\sin B \cdot \sin \theta}{\sin C \cdot \sin \phi}.$$

$$\frac{\sin D (\sin \theta \cdot \cos F + \cos \theta \cdot \sin F)}{\sin E (\sin \phi \cdot \cos A - \cos \phi \cdot \sin A)} = \frac{\sin B \cdot \sin \theta}{\sin C \cdot \sin \phi}.$$

$$(5) \therefore \frac{\sin D \cdot \cos F}{\sin B} + \frac{\sin D \cdot \sin F}{\sin B} \cdot \cot \theta =$$

$$\frac{\sin E \cdot \cos A}{\sin C} - \frac{\sin E \cdot \sin A}{\sin C} \cdot \cot \phi$$

$$\text{Let } \frac{a \cdot \cos H}{\sin B} = h$$

$$\frac{a \cdot \sin H}{\sin B} = l$$

$$\frac{b \cdot \cos G}{\sin C} = m$$

$$\frac{b \cdot \sin G}{\sin C} = n$$

$$\frac{\sin D \cdot \cos F}{\sin B} = o$$

$$\frac{\sin D \cdot \sin F}{\sin B} = p$$

$$\frac{\sin E \cdot \cos A}{\sin C} = q$$

$$\frac{\sin E \cdot \sin A}{\sin C} = r$$

Substituting in equations (4) and (5) respectively

$$\left. \begin{array}{l} (6) \ h \cdot \cot \theta + l = m \cdot \cot \phi + n \\ (7) \ o + p \cdot \cot \theta = q - r \cdot \cot \phi \end{array} \right\} \text{To obtain } \cot \theta \text{ and } \cot \phi.$$

$$(8) \cot \theta = \frac{h \cdot \cot \theta + l = m \cdot \cot \phi + n}{h} \left| \begin{array}{l} o + p \cdot \cot \theta = q - r \cdot \cot \phi \\ p (m \cdot \cot \phi + n - l) \\ o + \frac{p (m \cdot \cot \phi + n - l)}{h} = \\ q - r \cdot \cot \phi \\ o \cdot h + p \cdot m \cdot \cot \phi + n \cdot p - p \cdot l \\ = q h - r h \cdot \cot \phi \end{array} \right.$$

$$(9) \therefore \cot \phi = \frac{q h - o h - n p + p l}{p m + r h}.$$

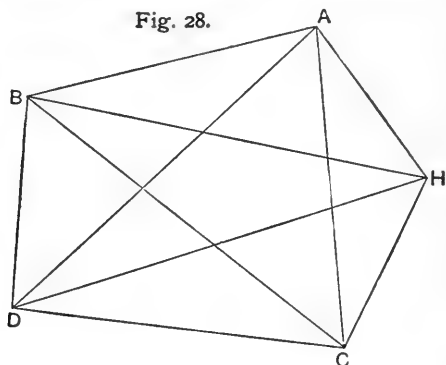
The angles ϕ and θ being obtained from the equations (9) and (8) respectively, we have the mercatorial bearings of O

from G and W, and the diff. lat., from which we can obtain the distances G O, O W, and thence the mercatorial bearing and distance G W.

EXAMPLE V.—*Two inaccessible peaks visible from three positions, the intermediate position being visible from the other two, which are invisible from each other.*

In Fig. 28, A, C are two stations at the extremes of a survey lying more or less in a meridional direction and invisible from each other.

Fig. 28.



H is an intermediate station visible from A and C.

B and D are two inaccessible peaks visible from A, H and C.

Angles are observed at A, H and C.

True bearings and latitudes at A and C.

$$\text{In } \triangle ABH, \quad BH = AH \cdot \sin BAH \cdot \operatorname{cosec} ABH$$

$$\text{In } \triangle CBH, \quad BH = HC \cdot \sin BCH \cdot \operatorname{cosec} HBC$$

$$\therefore \frac{AH}{HC} = \frac{\sin BCH \cdot \sin ABH}{\sin BAH \cdot \sin HBC} = M$$

$$\text{In } \triangle ACH, \quad \frac{AH}{HC} = \frac{\sin ACH}{\sin HAC} = M$$

$$\text{Now, } ACH + HAC = 180^\circ - AHC = a$$

$$\therefore ACH = a - HAC$$

$$\text{and } \sin ACH = \sin \overline{a - HAC}$$

$$\therefore \frac{\sin ACH}{\sin HAC} \cdot \sin HAC = \sin \overline{a - HAC}$$

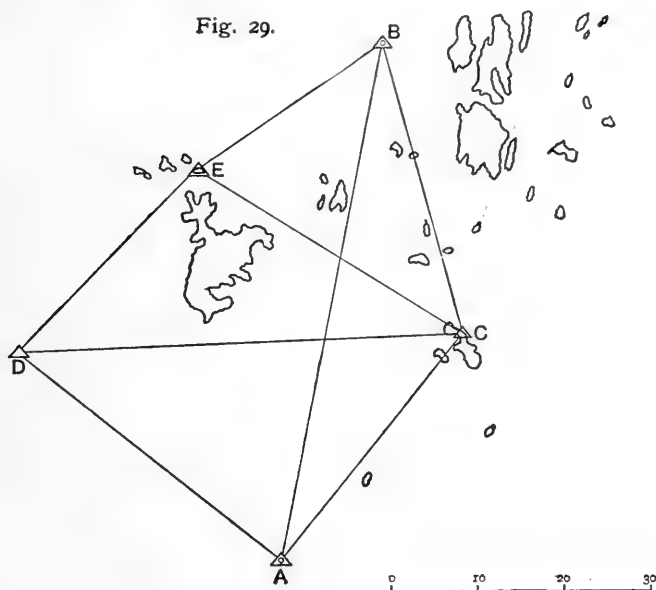
$$\text{or } M \cdot \sin H A C = \sin a \cdot \cos H A C - \cos a \cdot \sin H A C$$

$$\therefore M = \sin a \cdot \cot H A C - \cos a$$

$$\cot H A C = \frac{M + \cos a}{\sin a}.$$

Whence true bearing of $A C$ is found, and from the diff. lat. the distance $A C$ follows, together with all the sides and angles of the triangulation.

EXAMPLE VI.—The following sketch of the Anamba Islands (Fig. 29) is an illustration of an astronomical base derived from the diff. lat. between A and B , which are about 60 miles apart and invisible from each other. The angles in the triangles $B E C$, $E C D$, $D C A$ are all observed.



Assuming $D C = 10,000$ units, calculate in the triangulation the sides $C A$, $C B$.

In $\triangle A C B$, given $C A$, $C B$, and $\angle A C B$, find $\angle^s C A B$, $C B A$, and $A B$.

Apply these angles to the observed true bearings of C from

A and B respectively, and we have the mercatorial bearing of A B.

From the mercatorial bearing of A B and the diff. lat. calculate the distance A B. Having the length of A B in units and also in miles, we can turn the lengths of the other sides from units into miles, and proceed to plot the points on the side A B on the required scale, either by distances or by chords.

EXAMPLE VII.—Fig. 30 illustrates the case of a long stretch of coast-line of convex form, on which two stations, A and B, separated by a distance of many miles, have been fixed in the

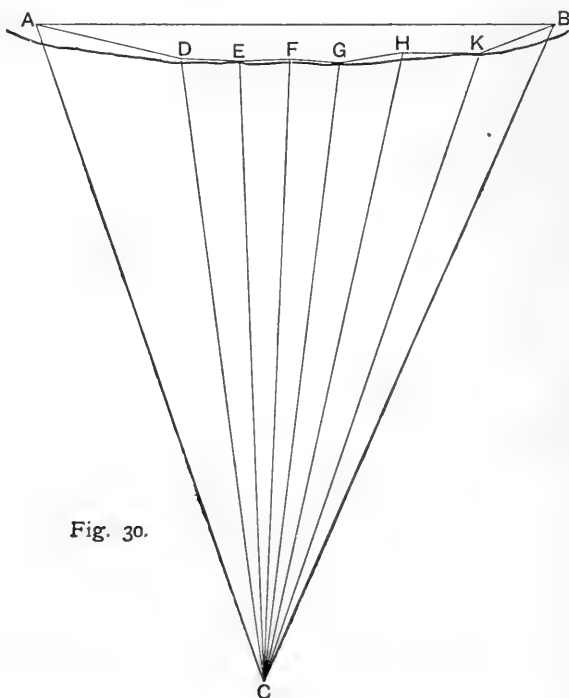


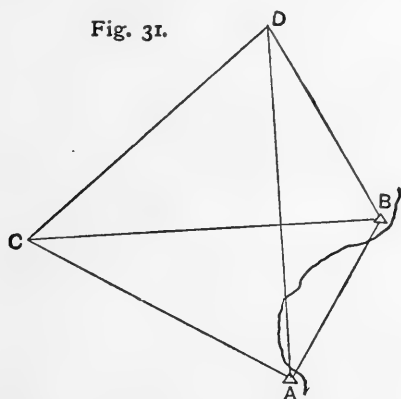
Fig. 30.

main triangulation. The curve taken by the coast, and the obstruction to view by wooded summits, prevent stations along the coast being seen from either A or B, except for a comparatively short distance. C is an inaccessible and very

distant mountain-peak on an island or across a strait fronting this portion of the coast, also fixed in the triangulation, and observed from A and B, which are visible from one another.

The intermediate stations D, E, F, G, H, and K, which are so placed as to be visible from each other, may be fixed on the side-shots and the calculated angles at C; but, being very close to one another as compared with the distance of C, the calculated angles will be liable to error unless great care be observed, and the errors, moreover, will be cumulative. In such a case true bearings of C from the intermediate stations are preferable to calculated angles, and are laid down as reversed bearings from C with the convergency applied. If C is at such a distance from the coast-line in question that it cannot be plotted on the sheet, the several distances A D, D E, E F, etc., can be calculated and laid down on the side-shots.

EXAMPLE VIII.—*Given two inaccessible and comparatively distant objects, the distance between which is known either from the chart or from some previous triangulation, the long side of a small plan may be obtained from them as follows, with a degree of accuracy proportionate to the distance and direction in which the distant objects lie relatively to the direction of the long side of the plan.*



In Fig. 31, A and B are theodolite stations at the extremes of a small plan. C and D are two inaccessible points the distance between which is known. Angles being observed at A and B,

Assume $AB = 10,000$ units.

In $\triangle DAB$, calculate AD .

In $\triangle CBA$, calculate AC .

In $\triangle DCA$, calculate DC .

Knowing DC in units and also in feet, and AB in units, find AB in feet by proportion.

EXAMPLE IX.—*In this case it was desired to correct the published chart of Bahia Blanca, and to plot points on the same scale.*

In Fig. 32, A and B were two well-defined summits about 20 miles apart, not visible from each other, being the only two points on the chart that were recognisable. It was inconvenient

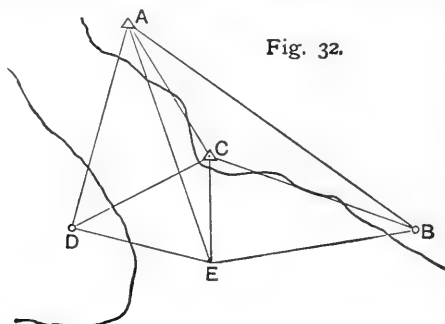


Fig. 32.

to visit B . D was a conspicuous tree. C is a station on a coast-hill, and E a station on a sandbank, from both of which stations all points were visible. At A angles to D , E , and C were observed, and all angles observed at C and E .

Assuming $AD = 10,000$ units,

In $\triangle ADC$, DEC , CEB , calculate AC , CB .

In $\triangle ACB$, given CA , CB , and $\angle ACB$; find $\angle CAB$, $\angle CBA$.

All the angles now being known, the points may be plotted on the side AB taken from the published chart.

ILLUSTRATING THE USE OF THE SHIP FOR TRIANGULATION.

EXAMPLE X.—Given the angles and lengths of the sides of a well-executed trigonometrical survey, with stations near the coast from 20 to 25 miles apart, successively visible from each other, intermediate points may be fixed with all the precision necessary for a coast survey, by means of angles from the ship tautly moored in positions where the receiving angle from two of the trigonometrical stations is not greater than 110° . In Fig. 33, A and B being two trigonometrical stations

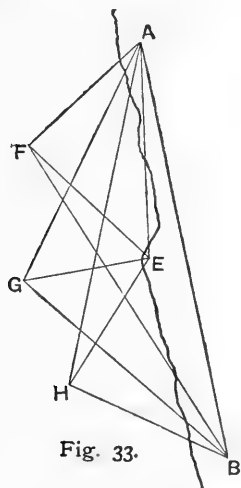


Fig. 33.

near the coast, the ship is shot up in three suitably selected positions, F, G, H, by observers at the two main stations. Angles are in each case observed simultaneously at the ship to an intermediate station E on the coast-line, which, if possible, is so placed as to see one of the two main stations. This intermediate station may then be used in conjunction with either of the two main stations for shooting up the ship in such positions as are necessary to fix coast-marks for sounding, etc.

The intermediate station is desirable on account of the distance at which the ship must necessarily place herself from one or other of the main stations in order to fix the coast-marks.

All complete triangles in which the ship is involved should be tested before plotting.

This use of the ship is permissible because the points fixed by means of her angles lie between regularly triangulated stations, and thus cumulative errors are avoided.

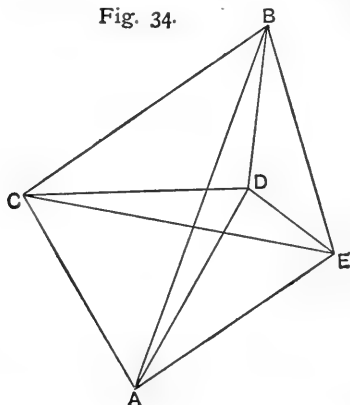
EXAMPLE XI.—*An inaccessible object visible from both terminal astronomical positions, which are invisible from each other, with an intermediate station visible from one of them, and from which the inaccessible object is also visible. From the ship at anchor midway between the astronomical positions all points are visible.*

In Fig. 34, A and B are the terminal astronomical positions from which the inaccessible object C is visible.

D is an intermediate coast-station from which B and C are visible.

From the ship tautly moored without the swivel at E, all points are visible.

Fig. 34.



Observers being at the masthead and provided with Galton's sun-signals for flashing, if necessary, other observers with theodolites and heliostats are stationed at A, D, and B.

At a preconcerted signal (the dipping of a flag, letting fall square sails, or at a given moment by comparison of watches beforehand) simultaneous angles to the ship are observed at A, B and D, and the observers at the mast-head take angles to all the points.

Repeat the operation three or more times as a check, and test each of the triangles ADE , BDE when the observers return to the ship. True bearings of C are obtained from A and B .

Assume $BC = 10,000$ units.

- (1) In $\triangle BCD$, find CD .
- (2) In $\triangle BCE$, find CE , EB .
- (3) In $\triangle ACE$, find CA , EA .
- (4) In $\triangle ACD$, find CA .

Compare values of CA as found in (3) and (4) as a check.

(5) In $\triangle AEB$, given EA , EB , AEB , find $\angle^s BAE$, ABE , and AB .

(6) Knowing true bearing of C from A , apply $\angle^s CAE$ and BAE , and obtain true bearing of B from A .

(7) Knowing true bearing of C from B , apply $\angle^s CBE$ and ABE , and obtain true bearing of A from B .

(8) The mean of (6) and (7) is mercatorial bearing of AB , as found through E .

(9) Apply convergencies to true bearings of C from A and B respectively, and find the reversed true bearings.

$\angle ACB =$ difference of these reversed true bearings.

(10) In $\triangle ACB$, given CA , CB , $\angle ACB$, find $\angle^s CAB$, CBA , and AB .

(11) To the true bearing of C from A , apply $\angle CAB$, and obtain true bearing of B from A .

(12) To the true bearing of C from B , apply $\angle CBA$, and obtain true bearing of A from B .

(13) The mean of (11) and (12) is mercatorial bearing of AB , as found through C .

(14) Compare results of (8) and (13), and adopt the mean.

(15) Compare the values of AB as found in (5) and (10).

(16) Calculate mercatorial bearing and distance AB from the observed astronomical positions.

The mercatorial bearing thus found will almost certainly differ from that found by triangulation in (14); but if the angles have been carefully observed, the result by triangulation should be preferred. Plotting on the line AB , using the angles found in the calculation, the accuracy of the work will be proved by the intersection of the points. This could not be

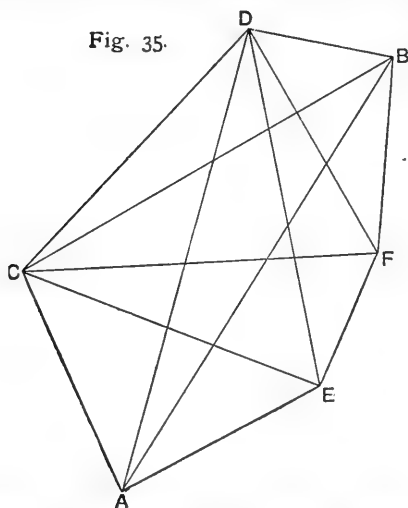
attained by using the true bearing resulting from the astronomical positions.

Intermediate points may be shot in from other ship's positions fixed on theodolite lines from A and D or from D and B, with a third shot back from C derived from the calculated angle through either station.

The observed astronomical positions should be adjusted to the results by triangulation in respect to bearing.

EXAMPLE XII.—*Two inaccessible objects visible from both astronomical positions, invisible from each other, and at such distance apart that the ship cannot see both from a single position.*

In Fig. 35, A, B are the two terminal astronomical positions; C, D, two inaccessible objects visible from A and B; E, F, ship's positions visible from A and B respectively. The ship being



shot up in her two positions, and angles at the ship being obtained simultaneously between the shooting-up station and C and D, we have sufficient data for calculation from which the main points may be subsequently plotted.

Assume $BF = 10,000$ units.

- (1) In $\triangle BFD$, find DB , FD .
- (2) In $\triangle BFC$, find CB , FC .

(3) In $\triangle DBC$, given DB , CB , and $\angle DBC$, find $\angle^s BDC$, BCD , and DC .

(4) In $\triangle DFC$, given FD , FC , and $\angle DFC$, find $\angle^s FDC$, BCD , and DC .

(5) $\angle BDC = BDF + FDC$.

Compare BCD as found in (3) and take the mean.

(6) $\angle BCD = FCD - FCB$.

Compare BCD as found in (3) and take the mean. Apply convergencies to the true bearings of C and D from A and B respectively.

(7) $\angle ADB$ = difference between the reversed bearings of D from A and B .

(8) $\angle ACB$ = difference between the reversed bearings of C from A and B .

(9) $\angle EDF = ADB - (ADE + BDF)$.

(10) $\angle ECF = ACB - (ACE + BCF)$.

$\angle FDC = BDC - BDF$. Compare also with (4).

$\angle FCD = BCD + BCF$. Compare also with (4).

$\angle DFC$ has been observed direct.

Test the triangle DFC .

(11) In $\triangle DFC$, given FD and all the angles, find CF , DC .

Compare the values of DC as found in (3), (4), and (11), and take the mean.

Compare the values of CF as found in (2) and (11).

$\angle ACD = ACE + ECF + FCD$.

$\angle ADC = BDC - ADB$.

$\angle CAD$ has been observed direct.

Test the triangle ADC .

(12) In $\triangle ADC$, given DC and all the angles, find AC , AD .

(13) In $\triangle ACE$, find AE , CE .

(14) In $\triangle ACB$, given AC , CB , and $\angle ACB$, find $\angle^s CAB$, CBA , and AB .

(15) In $\triangle ADB$, given AD , DB , and $\angle ADB$, find $\angle^s DAB$, DBA , and AB .

(16) Compare the values of AB as found in (14) and (15), and take the mean.

(17) Apply $\angle CAB$ to the true bearing of C from A , and obtain the true bearing of B from A .

(18) Apply $\angle CBA$ to the true bearing of C from B , and obtain the true bearing of A from B .

(19) The mean of (17) and (18) gives mercatorial bearing of A B as found through C.

(20) Apply $\angle D A B$ to the true bearing of D from A, and obtain the true bearing of B from A.

(21) Apply $\angle D B A$ to the true bearing of D from B, and obtain the true bearing of A from B.

(22) The mean of (20) and (21) gives mercatorial bearing of A B as found through D.

(23) Compare the result of (19) and (22), and take the mean.

From the astronomical positions of A and B calculate mercatorial bearing and distance A B.

The mercatorial bearing thus found will almost certainly differ from that found by triangulation in (23) ; but if the angles have been carefully observed and the inaccessible objects C, D are favourably situated, both with respect to each other and to A and B, the result by triangulation should be preferred, and plotting on the line A B, using the angles found in the calculation, the accuracy of the work will be proved by the intersection of the points. Intermediate points may be shot in from the ship's positions.

The observed astronomical positions should be adjusted to the results by triangulation so far as bearing is concerned. The necessity for so doing in this and in the preceding example is greatest when A and B are nearly north and south of each other. It diminishes as their bearing approaches an east and west direction, on account of the probability of their difference of latitude being less liable to error than their meridian distance. When nearly north and south of each other, it is unnecessary to observe the meridian distance.

EXAMPLE XIII.—To find the proper position in which to place the ship in order to obtain the most favourable geometrical conditions for fixing a point on shore from two other known stations on the same shore to the right and left, or on each side of it, as the case may be.

Reference may advantageously be made to Shortland's "Nautical Surveying," p. 188, on which will be found the proof of the following proposition :

In Fig. 36 A and B are two fixed stations on the coast. It

is required to fix another station, C, further along the coast, no inshore points being available to carry on the triangulation, and C being visible from B.

To find D, the *best* position for placing the ship, D B should roughly bisect angle A B C; and $A D C = 180^\circ - \frac{A B C}{2}$, or $180^\circ - A B D$.

Observers being stationed at A, B, C, the observer at B puts up transit marks, if necessary, to give the ship the direction of B D.

The bearing of A B and D B being known, angle A B D is the difference of those bearings, and the position on the line D B can be picked up by the ship.

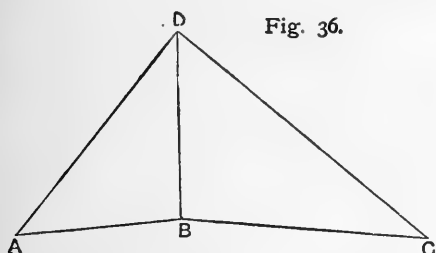


Fig. 36.

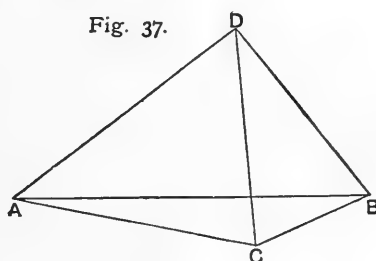


Fig. 37.

The ship hoists flag and dips in the usual manner when simultaneous observations are made at A, B, and C, and repeated as often as necessary. B C is calculated from A B through D B, using the calculated angles at D.

In the case of the station C, which it is required to fix, being situated between the fixed stations A and B, as in Fig. 37, the ship should be placed at D, so that D C bisects A C B and $A D B = 90^\circ$.

EXAMPLE XIV.—*Illustrating the triangulation of a low wooded coastline destitute of natural features, and of a general convex form, by means of the ship and one floating beacon.*

In Fig. 38, A, B, C, D are \triangle^s on the coastline, the general convex form of which prevents their being visible from each other, except in the case of A and B.

The proper positions for the floating beacon and the ship require careful consideration to obtain the best results. E is a floating beacon, with bamboo 30 feet high and large black flag, well moored and placed off the coast in such a position as to satisfy as nearly as possible the following conditions :

- (a) As near the coast as possible consistently with being visible from C.
- (b) Within visual range of C (7 miles is usually a safe distance).
- (c) In such a direction from B as to form approximately an isosceles triangle whose apex is the first position of the ship at F ; bisecting the angle A B F will be a guide to this direction.
- (d) At as great a distance from B as is consistent with obtaining a sufficiently large receiving angle between A and B.

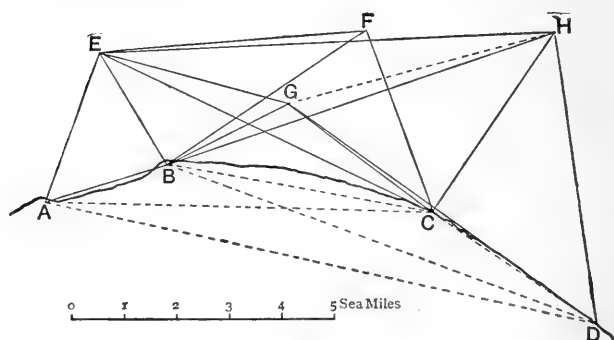


Fig. 38.

F is the first position of the ship, moored taut without the swivel, and taken up with a view to making each of the receiving angles B F E, E C F sufficiently large ; neither should be less than 30° . $\angle E C F$ is roughly known by the difference in the bearings of C from E and F.

G is the second position of the ship, situated as near the coast as possible consistently with both C and D being distinctly visible, and the receiving angle B G C not greater than 120° or 130° , whilst keeping the receiving angle E G B as large as possible.

H is the third position of the ship, in a direction from C

bisecting the angle $G C D$, and at such a distance from C as to make the receiving angles $E H C$, $G D H$ not less than 30° or 40° , taking care to keep within easy visual range of E . The angle $G D H$ is roughly known by the difference in the bearings of D from G and H .

It will materially assist the ship in picking up her proper positions, if she first steams along the coast and cuts the marks in by means of a running survey by patent log and compass. Having laid down the floating beacon E , a base may be measured either by sound or range-finder between E and B , or by chaining and traversing between A and B ; or in the case of an extended survey by this method, the scale may be eventually determined from the astronomical observations at either end of the survey, in which case it is not necessary to measure a base.

Observers with theodolites and heliostats being landed at A , B , C , and D , the ship takes up her positions successively at F , G , and H . At preconcerted signals, the ship is shot up simultaneously by each observer at each successive position, the angle to the beacon being also noted on each occasion. The observers in the ship at F take the angles $E F B$, $B F C$, and at G the angles $B G D$, $B G C$, $E G B$ are observed in the order named, for reasons that are stated on p. 168; at H , the angles $B H D$, $C H D$, $E H B$ are also observed in the order named.

The ship has thus never more than three angles to observe at each position, and they should be taken as quickly as possible to avoid any error creeping in through a slight movement of the ship between the observations. Two observers will materially diminish any risk of this sort.

The object of observing the floating beacon on each occasion from the shore stations, is to detect any slight movement from the position it occupied at the moment of observing at F . The theodolite lines to E , from A and B , effectively control its position. From the difference of the angles to it at those stations at the instant of observing F and G respectively, the false station which E occupies when the angles at F and G are observed can be calculated on the principles stated on p. 92, and a correction for it can be applied to the angles $E F B$, $E G B$ as actually observed, to reduce those angles to what they would have been if the beacon had not moved. A cor-

rection found in a similar manner should be applied to the angle $E H B$.

With these precautions considerable accuracy should be attained.

In order to ensure that no mistakes have been made in the angles, it is advisable to repeat the observations three times at each position.

Calcula-
tion.

Having corrected and tabulated each set of angles taken for each position, three separate values of the final results for plotting A , B , C , and D can be deduced, and the mean adopted.

In $\triangle A B E$, given $A B$ or $B E$ as a base, find $E B$.

In $\triangle E B F$, find $E F$, $B F$.

In $\triangle E F C$, find $E C$, $F C$.

In $\triangle B C F$, given $B F$, $F C$, and $\angle B F C$, find $B C$, $\angle^s F B C$, $B C F$.

In $\triangle E B G$, find $G E$, $G B$. } Compare $G B$ found in
In $\triangle B G C$, find $G C$, $G B$. } different triangles.

At B , the angle to C being known, and at C the angle to B also being known, we can test the angles in the triangle $B H C$, which affords a good check; similarly, $\triangle B G C$ can be tested.

In $\triangle E G C$, given $\angle^s E G C$, $E C G$, and $E C$, find $G C$.

Compare $G C$ with the value found in $\triangle B G C$.

In $\triangle E H C$, given $E C$, $\angle^s E C H$, $E H C$, find $C H$, $E H$.

In $\triangle E H B$, given $E H$, $\angle^s E H B$, $E B H$, find $B H$.

In $\triangle B H C$, given $B C$ and the angles, find $B H$, $C H$.

Compare values of $B H$, $C H$ found previously.

In $\triangle G C H$, given $C G$, $C H$, and $\angle G C H$, find $G H$, $\angle^s C G H$, $C H G$.

We have now the means of testing $\triangle D G H$, having all its angles.

In $\triangle D G H$, given $G H$, and the angles, find $H D$, $G D$.

In $\triangle H C D$, given $C H$, $H D$, and $\angle C H D$, find $C D$ and $\angle^s H C D$, $H D C$.

We can now test $\triangle G C D$, having all its angles.

In $\triangle G C D$, given $G D$, $C G$, and all its angles, find $C D$. Compare the value of $C D$ with that found previously.

In $\triangle B C D$, given $B C$, $C D$, and $\angle B C D$, find $B D$ and $\angle^s C B D$, $C D B$.

In $\triangle A B D$, given $A B$, $B D$, and $\angle A B D$, find $A D$ and $\angle^s B A D$, $B D A$.

We have thus found all the sides and angles in the quadrilateral $A B C D$, which can be plotted on any required scale. Unless subsidiary marks have been shot in from the ship's stations, there is no occasion to plot the latter.

If necessary, the survey may be extended along the coast in either direction on similar principles.

The foregoing method was used with success on the East Coast of Africa, and on that occasion no floating beacon was used. In such a case it will be seen that, the first position of the ship being at E , the second position should be somewhere intermediate between E and F . It must be taken up with a view to obtaining a sufficiently large receiving angle between A and B , and at the same time to give a somewhat larger receiving angle at C between the lines to it from the two positions of the ship. The receiving angle at C can always be known roughly by the difference of its bearing from the two positions of the ship.

We are now necessarily dealing with receiving angles much smaller than we should wish to use, and the case is merely mentioned as an exercise of ingenuity when hard pressed. It will, of course, be noted that the conditions of the problem materially improve if the trend of the coast takes a more favourable direction than that assumed in the example, enabling the shore stations to see each other.

EXAMPLE XV.—*Triangulation of a low wooded coast destitute of natural features, on which landing is practicable, using the ship and floating beacons.*

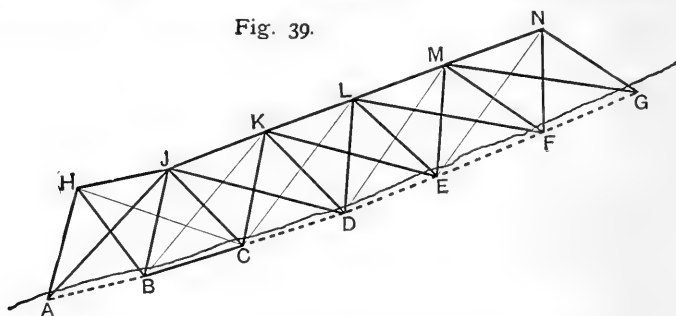
If the stations on the coastline are visible from one another, the triangulation is simple enough; but if, as frequently

happens in practice, the curve of the coast is such as to prevent one station from seeing the next, except occasionally here and there, in order to avoid placing stations unnecessarily close to each other, it may sometimes become necessary to resort to the principles adopted in the previous example.

It is proposed to illustrate such a case as follows :

In Fig. 39, stations are erected along the coast about 5 miles apart at A, B, C, D, etc., in such positions that at least two of the stations are visible from each other. Four of these stations are occupied at one time by observers with theodolites

Fig. 39.



and heliostats. Beacons are laid down at H, J, K, L, etc., forming approximately equilateral triangles with the shore stations. The ship is then anchored as near each beacon in succession as may be convenient without fouling it, and without obstructing the view of the beacon from the two shore stations adjacent to it.

At each position of the ship, at a preconcerted signal, the observers at the mast-head are shot up by each of the observers on shore, who immediately afterwards note the angles to the beacons. Simultaneous angles are observed at the ship between the shooting-up stations and the adjacent beacons.

As the ship moves from one position to another, each observer on shore moves on to the next station, where he should find a note from the officer who has vacated it, stating the zero he has used.

The theodolite lines to the beacons effectively control any small movement due to their being floating objects, and give the means of referring their positions at the moment of making each ship's position to those originally occupied by them when

the first station was made. For this purpose a rough plot may be made to get the approximate distances, from which, as explained on p. 92, the "false station" can be calculated and the necessary corrections applied to the angles obtained at the ship to the beacons at each of her positions.

Similarly, the ship in each position may be referred to the original spot that the beacon occupied, and the necessary correction applied to each of her angles.

Thus the ship's angles to the beacons will receive a double correction—(1) that due to small movement of the beacons; (2) that due to the angles at the ship not having been observed at the original positions of the beacons.

The ship's angles between the shore stations will require correction for the latter cause only.

The more frequently pairs of stations along the coast are visible from each other, the less troublesome and more correct the calculation becomes.

True bearings observed from one coast station to another on those occasions when they are visible, compared with the true bearings of those stations worked up from former true bearings through the calculation, will keep a satisfactory check upon the work.

Having corrected the angles for "false station" and tested Calculation. all the complete triangles, the calculation may be proceeded with.

Assuming the stations B and C to be intervisible, and the side $BC = 10,000$ units,

In $\triangle BJC$, find BJ , CJ .

In $\triangle HJA$, find HA .

In $\triangle HBJ$, find HBJ , HJ .

In $\triangle HAB$, given HA , HB , and $\angle AHB$, find $\angle^s HAB$, HBA , and AB .

In $\triangle JCK$, find JK , CK .

In $\triangle JKD$, find KD .

In $\triangle CKD$, given CK , KD , $\angle CKD$, find $\angle^s KCD$, KDC , and CD .

In $\triangle DKL$, find KL , DL .

In $\triangle KLE$, find LE .

In $\triangle DLE$, given LD , LE , $\angle DLE$, find $\angle^s LDE$, LED , and DE .

In $\triangle LEM$, find LM , ME .

In $\triangle L M F$, find $M F$.

In $\triangle E M F$, given $M E$, $M F$, $\angle E M F$, find $\angle^s M E F$, $M F E$, and $E F$.

In $\triangle M F N$, find $M N$, $N F$.

In $\triangle M N G$, find $N G$.

In $\triangle F N G$, given $N F$, $N G$, $\angle F N G$, find $\angle^s N F G$, $N G F$, and $F G$.

Having thus obtained the true bearing and distance in units of each station from the next, from one end of the survey to the other, and having calculated the true bearing and distance of the terminal stations from their observed astronomical positions, the distances in units can be turned into miles and all the stations plotted on the required scale. Intermediate points for sounding, etc., may be shot in from the ship's stations directly after the main angles are observed, or they may be fixed subsequently by means of the ship when the main points are plotted.

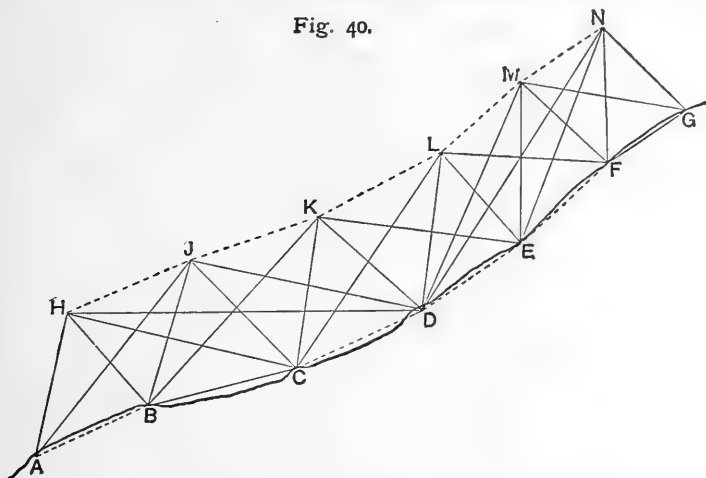
It will be noted that the accuracy of the work depends on the size of the receiving angle at each shore station between the ship and the next beacon in advance of her. This angle should in all cases be about 30° . In Fig. 39, the thick lines show the sides entering into the calculation, and consequently indicate those angles that are most important; the thin lines show the shots that are taken as checks, and the dotted lines connect stations that are assumed to be invisible from each other.

EXAMPLE XVI.—*Triangulation of a coast off which anchorage is impracticable, but on which landing can be effected.*

A rapid triangulation of a coast can be carried out by means of the ship under way with three observers in the foretop, and four observers with theodolites on shore at stations on the coast 3 or 4 miles apart. The shore stations may or may not in all cases be visible from each other; but it is assumed that occasionally at least the trend of the coast is such as to permit of two stations being in sight from one another, and the more frequently this occurs the more accurate is the work likely to be.

In Fig. 40, A, B, C, D, E, F, G are stations along the coast, of which B and C are visible from each other. H, J, K, L, M, N are ship's stations, at each of which simultaneous angles are observed between the four theodolite stations nearest to the ship from which she is at that moment shot up. The essential conditions are that the two first and the two last ship stations shall include observations to and from the first four and the last four theodolite stations, and that proper care and judgment are exercised in placing the ship so as to secure the best

Fig. 40.



results, as explained in Example XIV. The method of calculation is precisely the same as in that example, the principle being the same in both cases.

Having obtained by calculation the lengths of the sides, and the angles connecting the shore stations, they may be plotted without further calculation on an assumed scale which may be verified later, when the astronomical positions of the extremes of the survey are obtained.

As the ship moves from one position to another, the officer in the leading boat selects his next station, and occupies it while the remainder close up in succession to the vacated stations in readiness for shooting up the ship at the next position. On vacating a station, the officer should leave a note for the information of his successor, giving a description of the zero he has used.

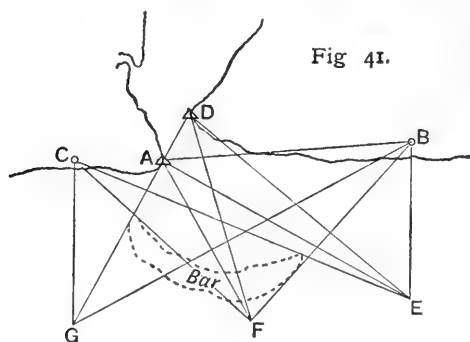
EXAMPLE XVII.—*Triangulation of a low wooded coastline by means of the ship and floating beacons, landing being impracticable.*

Detailed description of this is unnecessary ; it need only be remarked that the beacons should be laid down so as to form equilateral triangles, and that very satisfactory results can be obtained. Long stretches of the Liberian seaboard on the West Coast of Africa have been triangulated in this manner, observation spots on shore being selected at intervals of about 50 or 60 miles, where landing could be effected, and true bearings obtained of objects as distant as possible wherever practicable.

Advantage should be taken of any favourably situated conspicuous mountain-peak to revert to the methods described in former examples.

EXAMPLE XVIII.—*This illustrates the survey of the bar of a river on the West Coast of Africa, such as that of the River Brass, the ship and two observers only being available.*

In Fig. 41, A is a theodolite \triangle on the shore, so placed as to see as far as possible along the coast in the direction of B, and visible from D on the opposite entrance point of the river.



A D will form the base for continuing the triangulation up the river. B is a conspicuous tree about 6 miles distant, and clearly recognisable from A, and also from the ship's stations at E, F, G. C is another tree, also recognisable from the ship's stations.

An observer is landed at A, and the ship anchors or moors at F. A base by sound or range-finder is then measured between A and F, and at a given signal the ship is shot up from A, and the angles A F B, A F C, A F D observed at the ship.

The ship then moves to E, and afterwards to G, simultaneous angles being observed to the same points as before at both stations.

Calculate A B in $\triangle A F B$, and G B in $\triangle A B G$.

The true bearing of B will be observed from A.

Intermediate points between D and B and A and C may be shot in from the ship's stations, the observer being careful to use main points as zeros which will give the least change in the angle due to any small movement of the ship whilst the angles are being taken.

EXAMPLE XIX.—*Triangulation of a strait with good natural objects, by means of the ship under way and one theodolite station on shore.*

The method illustrated by the previous example may be used with good effect in the case of straits, such as Ward Hunt Strait in New Guinea, where the summits of the hills are sharp and well defined, though densely wooded.

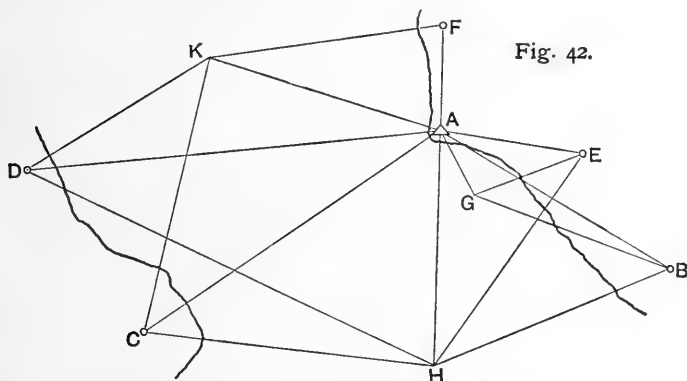


Fig. 42.

If there are three or four observers on board the ship, the main angles may be taken simultaneously, and the ship need not anchor. A very large area may thus be rapidly triangulated, its limitation being practically determined by the dis-

tance at which the shore stations can distinguish signals from the ship.

An observer with theodolite and heliostat and brass Cohorn mortar is landed at A. B, C, D, E, and F are well-defined peaks (see Fig. 42).

From the ship at G a base by sound (or range-finder) is measured by firing from the ship and from the shore alternately; at each discharge the ship is shot up from A, and the angle at the ship between A and E observed simultaneously. A separate value for A E is obtained for each discharge, and its final value determined from consideration of the formula $T = \frac{2 \ tt'}{t + t'}$.

Ships' stations at H and K provide all the data necessary for plotting the points on the side A C or the longest convenient side.

EXAMPLE XX.—For the purposes of a rapid sketch survey of a volcanic group of islands having well-defined summits, a view may perhaps be obtained of a sufficient number of peaks, from two theodolite stations visible from each other, and situated on coast hills easily accessible on islands at a considerable distance apart. From these stations several peaks may be fixed with precision on two shots.

The ship, taking up positions in different parts of the group, proceeds to fix herself on those peaks, and obtains third shots to the remaining points; the correctness of the work is thus tested. Fixing on these points, others still more extended may be shot up from different positions of the ship. If time permits intermediate points may also be shot in, with a view to laying down the details of the islands and plotting soundings. The scale may be determined by patent log distances between positions fixed on the points already plotted, or by astronomical observations. By methods such as this, much valuable information may be obtained by the officers of any ship visiting regions of which little is known.

Fig. 43 is an illustration of the above method. A is a theodolite \triangle on a low coast hill conveniently accessible, and B is also a theodolite \triangle on a summit near the coast of an off-lying island. From these \triangle s the conspicuous peaks E, F,

and the summit of the island G are shot up and fixed on two lines.

The ship taking up a position at C and fixing herself on the three points F, A, and B, a third shot is obtained to fix E.

Taking up a position at D, and fixing on the three points E, A, and B, third shots are obtained to fix F and G.

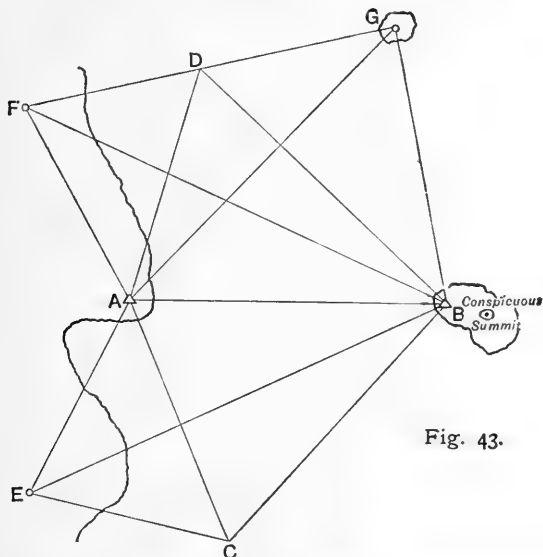


Fig. 43.

The conspicuous summit of the island B is also shot in from A, C, and D.

It will be noted that the positions of the ship at C and D are so placed that F and E are respectively nearly in line with A in each case, and therefore, however acute may be the first two lines to those peaks, the ship's position is not materially affected by any uncertainty in their exact positions arising therefrom.

EXAMPLE XXI.—*Survey on large scale of a shoal from which a distant conspicuous peak is in sight.*

In such a case, if the shoal happens to be a long narrow ridge, the usual method of anchoring on it and starring round the ship is inapplicable.

In order to obtain the least water on such a ridge, it must be crossed by lines of soundings at close intervals at right angles to the direction of the ridge. Lines of soundings radiating from the ship do not fulfil these conditions.

Mooring the ship between one end of the shoal and the peak, observe the bearing of the latter. Knowing the distance of the peak approximately, calculate its change of bearing due to a movement of 1 mile at right angles

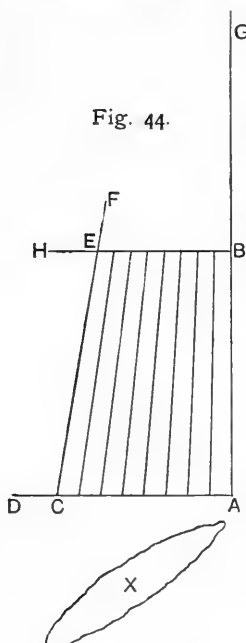
to its direction, and call this angle a .
In Fig. 44 from the ship at A draw A D at right angles to A G, the line of bearing of the peak. On A D set off A C = 1 mile on the required scale. At a point B near the edge of the paper draw B H parallel to A D. At C lay off angle A C F = $90^\circ - a$, intersecting B H at E. Subdivide A C and B E into the same number of equal parts at close intervals, and join the corresponding points on each line.

The lines thus drawn converge in the direction of the distant peak, and, spreading out fan-like on the paper, they can be used for fixing equally well as the peak itself, the limits of the paper not permitting the peak to be plotted on the scale adopted.

The shoal X can now be satisfactorily sounded out, running the lines of soundings in the required direction, and fixing the boat by observing the elevation of the mast-head and the angle subtended between the ship and the peak.

Each fix is plotted by drawing the arc of a circle, with the ship as a centre, at the distance corresponding to the mast-head angle; and setting the observed angle between the ship and peak on the station pointer, with its centre resting on the arc of the circle, bring one leg to pass through the position of the ship, and the other to correspond with, or parallel to, one of the lines radiating from the peak. The point thus found on the arc of the circle is the position required.

Fig. 44.



The error involved in this method is due—

- (1) To any movement of the ship.
- (2) To the error in the assumed distance of the peak.

As regards (1), the error is inherent to any method of floating triangulation, and cannot altogether be avoided, although it may be minimised by mooring the ship taut.

As regards (2), the following investigation shows what it is likely to amount to at any given distance from the ship :

In Fig. 45, A being the ship, B the true position of the peak, D its assumed position, draw A C at right angles to A B, the

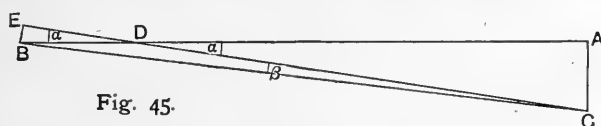


Fig. 45.

point C representing the limit of distance to which the soundings extend from the ship.

Join B C, C D, and from B drop the perpendicular B E on C D produced.

Let angle A D C = α , and B C E = β .

$$\text{Then, } \tan \alpha = \frac{A C}{A D}.$$

$$B E = D B \cdot \sin \alpha$$

$$B C = \sqrt{A B^2 + A C^2}$$

$$\begin{aligned} \sin \beta &= \frac{B E}{B C} = \frac{D B \cdot \sin \alpha}{\sqrt{A B^2 + A C^2}} \\ &= \frac{\text{error in assumed distance of B} \times \sin \alpha}{\sqrt{A B^2 + A C^2}}. \end{aligned}$$

β represents the difference between the correct bearing of A from C, and its bearing as deduced by erroneously assuming the position of the peak to be at D.

The resulting error in the fix will be represented by the arc of the circle at the distance from the ship corresponding to the mast-head angle, intercepted by two radii separated by the angle β .

The angle β , upon which depends the resulting error in the fix, varies as the sine of the angle between the ship and the peak, being greatest when the angle is 90° and nil when ship and peak are in transit.

If the true distance of the peak is 30 miles, with an error of 3 miles in estimating the distance, $\beta = 12' 45''$, producing a maximum error of 22 feet in the fix, at a distance of 1 mile from the ship. At a distance of 20 miles, and an error of 2 miles in the distance, $\beta = 0' 19''$, producing a maximum error of 33 feet in the fix. The greater the distance of the peak, the less will be the effect of any given error in estimating its distance. The distance can be found with sufficient accuracy for practical purposes by the intersection of bearings of the peak from each end of a patent log base run both ways.

If the shoal is of considerable extent, a better position for the ship to take up would be between the centre of the shoal and the peak, and lines converging on the peak should be drawn in a similar manner on each side of its line of bearing.

Graduation of Sheet before Plotting.

In some extensive surveys on a small scale it may be necessary to graduate the sheet first, when positions can be placed on it by their latitudes and longitudes, and the intervening parts plotted or triangulated by means of bases measured at each of these astronomical positions. This will be done when coasts are low and marks scarce. We can scarcely hope that when these different bits meet, they will agree exactly; but with a small scale, say $\frac{1}{2}$ inch to the mile, the discrepancy ought not to be sufficient to introduce much error, if we square in 5 or 6 miles of the points worked up from either end, when they meet and disagree.

This undoubtedly partakes of the nature of "cooking"; but when we undertake to map a coast on such a small scale, we cannot pretend to much accuracy in detail, and shall only do this when it has been considered advisable to lay down a large extent of coast in the time available, with the intention of presenting its more salient features as correctly as we can.

Work amongst islands (as portions of the Pacific) would be done in this manner.

FIXING MARKS.

It is not possible to lay down any dogmatic plan for fixing the marks which have to be erected. In many cases it is well to put them all up first, and then get angles to them afterwards; but if non-surveyors are deputed to make the marks, they will seldom be placed in the right spots. A whitewash, for instance, will be so placed that it cannot be seen in certain directions. A tripod or pole will not be in the most convenient position for the officer who afterwards puts in the coast-line, and numerous small errors of this description will be made by one who is not capable of taking in all the little requirements.

It is therefore more satisfactory to send a surveyor to do this, and while he is there he may just as well take angles, so that the writer has found it saves time in the end, in general, to have a surveyor at some main or secondary station, whence he can see most of the marks, and let the officer who erects the mark take angles at it to the above station, which we may call the "shooting-up" station, and to a sufficient number of other stations which can be seen from the shooting-up station also, to fix himself. The angles from these other stations can then be calculated. In this way two or three officers can be at work putting up marks, and fixing them at the same time. The officer who erects a mark gives it a name, and notes the time by his watch when he is there. The officer at the shooting-up station also takes the time, and notes the position and kind of mark put up, to which he takes his angles, writing the name against it in his book when he returns to the ship and meets the other officers.

System-
atic fixing
of Marks.

When dealing with a large number of marks placed on the coast at short intervals apart, it is a great convenience in plotting, and afterwards in sounding, to give names to them in alphabetical order.

Marks in
Alph-
abetical
Order.

For this purpose several lists of names of one syllable should be selected from a dictionary, each list referring to a particular class of name, such as those of insects, animals, birds, fishes, etc.

Judgment is necessary in deciding where to place marks on the coast.

Selection
of Posi-
tions for
Marks.

They should be distributed at fairly equal distances, and as a general rule the more difficult it is for the coast-liner or officer sounding to fix themselves, the more numerous should be the marks to help them. But it is useless to put up a mark in a position where it cannot be fixed, and a sufficient number of subsidiary stations must be fixed to ensure every mark having a sufficient number of angles giving a good cut to fix it.

Projecting points and bights and bays require particular attention in this respect, and every endeavour should be made to secure side-shots from mark to mark. When this cannot conveniently be achieved, a false station made with a sextant a short distance out from the coast will frequently give the required angle.

Officer
marking
respon-
sible for
Suffici-
ency of
Angles.

The officer marking must think for himself whether he has enough angles to fix the point; and in case any mark cannot be seen from the shooting-up station, he must get an angle from some other of his marks, which will be then used to calculate the other angles in the same manner.

Tangent
of a Point
as a
Station.

A well-defined tangent of bushes, or the high-water line of a point, having been shot up from some fixed \triangle , the line may be utilized to fix another \triangle that it may subsequently be necessary to make on the point, though not on the particular tangent shot up, which may be at a considerable distance beyond. The required line may be found by standing between the tangent of land or bushes and the \triangle from which it is shot up, and sending a man to a little distance, waving him to one side or the other, moving yourself under his direction, until you have the shooting-up \triangle , the man, and the tangent, in line from where you are standing. The line being thus determined, the required \triangle can be made on it, or at a measured number of feet on either side of it. This is a rough-and-ready way of measuring 180° , and is frequently useful in various ways when marking the coast to ensure that marks are visible from any particular direction.

If it is necessary to place the theodolite at some distance *beyond* the particular tangent observed, the line is obtained by bringing the latter in line with the shooting-up station, and measuring the distance from the theodolite at right angles to that line.

A heliostat is often invaluable. In hazy weather, and when the shooting-up station is distant especially, a flash will be seen when neither mark, nor boat, nor anything to direct where to look for the mark, will be visible. The officer shooting up should also return the flash, to show he sees the station, as well as give a well-defined object to get the angles to.

Use of
Heliostat

Of course, circumstances may not render this system advisable, but it is here suggested as having worked very well in many places, a long extent of coast being "marked" and all marks fixed in a short time.

Frequently the minor marks must be fixed by angles from the ship, or a boat at anchor, as on a straight coast where nothing behind can be seen from the marks. When this is necessary it will often be also necessary to carry on the main triangulation as well by means of ships and boats, so that a description of one serves for the other.

Triangu-
lating,
and fixing
Marks by
Ship.

The ship, anchored short, or moored if necessary, should be shot up from one or more shore stations. If the angles taken from the ship are indispensable to fix her own position, try calculating the back angles from other objects first, and lay them off as cuts to the position, as if they agree it will be the most satisfactory manner; but often back angles, calculated from sextant angles, will not be correct enough to give a good intersection, especially if the points are distant. In this case let all the angles taken at the ship or boat be plotted on tracing-paper as before described, and the position pricked through on the guiding line from the shore station. A signal should be made when the angle to the ship is to be observed, and the angles from the ship taken at the same time.

The ship angles should be observed from the fore part of the ship, and frequently the foretop will be found the best place. Whatever spot is used it must, of course, be arranged beforehand, so that the observer's exact position on board may be taken from the shore station.

From the ship the main angles—that is, the angles to the positions already plotted—which are to be taken for the purpose of fixing the ship, must be observed first, using some well-defined station as zero, and measuring all the main angles from this with the sextant. Some other station must be chosen as the zero with which to measure the angles to the

Taking
Angles
from Ship.

marks, and the angle to this second zero observed from the main station zero.

This second zero is wanted to be in such a position with regard to the marks that any slight movement in the ship will make the least possible difference in the angles to be observed between it and the marks. It must be, therefore, at about the average distance of the marks. It will not do to choose some object miles away behind the marks, as the least swing of the ship will at once alter the whole of the angles. Generally speaking, the central mark to be fixed will answer the purpose best, but in many cases it will be found necessary to change this zero for some marks, measuring from some other object at an equal distance from the ship.

If discretion be used in the selection of subsidiary zeros, such zeros being connected simultaneously with the main points used for fixing (of which *one* should usually be very distant), any small movement of the ship will have but a small effect on angles taken subsequently.

The governing principle is that a small angle between the objects at nearly the same distance is not sensitive to small movements in the position of the observer.

All angles that have a tendency to be sensitive should be observed simultaneously with the fixing angles; those that are less sensitive afterwards.

The objects selected for observation should be divided into groups, each group having its own zero.

The degree of accuracy obtainable in triangulation in which the ship is used as a "point" is largely dependent upon a clear perception of these considerations.

**Repeating
Angles.** When the minor angles have been taken, repeat the main angles to see if the ship has moved, giving another signal to the shore station for another angle from it. All mark angles should then be observed again to check errors.

It need scarcely be said that the more rapidly these angles are taken, the less the chance of any error arising from variation of ship's position, by change of direction of current, wind, etc. An experienced hand should therefore be chosen for this work.

**Telescope
of Sex-
tant.** A sextant with a telescope of high magnifying power is most useful. On this head, see p. 12.

Given the angles between three known objects so placed as to give a good fix, the position can readily be found within a few yards by means of the sextant.

Setting up the theodolite as near as possible to the supposed position of the \triangle , we must consider the three circles involved in the fix, and select that which is the most sensitive.

(1) Measure the angle between the two objects lying on that circle; plumb the centre of the theodolite, and mark the spot with a peg.

(2) If the angle is found to be too small, a slight movement of the theodolite towards the fixing objects in a direction at right angles to the arc of the circle we are concerned with will give a second position with an increased angle.

(3) Measure the angle again in the new position, and mark the spot with another peg.

(4) Find a point A on the ground between the two pegs, corresponding to the correct angle, by dividing the distance in the proportion of the difference between that angle and the angle observed at the two positions.

(5) Now move the theodolite several feet along the arc of the circle we are considering.

Repeat the operations (1), (2), (3), and (4), and find a point B, as already described.

(6) The line joining A B will represent the arc of the circle we are endeavouring to find.

(7) We must next consider the other two circles involved in the fix, and choose that which cuts the first circle at the broadest angle.

(8) Repeat the operations in connection with the two objects lying on the second circle that we are now concerned with, and find two other points, C and D. The intersection of the lines A B and C D will be the exact position of the \triangle required.

Select two other stations, B and C, at both of which at some former period angles have been observed to the \triangle (A) which it is required to locate. See Fig. 46.

Place a mark A' as near as possible to the supposed position of A.

The stations B and C should be so placed with reference to A as to subtend a broad angle, and the distance of each from A should be as short as possible.

To Recover the Exact Position of a \triangle from its own Angles between Known Objects.

To recover the Exact Position of a \triangle by Means of Angles observed to it from other Stations.

At each of the stations B and C, the angle to A' being observed, the difference between those angles and the angles to A, which were recorded formerly, gives the distance in feet A' b and A' c, from the formula—

$$\text{Angle in seconds} = \frac{\text{No. feet subtended} \times 34}{\text{distance in miles}}$$

in a direction at right angles to B A' and C A' respectively.

Having found the points b and c on the ground from these measurements, the point A readily follows.

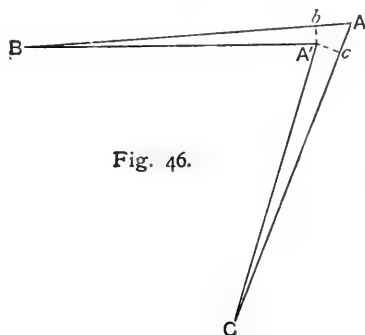


Fig. 46.

CALCULATING A POSITION FROM TWO ANGLES TO THREE KNOWN OBJECTS.

It may be sometimes required, in the course of a survey not regularly triangulated, to calculate the distance of the observer from an object, from the two angles he has observed between three known "points," one of them being the object whose distance is required. Or he may require the angle, at the object observed, to him, from the same data.

This is, perhaps, best accomplished by using the one-circle method, so called in contradistinction to the method of protraction by three circles already explained under "Station Pointer."

The three Figures 47-49 give the three possible positions of the objects, viz.: When the observer is inside the triangle

formed by the objects; when he is outside, and the centre object is nearer than one of the others; and when, under similar circumstances, it is the farthest.

If the angles between the three objects are known, which

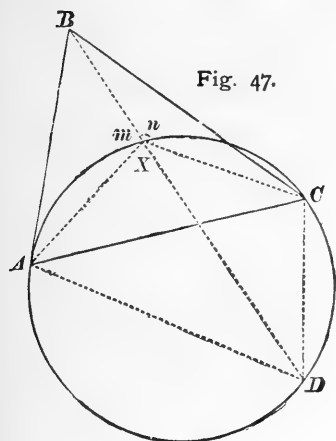


Fig. 47.

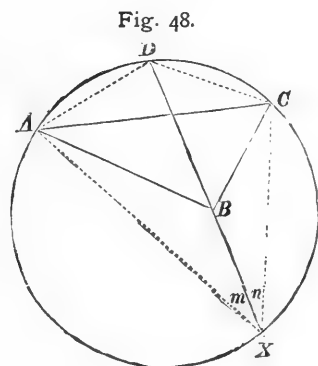


Fig. 48.

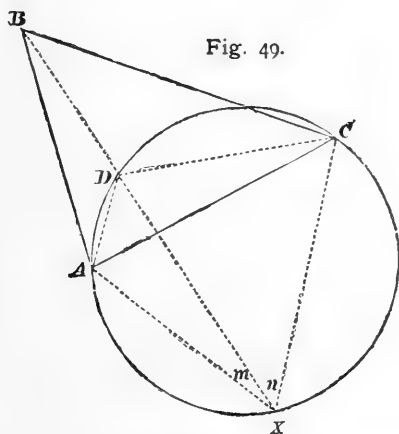


Fig. 49.

is most probable, the calculation of the second formula will be unnecessary.

Let $A B C$ be the objects observed. X the position of observer to be determined. $A B = c$, $B C = a$, $A C = b$, are the sides known, $A X B = m$ and $B X C = n$, the angles observed. Required $X A$ and the angle $B A X$.

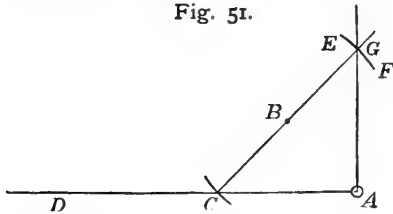
At A , in $A C$, draw, on the side remote from X , $A D$, making

From B and C draw, with a radius about half as much again as A B, short arcs intersecting one another. A line drawn through this intersection D, from A, will be at right angles to A B.

Take any point B, Fig. 51, in a direction about 45° from A, and from it as centre, with the radius B A describe a short arc

Erecting a Perpendicular to a Line from any Point not near its Extremity.

Fig. 51.



are intersecting A D in C, and likewise a short arc E F in the opposite direction. Join C B and produce it to intersect E F in G. A line joining A and G will be at right angles to A B.

Erecting a Perpendicular from the End of a Line.

In all careful work these operations should be checked by repetition, with different radii.

CHAPTER VI

RUNNING SURVEY

A **RUNNING** survey, the least accurate form of "sketch" survey, is one where the best part of the work is done from the ship running along the coast, fixing points, sketching in the coast-line and prominent parts of the land, and sounding, at the same time.

It is capable of many modifications, more especially with regard to the fixing of the main points.

**Roughest
Form of
Survey.**

The rudest form of running survey is where, beginning upon nothing, everything is eventually put on paper by observations, angles, and soundings taken from the ship without anchoring.

**Modified
Running
Survey.**

At the other extreme comes a running survey made upon some main points already fixed by triangulation of some kind, and which has for its object only the sketching of coast-line and detail of an inaccessible coast, which is assisted by occasional anchoring, and where sounding would be carried on in the boats as well as the ship, after enough natural objects have been fixed by the angles from ship stations.

**Graduat-
ing Be-
forehand.**

In making an extensive running survey of the simplest kind—*i.e.*, where we commence on nothing, and only run past the coast once—it is well to have the paper graduated (see p. 386), as astronomical observations from time to time will fix the scale of the chart, and it is easier to plot these positions when the sheet is graduated.

**Method of
Ordinary
Running
Survey.**

The course and the distance run by the ship between each position where series of angles are taken, as given by patent logs, will form a series of bases, which will have to be, however, modified afterwards to agree with the positions astronomically fixed, which must be taken as the fundamental points of the chart.

A running survey must be roughly plotted, and everything sketched in, as we go on, putting down position after position by course and distance, and cutting in the objects we choose for marks, giving them names by which to recognise them, and to record in the sounding book. Assistants should be told off for separate duties—one to look after the sounding; another to sketch in the coast-line and hills between each object chosen, on another sheet or sheets of paper; the chief and some assistants getting the angles; one writing down; another plotting the stations and drawing the lines to the points, so as to see what angles are wanted at the next station to objects already chosen, and how far on the next station should be. Bearings should be taken of all prominent points in transit, and patent log noted.

At each position, as laid down by course and distance, commence plotting by laying down the bearing of the object we have selected for zero for the round of angles. From this the other angles can then be laid down.

It follows that a bearing must be obtained, as a necessity, from each position. This should be taken to the zero selected.

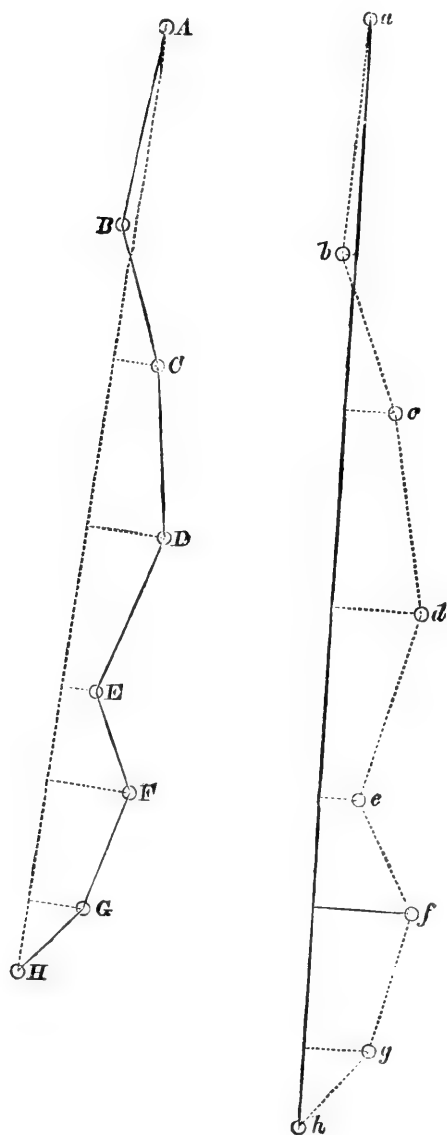
At each fix the most important angles and those that change most rapidly should be the first to be observed. If a distant object is connected by angles with other objects at the moment of the fix, it is recommended to take the bearing of the former, because its bearing will change more slowly than that of the others, and thus allows more time to decide on the exact bearing than can be the case when an object is changing its bearing more rapidly due to the ship's movement.

Distant hills are a great help in a running survey, as, when replotting from the astronomical positions, if these hills can be fixed by bearings (true or compass) from them, the angles taken to the hills, at a position now and then, may possibly be used as fixes, which may be plotted by station pointer, and so get intermediate positions independent of the patent log positions, which are so liable to error by the action of currents.

Hills of
Great
Value.

A running survey will nearly always have to be replotted, as the astronomical positions and those by patent log will never agree.

Fig. 52.



Having plotted the positions where astronomical observations have been taken, if the intermediate stations are to be put in by bearing and distance, they must be squared in so as to agree in total distance and bearing with the astronomical positions. Thus, in Fig. 52, let A be the position from which we start; B, C, etc., to H, are positions of the ship as plotted by course and distance on the rough chart; a , h , are the same positions as A, H, but as given by the astronomical observations.

Replot-
ting and
Squaring
in.

To bring the intermediate positions to agree with a , h , as plotted on the graduated sheet, we join A H and a h . Drop perpendiculars from B, C, etc., to the normal line A H. With the proportional compasses set to correspond to the different lengths a h , A H, measure the corresponding distances along a h for the points where the perpendiculars will cut, and lay off perpendiculars along which the corresponding distances can be measured, and so we obtain b , c , d , etc.

If any mountains have been observed both from A and H, their positions should next be put down by their true bearings. The angles taken from the first positions are now laid off, and as objects are fixed, they can be used as checks to the next positions. If we can rely upon the bearings taken to the mountains we shall use them to fix the intermediate positions in preference to course and distance, so that b , c , etc., may be again shifted, especially if the ship has not been accurately steered on her courses, or we have reason to think currents have varied at different parts of our run.

Nothing will agree exactly in a running survey of this kind, but a very fair approximation to the relative positions of conspicuous objects may be got.

No Ex-
actness
expected.

The amount of detail possible will not be very great, but will vary with the quickness and accuracy of eye and hand of the officer sketching it in. There is nothing that requires the knack which distinguishes a good surveyor so much as this sketching in fairly accurately of a coast-line in a running survey, and good judgment as to depth of bays, and other points that must be mainly put in by eye, is most valuable.

Amount
of Detail.

It is well to have one officer aloft, who will be able to get a better view of river-mouths, etc., and make little sketches of bits not seen from deck. He can also take angles to objects

that have sunk or not yet risen above the horizon of the deck.

Compass-bearings are of great use, as direction of valleys, etc., may be noted without making a position.

The whole course of a running survey will have to be one of compromise between discordant results, and only long practice will enable the surveyor to decide what to throw out and what to accept.

Modified
Running
Survey.

It may often occur in a survey, that a portion of the coast is inaccessible for landing by reason of heavy surf; or the shore is so cliffy or densely thick with jungle that stations cannot be made without loss of more time than they are worth. A running survey of this piece may be as much as is requisite, but the probability is that we shall be able to fix on some main points from the triangulation of the other and more important part of the survey, and these will greatly help us to make the best chart of the portion we can under the circumstances.

In such a case, the best course to pursue is to pass along the coast at some distance, stopping at convenient positions, where the ship can get station-pointer fixes by the main points, anchoring, if possible, for this purpose, and cutting in from these positions other secondary points nearer together, and nearer the coast than the first. Then pass along again closer to the land, and fix points on the shore itself, using the secondary points to fix the ship with. Boats may then be sent to sound, if required, or to sketch in more details of little bays, etc., if they can get near enough. Compromise will be required here too, probably, in plotting the points, as, unless the ship is absolutely motionless, it is unlikely the angles will intersect exactly, but it is astonishing what good results can be obtained with a number of officers taking angles at the same time, with the ship's way stopped, each being told off to take two or three angles as quickly as possible; the most important angles, and those that change most rapidly, being taken first.

Advantageous use may be made of beacons in a running survey.

As an example, we will give some details of a method employed with success on the south-east coast of Africa on an open coast, beacons being dropped in from 20 to 60 fathoms.

At a distance of from 2 to 4 miles from the shore drop a beacon abreast of some conspicuous object, called the First Breastmark (1 B in Fig. 53), which should be if possible some 3 or 4 miles back from the coast. Note the time, and shape a course parallel to the coast. Put over logs, and steam about 10 miles down it, sounding and fixing with objects selected, until the beacon gets indistinct from aloft, and you are abreast the Second Breastmark. Stop, haul in logs, note time, and drop Beacon II. During the run down the coast, three primary objects and other secondary ones have been selected and named, and on arriving at II. a Provisional Breastmark, 10 miles or so ahead, must be selected, and also the middle Primary Point of the next fleet.

At II. simultaneous angles are taken between I., the First Breastmark, A, B, C, Primary Marks, Second Breastmark, Provisional Breastmark, and Middle Primary of Fleet III. The secondary objects are next taken, using any of the above-mentioned points which are most conveniently situated as regards distance, so that any small change of position of the ship shall make the smallest possible alteration in the angle. Each officer is told off to a primary object and some secondary ones, and is responsible that his secondary objects are taken to a suitable zero.

On taking angles, the bearing of Beacon II. (which will be as close to as is safe), and also its distance by elevation of its staff, is noted. Take also compass bearing of Breastmark I. as a check.

Now steam straight out for, say, a mile or so. Turn ship's head ready for the run back, and stop. Take simultaneous angles as before, at Corner II., Breastmark I. being observed instead of Beacon I., which there is no occasion to look for.

Note time and bearing of Beacon II., put over logs, and shape a course parallel to first run.

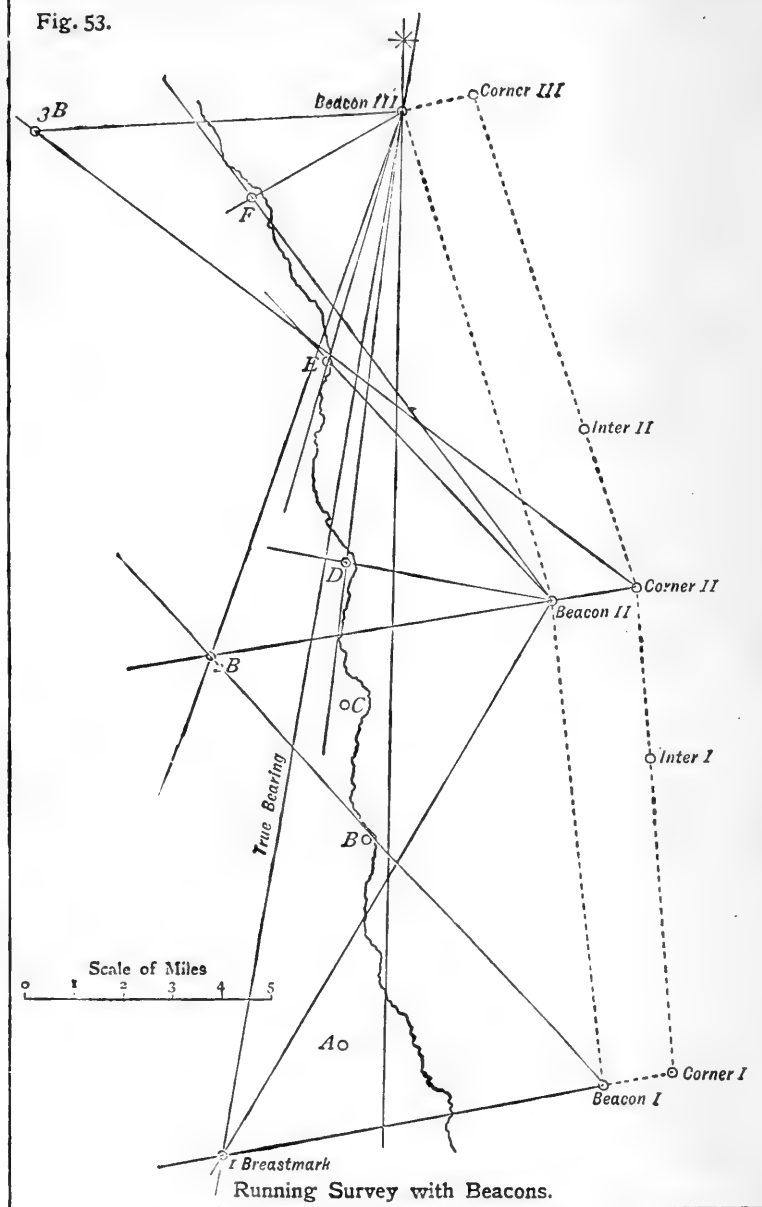
Run back about one-third of the distance. Stop, and make Intermediate Fix I.

When I. is on the same bearing as was II. when logs were put over, note time, read logs, and stop for the Corner Fix I.

Then run into I., take angles, etc., as at II., and pick up Beacon I.

The primaries and breastmark can be plotted roughly from

Fig. 53.



Walker & Boutall se.

what we have whilst beacon is being picked up, and will furnish enough to sound upon, and ensure filling up properly and not crossing the old line, for the distance between Beacons I. and II. is obtained from the runs up and down.

Whilst sounding up to II., the coast and topography are shot in, and a rough sketch of the coast and hills is put into the deck-book, which at the end of the fleet is sent down to the officer in the chart-room for plotting.

On passing close to Beacon II., put over logs and shape a straight course roughly parallel again to the shore, until abreast of Breastmark III., when stop, put over Beacon III., and take angles as described at II.

Turn outwards again and get Corner Fix III., and run back parallel to old course to Intermediate Fix II., which, and all subsequent intermediate fixes, should be at two-thirds of the distance back. At this position it is important to be able to get a good fix on the points of Fleet II.—in other words, that the angle between the Breastmark II. and Beacon II. shall be over 30° . It matters not whether Beacon II. is outside or inside of Breastmark I.

On dropping a beacon, the essential angles are the primary and breastmark of the fleet, and the other beacon, the provisional breastmark ahead and the middle primary ahead.

At the corner fixes, obtain the same angles as in dropping the beacon, only using the beacon just dropped instead of the other beacon. Take elevations for heights.

At the intermediate fixes, get the primary and breastmark of the fleet, the beacon towards which you are running, the breastmark of that beacon, and the next primary behind.

On picking up a beacon, get the primary and breastmark of the fleet ahead, the other beacon, the breastmark abreast of you, and, if visible, the breastmarks behind, as well as primaries.

It will not always, of course, be possible to distinguish the breastmarks of fleets so far ahead and astern, but whenever possible they should be taken, as points so far distant give excellent zeros for plotting, and the bearing is preserved. Any conspicuous object, whether used as breastmark or not, will answer this purpose.

A continuous series of true bearings is necessary, and it is more convenient if they are taken at the beacons.

Simultaneous angles must be observed at the same time to correct the bearing to points as far as possible up and down the coast.

With a Thomson's compass in a favourable position—*i.e.*, a position where it is not liable to change—a compass bearing will be admissible now and then.

The first true bearing should be taken from the last beacon from which the first breastmark is visible. No other bearing is necessary till the points fixed from this position are passed, when another is wanted to carry on and preserve the bearing.

When the coast trends nearly north and south, latitudes by twilight stars north and south are advisable every 30 or 40 miles to check the scale ; and similarly when coast is nearer east and west, longitude observations east and west.

Obtain shore observations when possible.

To plot, begin when the first true bearing is attained. Draw the meridian through the beacon from which it is observed, and lay off the true bearing of the first breastmark. Let us suppose we begin at Beacon III., and we are going to plot just the Fleet III. to II., and then II. to I.

Lay off the angles to II. and breastmark 2 B, drawing long lines. Calculate the patent log base III.-II. from the two runs, and prick off Beacon II.

Lay off all angles from III., first making sure that the whole angle between first breastmark and the breastmark ahead is the same both at dropping and picking up the beacon.

Then lay off the angles from II., and prick in the two breastmarks and primary points.

Lay off on tracing-paper the angles from intermediate Fix II., and all points should intersect and be pricked in. Angles to points in Fleet II. to I. can be laid off from this position.

Plot intermediate Fix I. by the points already fixed, and lay off angles.

Lay off angles from II., using III. as zero.

Prick in Beacon I. by distance, and test it by a station-pointer fix, using II. and Breastmark 2, the centre being on the line drawn from II. to I. If this agrees, there should be

an intersection at Breastmark I. of five lines—viz., from III., II., the intermediate fixes, and from I.

Plot the corner fixes and lay off all angles to secondary points.

All points being down, the soundings can be fixed and coast-line and topographical features sketched in, getting additional angles when necessary.

It is desirable to use two deck-books, entering all angles for each alternate fleet in one or the other. The plotting can then go on in the chart-room, while the angles for the next fleet are being obtained and recorded.

It is necessary to remember that on obtaining the true bearing of an object a long way ahead, such object is eventually plotted on that line.

If care be taken over the details described, the objects should plot very closely, when the tide is not too strong, and precautions are taken not to run the bases when the streams are changing.

All angles taken near a beacon should be reduced to the position of that beacon, the bearing and distance of it having been noted for that purpose. The reduction is readily effected by writing the angles down as if they had been taken with a theodolite, and using the method explained on p. 91.

At the extremities of such a survey as this shore observations should be obtained, if possible. From these positions and from the true bearings obtained corrections to scale and bearing can be made as already explained on p. 108.

In calculating the length of the different patent log bases the following formula should be used :

$$X = \frac{t(b-a)}{t+t'}$$

Where X is the current in time t with the stream,

a is distance shown by the log with the stream,

b is distance shown by the log against the stream,

t' is time occupied in run against the stream.

From X the true distance can be deduced.

The survey of the coast of Zululand, 200 miles in extent, carried out on these principles on a scale of half an inch to the mile, was found to agree precisely with the results of the

Colonial Trigonometrical Survey when the latter was completed several years afterwards.

An instance is thus afforded of the remarkable accuracy that may be attained by a floating beacon survey, notwithstanding the fact of landing being impracticable between the terminal points.

Sketch
Survey by
Compass
Bearings
and
Vertical
Angles.

In the case of an island culminating in a high, well-defined summit, visible from all directions, a useful and accurate method is to steam round it at a sufficient distance to obtain a true horizon, stopping to make as many stations as may be desirable, and fixing by compass bearing of the summit and its vertical angle.

The height is roughly obtained by shooting in its summit from two positions on a patent log base whilst approaching it.

With this approximate height and Lecky's Vertical Danger Angle Tables, each station may be plotted on its bearing of the summit.

From these stations the island is shot in by angles between its tangents and its summit, and angles to any other natural features, plotting the work as we go on any convenient scale, which must be considered only as provisional.

On completing the circuit of the island, the true scale is found by measuring the total distance in inches on the plotting-sheet, from the first to last stations, and dividing it by the distance between them in miles as shown by patent log.

The final height of the summit bears to the rough height used in plotting the direct proportion of the provisional scale to the true scale.

This method may be utilised for the sketch survey of a coast where there are well-defined peaks of sufficient height at convenient intervals, and would be superior to an ordinary running survey.

From positions of the ship fixed by bearings and elevations of one peak, another farther along the coast is shot in and its height determined; this second peak is then used in its turn to fix a third, and so on.

The smaller the vertical angle, the more liability there is to error; but a glance at Lecky's tables will show what effect an error of, say, 1' in altitude will produce for any given height and distance, and the limit of distance must depend upon this consideration.

CHAPTER VII

COAST-LINING

WHEN and how the putting in of the coast-line is done must depend much upon circumstances.

If making a chart with pretensions to accuracy in the details, it is better to do it before the soundings are taken, as, for the inshore soundings, the little points and bays, not distinguished by marks, will be very valuable. In this case, too, every yard of the coast that can be walked over should be. If the surveyor pull along the coast in his boat, from one spot to another, he will be liable to miss little details, such as stream entrances, which may be blocked by the sand beach in summer, lagoons behind the shore, etc. The boat should therefore only be used to pass rocky points and cliffs that cannot be walked along, or to make stations in, at anchor off the coast, if it is necessary to do so, to shoot up the details.

In a
Detailed
Survey.

The method of putting the coast-line on to the sheet also varies. The angles can be taken, and the details between subsidiary fixes on the beach sketched into the angle-book, using always a larger scale than that of the chart, and then these fixes and angles plotted on to the chart after return on board ; or the surveyor can take a field-board, with the points on it, with him, and plot the coast as he goes along it on to his board.

Plotting
the Coast-
line.

Of these two the latter method is by far the better, and should always be employed if possible. There is no chance of having necessary angles omitted if the fixes are plotted at the time, and any little error is easier detected on the spot than when plotting afterwards on board. Of course, rainy weather or other circumstances will sometimes prevent the work being plotted at the time, but unless some good reason exists, it should be done.

Plotting
on the
Ground.

Instru-
ments re-
quired.

If conveniently situated marks are plentiful, the coast-liner will only want his theodolite or sextant, or both, to take his angles, and a station pointer and tracing-paper for plotting, with protractor, etc. But if the coast has no objects off it to seaward, and landmarks are also short or invisible from the shore, he will require, very probably, a pole of measured length, whereby to ascertain, by observing the angle subtended by its extremities, the distance of points, etc., from one another.

A convenient form of this pole is described under "Ten-Foot Pole," p. 40.

Each assistant should have a copy of the Ten-Foot Pole Table,* on a piece of cardboard, always in his angle-book, ready for reference in the field.

General
Method of
Coast-
Lining.

Let us suppose an officer landed with his board of points to do coast-line.

He will start at some point already plotted on the chart, and will take angles from it to all the objects he can distinguish between him and the next fixed point, and beyond, if necessary.

He will then walk on to another spot, where he will make a supplementary station, fixing himself by angles to known points, either by theodolite or sextant, according to circumstances.

He will then plot this, his No. 2 \triangle , on his board, by station pointer or tracing-paper, taking care to check his position by his line from the first \triangle , or by a third or "check" angle from his present position. His No. 2 plotted, he will sketch in on the board, the coast-line between that and the first, having noted any peculiarities as he walked along.

The scale of the chart will largely influence the distance between the subsidiary stations to be made by the coast-liner, as will also the character of the shore-line, and the intended nature of the chart as to exactitude of detail.

If the work is to be plotted on return on board, the system is precisely the same, only the detail of coast between the stations must be sketched in the angle-book, instead of directly on to the board.

Coast-line may occasionally have to be sketched from a

* Appendix R.

boat pulling along the shore, fixing and shooting up any natural object on the beach from positions at anchor. Care is necessary to select suitable points as zeros, so that any small movement of the boat will produce the least amount of error (see p. 168).

When the coast-liner sees that at the next station he will not be able to fix himself by angles, he must use his ten-foot pole, sending a man on with it with instructions where to stand, or going on himself to some selected point to which he will first take the angle; leaving the pole behind with a man at his present station, with directions, when signalled, to hold the pole horizontal, and at right angles to the observer.

Using
Ten-Foot
Pole.

To ensure the latter, either a rough pointer of some kind can be attached to the centre of the pole, so as to project at right angles, in which case the holder will be directed to point this to the observer, or he will be told to sway it gently backwards and forwards, and the observer will read the largest angle he can measure.

The angle observed, and the corresponding distance looked out of the table, the latter is measured on the scale of the chart, and applied by a pair of compasses, as a distance from the last station along the line laid off from that last station in the direction of the required station.

If necessary, the whole coast can be carried on in this way; but if the marks are a long way apart, great care must be taken in observing the angles on to the positions to be measured, as there is no check on the work, and each error will be accumulative. In this case the man must be sent on, and must mark the exact place he stood when the angle was observed to him, and the coast-liner must make his next station precisely on that spot.

A distant station or well-fixed object, not many degrees from the line of coast, should, if possible, be used as a zero at each position for shooting up the ten-foot pole. Errors in bearing resulting from faulty distances are thus minimised. A range-finder is preferable to a ten-foot pole for long distances.

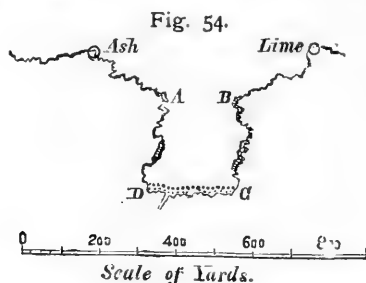
The azimuth compass may sometimes be employed in this work with advantage. Any little error, when a properly fixed station is reached, can be squared in.

It will be understood that this ten-foot pole method is only

used for the smaller detail, where sufficient angles to fix cannot be obtained. It is especially useful in delineating the shores of islands, or of small bays which have no fixed point in them.

For instance, in Fig. 54, let us suppose the two points, marked Ash and Lime, are fixed, but in between them is the small bay shown.

At Ash we obtain the angle between Lime and A, the next point visible, and also the distance by our ten-foot pole. If



we can make out that B is a point, and can see any prominent spot on it, we shall get an angle to that also.

We then go to A, sketching in between on the way. At A we become aware of the little bay, and we send the pole over to C, pointing out to the man with it where to stand, and telling him to put a stick or stone there, when he is signalled to go on to B.

At A we get all we can, angles from Lime as zero, to Ash, B, C, tangent of bay on towards D, and anything prominent, and the distance to C by the pole.

Leaving a little mark at our station at A, we go to Lime, and take angles from Ash to A, B, and distance to B by the pole now there.

We then go back to B, and send the pole over to D, and again get all angles we can, and distance to D.

We now sit down and plot our data. We have two angles to A from Ash and Lime, and a distance to A from Ash. These ought to agree, and we prick in A. We have the line to B from Lime, and perhaps from Ash as well, but we will suppose not, and will plot B by the distance from Lime. Then, placing our protractor on B, lay off the angle observed there to Ash, which ought to go through, and make a check for B.

We plot C and D by their distances on their respective lines from A and B. We then walk round the bay, sketching it in, and can get an angle at C, from A to D, as another check, and any other angles to assist in sketching in details.

The coast-liner will generally be responsible for all the details of topography close to the coast such as follow, the scale of the chart being taken into consideration as to with what degree of accuracy detail can be laid down.

Coast-Liner to do Topography near to the Shore.

Heights of cliffs must either be measured with a lead-line, or by getting an elevation to some definite point, which must afterwards be fixed, from one of the stations, or may merely be estimated and entered in the angle-book.

The height of a cliff or rock can be readily calculated on the spot from an angle, by the formula :

$$\text{Height in feet} = \frac{\text{Angle in seconds} \times \text{distance in miles}}{34}.$$

Cliffs have generally to be exaggerated on the chart, to show distinctly. The height in feet should be written against them.

Whilst coast-lining, the height of small hillocks, rocks, etc., along the shore can often be sufficiently approximately measured in the following manner, viz. :

Standing at the high-water line, notice when the sea horizon cuts the slope of the hillock or rock, etc. ; walk to this spot, stand there, and repeat the operation as often as necessary until the summit is reached. The number of times this is repeated \times height of observer's eye above the ground, + the odd feet of its summit above the last level observed, = height required.

The directions of lower parts of streams, or rivers, must either be walked up, and fixed, a certain distance back, or can merely have their entrances fixed, and an angle taken up for their general direction.

Lower spurs of abrupt hills must be sketched in, assisted by angles to them from different points.

Houses standing back from the shore must be put in. These can usually be fixed by angles to them without visiting them, unless it is necessary to get their dimensions, names, etc., or perhaps to ascertain if a good well or spring of water may be near, that would do for watering on an emergency.

Swamps near the coast should be sketched in as far as necessary, and a look-out kept for evidences of any extension of their area in winter. Information on these points can be picked up from passing inhabitants.

Angles should be got also to any conspicuous objects farther inland, as they will be very useful when the topography is sketched, and the surveyor should always look ahead, and seize any opportunity of the kind for helping on other parts of the work than those he may be immediately engaged in.

Roads near the coast should be walked back to, and fixed here and there, sketching in between.

Rocks above water, or breaking, should be fixed. Though these come into the province of the sounding, it is often useful to have them down first; and in the case of a break only, it may be very much so indeed, as it may be an isolated head, which a boat sounding near high water may miss.

Low-
Water
Shore-
line.

Though it is the high-water line that the coast-liner is more immediately concerned with, he should mark at low water the position of the dry line, especially where this runs off a long way at points, etc.

In a detailed survey on a large scale, it will be necessary to send some one round the water-line at low tide to get it accurately, but on small scales this is more usually obtained by the soundings, for by reducing these to the low-water level of springs, a series of points will be obtained, where each line of soundings crosses the low-water line, which can then be drawn in as a line passing through these points.

As the tide rises or falls, the time should be noted at which rocks and various parts of the shore cover or uncover. Reference to the tidal register will show their heights above low-water, or the amount they uncover, which should be expressed on the chart.

Every part of the survey should be seen at least once at low-water, and rocky heads that dry, with their heights stated against them, must not be omitted from the chart.

Eleva-
tions of
Hills.

Angles of elevation for heights of the hills should be taken when getting the angles for fixing the points of the chart, from main and secondary stations, or any well-fixed points; but if the coast-liner gets some more elevations from marks

on the water-line, they will never come amiss, as long as the position is well fixed.

The officer coast-lining will make a note of anything worth recording in the sailing directions, as little nooks for landing, convenient places for watering, etc., letting his captain know on return on board, in order that they may be, if necessary, again looked at, or entered in the latter's notes.

General
Informa-
tion for
Direc-
tions.

It may be convenient to keep a book for the purpose, in which any useful information can be entered.

It is very useful to know the angle subtended by different parts of the hand extended at arm's-length.

Approximate
Measure-
ment of
Angles by
Means of
the Hand
extended
at Arm's-
length.

For instance, from the tip of the thumb to the tip of the little finger will be found to subtend an angle of about 20° . With the fist closed and thumb extended the angle is about 15° ; and from the tip of the thumb to the nearest outline of the hand is about 5° . The fist closed subtends an angle of about 10° .

By the aid of some such measurements as these carefully determined and committed to memory, any angle may be measured within surprisingly small limit of error.

The entrance being fixed in the ordinary manner, the triangulated "points" are frequently lost to view on entering the sound. A bearing or angle being drawn to the farthest point of land visible up the sound from the innermost position where it is possible to fix on the outside "points," a patent log base run to that farthest point will give data upon which to sketch in the intermediate coast on each side, if proper angles are taken at each end of the patent log base. The survey of the sound is continued in a similar manner for the whole of its length on a succession of patent log bases. Great care should be taken to ensure that a sketch carried out in this way does not get out in bearing.

Sketch
Survey of
a Deep
and
Narrow
Fiord or
Sound.

As an instance of the application of the ten-foot pole method, we may mention the following, which is adapted for use on shores with fringing coral reefs, or broad sand or mud flats, which dry sufficiently at low water to enable people to walk on them, and when either the steepness of the hills or the denseness of the vegetation prevents marks being fixed on the coast.

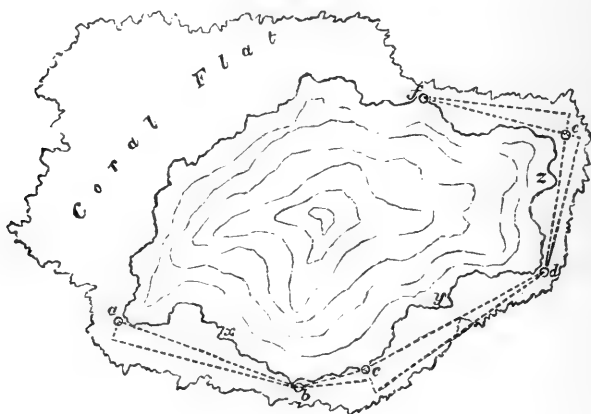
Further
Applica-
tion of
Ten-Foot
Pole
Method.

Let annexed diagram, Fig. 55, represent an island of this kind.

A long measured lead-line, say of 500 feet, is provided. This is taken by an officer we will call B, who has a prismatic compass. Another officer, A, is provided with theodolite, or sextant, or micrometer, and prismatic compass, according to circumstances, sextant and compass being quite sufficient.

Starting at *a*, B remains there while A walks to *b*. B stretches his line out at right angles to *a*, *b*, and plants a flag

Fig. 55.



at the extremity. A observes angle subtended by flag and $\triangle a$, with his micrometer or sextant, and both A and B observe the bearing of *a b*.

A waves to B, who goes on to *c*, when the operation is repeated.

A then moves on to *d*, B pivoting his line round *c*, so as to be rectangular to *c d*; and so on, until *f* is reached. We will here suppose that, from *a* to *f*, we have been able to triangulate, the reef being broader. We have therefore the correct bearing and distance of *a f*.

To plot this, the mean compass bearings and distances *a b*, *c*, etc., will be put on a separate sheet of paper on a larger scale than the chart, and the positions *a f* being joined on both, the other stations will be squared in on to the chart.

Marks will be left at each station, if required for sounding,

or delineating the outer edge of the reef. Subsidiary marks can be made at other points, as x , y , z , and fixed by angles from b , d , etc., with distances measured by the angle of the line.

The shore line can either be sketched by A, as he walks from station to station; or can be put in afterwards, if greater correctness is required, using the ordinary ten-foot pole to fill in between a , b , c , etc.

If a theodolite is used, which it is well to do in a case where we have not been able to get any measured base at all, and must consequently work back to a , it must be set up first at a , and the angle to b taken from some fixed object, whose true bearing we should obtain, as we in this case must not be dependent on the compass. B will be at b with his line, and when A has finished, will walk on to c , so that A, when he arrives at b , can take the angle from a as zero to c . With a theodolite, then, A must visit every station, unless B has one also.

At every new position, the last \triangle will be used as zero.

The readiest way for B to direct his line so as to be at right angles is to use the so-called "cord-triangle," which is simply a triangle formed of a piece of line whose sides are in the proportion of 3, 4, 5, the angles being marked by knots. When stretched on the ground, with the corner between 3 and 4 at the \triangle , and the 4 side coincident with the direction of the other \triangle , the direction of the 3 side is at the right angle required. Any similar contrivance will serve the purpose.

NOTE.—This method was largely used by Lieutenant W. U. Moore in the survey of the Fiji Islands, and is a good example of the dodges that have to be improvised to meet circumstances

CHAPTER VIII

SOUNDING

Boat Sounding—Ship Sounding—Searching for Vigias.

Import-
ance of
Sounding.

It is difficult to say that any one step in the construction of a chart is more important than another, as each is necessary for the completion of the whole, and an error anywhere may cause a disaster ; but if any particular item *is* to be picked out, perhaps the sounding should rank in the highest place.

The operation of sounding is the least pleasant part of a marine surveyor's work, especially when the weather is against him, and the sounding uninteresting—that is, where the depths are regular, and there is no excitement in the way of discovering and working out shoals and reefs ; but the notion that it is therefore always to be relegated to the juniors of a survey is not only hard upon them, but may introduce errors into the very part of the chart which, as we have already said, is the most directly important.

As soon as the points are down—*i.e.*, plotted—the sounding can be commenced ; but, as before remarked, on an intricate piece of coast it is better if the coast-line is put in first.

Ordinary
Method of
Sounding.

The ordinary main plan of sounding is thus. The boat proceeds in straight lines in a direction, of a length, and at distances previously decided on, with a man in the bow constantly sounding. Every so many soundings, as the case may be, the officer takes angles with a sextant to fix the position of the boat, always doing this at the beginning and ending of every line.

It is evident that this main plan may be largely varied in its details.

In the first place rises the question as to whether it is better to plot fixes, and enter soundings on the sheet, regularly, in

the boat, or leave them until return on board, merely putting down an occasional fix to see where you are. The writer says, certainly, as a rule, plot them at once. It can be done in ordinary circumstances just as correctly, and gives more information to the officer sounding as to little bits which may want additional casts, and it also gives the men at the oars a little rest from time to time. In very rough water it of course cannot be well done, and must be left till return on board to the comparatively motionless ship; but when you can, plot at once. In harbour work on large scales, again, it will be better to plot afterwards, as great accuracy will be required.

The extent to which the soundings themselves can be entered at the time on the chart depends, of course, upon the state of our knowledge of the tide. If the tidal range is small, or the motions of the tide are sufficiently known to form a table of reduction beforehand, the reduced sounding can be written on the board at once. If not, the soundings as taken can be written down, and reduced on inking on return on board, or only the sounding taken at each fix can be written against the prick of the fix, and intermediate soundings left to be entered on board. The latter will generally be found most convenient.

The pace at which the boat may go, and the necessity or not for stopping at the casts, will depend on the depth of water and the capacity of the leadsman.

Circum-
stances
guide
many
Details.

Whether it is necessary to stop to get the angles depends upon the convenience and visibility of the marks and the quickness of the angle-taker. A beginner will, of course, do everything deliberately, until he feels capable of combining speed with correctness.

Whether each fix shall be plotted at once, or whether to wait until two or three have been got, and then lay on oars or anchor for a few minutes, must also vary with circumstances.

If laying on oars, keep the lead on the bottom with a slack line, and let the coxswain keep the boat in position.

What the distance should be between each fix will depend largely upon the scale of the chart and the nature of the bottom. On an evenly sloping bottom many soundings can be got without another fix; but where depths vary or increase rapidly, the fixes must be closer together.

The soundings which will be joined together on the finished chart by fathom lines—*e.g.*, the three, five, ten fathoms, etc.—should always be fixed, and in doing this it must be remembered that it is the *outer* sounding of any of the same depth that will be on the fathom line, and also the tide reduction must be taken into consideration. This latter will, of course, be in many cases only approximately known, so that exactly the right sounding may not be fixed.

Direction
of Lines.

The sounding-lines should be in ordinary cases at right angles to the coast, and parallel to one another, as not only will a better line be got for tracing the fathom lines, but the boat will easier be kept in her right direction by observing two objects which have been seen to be in transit, in the right direction, at the commencement of the line.

A narrow ridge should be examined by short parallel lines of soundings run on transits, if possible, and crossing the ridge at close intervals, in a direction at right angles to its length.

Marks in
Transit
for Direct-
ing Lines.

In nice work on large scales it is generally necessary to place two marks in line for this purpose ; but, for ordinary surveying, changing them from one line to the other will take far too much time for the purpose, and marks to answer all practical purposes may usually be found placed by Nature already.

While running out from the shore on a line of soundings it is desirable to take off from the sounding-board the fixing angles for the end of the line, in order to know when it has been reached, without waiting to plot.

When fixing near the end of a line, measure by station pointer the angle between a point fixed on the sounding-board and the point at which the next line of soundings cuts the coast-line, remembering the fact that small angles between objects at about the same distance are least affected by the movement of the boat. Placing this angle on the sextant, endeavour to pick up some object on the beach where the next line cuts it, to serve as the front transit mark for running in upon. The back transit mark, if sufficiently distant, will remain nearly unchanged, and should be brought in line with the new front mark. Fixing again at the end of the new line, shoot up the proposed transit line, and amend it as necessary by taking the required angle off by station pointer and setting it on the sextant as before.

Before running in on the new line it is a wise precaution to pick up in a similar manner another object on the beach as nearly as possible at the end of the next line to be run out from the shore. When close in shore it is often impossible to fix, and the lines may therefore be irregular without some such assistance.

In sounding out a small harbour, circumstances must guide the direction of the lines.

A point of land or a reef is often prolonged under water by a narrow shoal ridge, which, unless carefully searched for, may escape detection. Radiating lines of soundings at close intervals are necessary, supplemented by cross-lines in cases where the geological features suggest the possibility of such prolongation occurring. All rocky points likely to be rounded closely by a ship should receive such close examination.

In planning lines of soundings it is desirable to call attention to one particular element of danger. This lies in the elbow formed by changing abruptly the direction of parallel lines of soundings, the area of unexamined ground thus being unduly great compared with the remainder.

Two observers being in a boat near the required spot, the fixing angles are clamped on their sextants. Manœuvring the boat to bring on the more sensitive of the two angles, the direction of the arc of that circle is estimated, and the boat steered to keep on it by preserving the exact angle between the two objects. The other angle will then gradually be brought on and the precise spot will have been reached.

In sounding off breakers it is frequently a good and safe plan to steam round them as closely as may be considered prudent, cutting in their tangents and sounding at the same time. The lines may afterwards be run off and on at right angles to the line of breakers, but it will not now be necessary to do more than *approach* the danger-line that has been first run, and the safety of the ship or boat is less imperilled by thus having a definite limit within which she must not pass.

In turbid waters pinnacle rocks are seldom discovered by the ordinary lines of soundings. If an indication is obtained, it is very possible that it may be by getting a *deeper* sounding than usual, the scour round the base of the rock making a

Sounding
off Points
of Land or
Reef.

To Pick
up a given
Position
by Sex-
tant
Angles.

Danger
Line.

Pinnacle
Rocks.

depression round it, or round one half of it, especially with a muddy bottom. In calm weather and tideway, the ripple on the surface may reveal the existence of a rock a few yards further upstream than the nucleus of the ripple.

Starring
off a
Light-
house or
round the
Ship.

In sounding off a lighthouse situated on an isolated rock or island, the lines may be run radiating from the lighthouse, the distance being fixed by its angle of elevation, and the direction of each line being maintained by preserving a constant angle between the lighthouse and some distant object (the more distant the better). The boat then travels along the arc of a circle which is very "sensitive," and the further the distant object, the more nearly will the boat's path approach to a straight line. The angle is altered for each line. Sounding round the ship at anchor may be carried out in a similar manner if a distant object is in sight and its bearing observed. The horizontal angle suffices to keep the boat on her line, and obviates the use of prismatic compass for the purpose.

Depth to
which
Boat's
Soundings
should be
carried.

The depth to which the boat soundings are to be carried will depend upon circumstances. When soundings of over 20 fathoms are taken from a boat it gives a great deal of labour, unless a small sounding machine is carried.

When the boat gets to the end of her line, and turns to pull along to the end of the next one to return, soundings should still be carried on, as before.

The method of using the station pointer has been explained under the head of "Station Pointer."

Construc-
tion of
Station
Pointer to
be remem-
bered.

It only remains to note that it must be recollected, in getting the fix, that the right or left angle (according to whether a right-handed or left-handed station pointer is in the boat) must be observed of a sufficient number of degrees to be measured on the instrument, if possible. If this cannot be got, recourse must be had to tracing-paper for plotting the position.

Entering
Soundings
in Book.

The sounding-book need not be ruled. There are several ways of writing down the objects used for fixing and the angles between them, but the best, if space permits, as it does in the sounding-book supplied by the Admiralty, is to put them down as you look at them, the right-hand object to the right, the middle one in the middle of the page, and the left one on the left-hand side. The hour, in Roman

numerals, and minutes should be entered from time to time, to know the reduction for tide ; the sounding at the fix goes on the extreme right, and subsequent soundings up to the next fix, in a row underneath, thus :

| | | | | | | |
|-------|--------|---------|-------|---------|-----|----|
| X. 14 | Pagoda | 28° 31' | Mat | 62° 14' | Can | 7½ |
| | | | | | | s |
| 7½ | 8 × × | 8½ | 9 × × | 10 × | | |
| | s | | m | m | | |
| | „ | 23° 02' | „ | 60° 08' | „ | 11 |
| | | | Pea | 41° 17' | „ | m |

The cross (×) signifies the same sounding as before ; and it may here be mentioned that *all* soundings must be put down, even though there may not be room for half of them eventually ; as, the man heaving regularly, if all his casts are not registered, the change of fathom will not come in its true place when interpolating between the fixes.

Space must be left under each line for the soundings, as reduced to low-water, to be written in in red ink.

A sliding scale for the reduction of soundings has been devised by Lieutenants Helby and Craven, which avoids the liability of arithmetical errors. This scale is supplied to surveying ships.

A check angle should be taken from time to time, to make certain that things are right, as is noted above at the last cast, in the example Can to Pea. This is especially necessary at the commencement of work with new points, as mistakes *will* occur in plotting points occasionally. A check will show at once if points are true and if the angles have been taken correctly.

Judgment is needed in selecting objects for check angles. If badly selected, a satisfactory check on the position may not be afforded by them. A sensitive angle should be chosen connected with the middle object of the primary fix.

The nature of the bottom must be taken every few casts, and recorded, the officer having a look at it from time to time himself, to make certain that the leadsmen is calling the stuff he brings up by its right name. For instance, many men will insist on calling “ stones ” rock, which is, of course, quite a different thing.

All Casts
to be
entered.

Space for
Reducing.

Check
Angles.

Nature of
Bottom.

Same
"Points"
to be used.

The same objects should be taken for the fix as long as possible. It tends to check errors in reading off, as the angles at each fix will bear a definite proportion to the last set. For instance, if we are pulling off shore with both Mat and Pagoda astern of us, the angle will be less each time, and a reading of, say, 33° instead of 23° would be at once detected as erroneous, before the disjoining of the line when the fix was plotted showed there was "something wrong somewhere."

The variation in the angles will also enable us to see if the fix is remaining good. This plan also saves time in setting the station pointer verniers.

Necessity
for Assist-
ance.

When assistants are not thoroughly used to the work of sounding, it will be necessary to have two in each boat, to ensure no mistake; but when not only officers but men get used to it, one officer will in most cases be able to carry on the work by himself, with the assistance of a man to write down for him. Now that seamen are all taught to write, there is seldom any difficulty in finding one of the boat's crew—the coxswain, if possible—to write down fairly. The same man will steer generally, and so permit the officer to keep his eyes for other matters.

In deep water the boat must, of course, be stopped, and the leadsman will only heave when told. The interval can be timed by watch, or, in very open deep soundings, by the Massey's log towing astern, fitted as described on p. 53.

Distance
Between
Lines of
Sound-
ings.

The distance between the lines of sounding will depend upon the scale and the character of the survey, also upon whether the place is inhabited or not, for where there are natives information can be picked up as to shoals, etc., from the fishermen. The value of this, however, largely depends upon the intelligence of the informant, and often cannot be trusted.

If the coast or harbour be unknown and the land of certain geological formations, it takes a great deal of sounding to be certain no stray rocks exist undiscovered; and, as was pointed out in our preliminary remarks, the majority of marine surveys are not on a sufficient scale, nor will time at disposal allow us, to sound so close as to be absolutely certain nothing is missed. The surveyor must make up for this by keeping his eye ever on the look-out for discoloured water, and by examining every suspicious spot.

It must always be remembered that on the ordinary scales used for surveying figures may look close together, and yet be in nature quite far enough apart for a rock or bank to exist, without giving any indication in the lines of soundings passing on either side of it. On a scale of 3 inches to the mile each figure will occupy a space of 50 yards nearly.

It is difficult to ensure the detection of all hidden danger except on large-scale surveys. Five lines to the inch is about as close as lines of soundings can be run without overcrowding ; if closer lines are required, the scale must generally be increased. Cross-lines will to some extent compensate for deficiency of scale. The decision as to the scale practically governs the degree of minuteness with which the ground is examined. In order to realize fully the imperfection of small-scale charts, it is only necessary to enlarge to the moderate scale of 6 inches a chart apparently closely sounded on a scale of 1 inch to the mile. The effect is striking, and shows how easily a rock may have been missed. Small-scale charts are, therefore, misleading in this respect. The time occupied over a survey and the consequent expense varies roughly as the square of the scale.

Though insisting upon close sounding in circumstances where it is required, it should be noted that much valuable time and labour may be uselessly expended in running lines of soundings closely in depths from which a shoal cannot possibly rise without giving some indication of its presence on one side or the other of lines at a reasonable distance apart. The lines and the distance apart of the soundings should be spaced relatively to the depth of water in order to catch these indications. When found, they should be examined. These remarks do not apply to depths under 20 fathoms, but in greater depths in surveys on comparatively large scales it is possible to sound too closely, and to do so indicates want of judgment and supervision. The nature of the bottom and the geological character of the country must always be taken into account in spacing soundings, due consideration being also paid to the angle of slope which a shoal must attain to escape detection in a given depth with lines of soundings at certain intervals. A diagram drawn to a true scale vertically and horizontally will indicate the requirement.

Scale of
the Chart

Unneces-
sary
Sound-
ings.

Sus-
picious
Ground.

It will depend upon the orders received from the chief of the survey whether suspicious ground is searched at once, or merely pointed out on return on board for further examination. As a general rule, whenever the soundings, in pulling offshore, say, *decrease*, it is suspicious, and the spot must be examined by intermediate lines, which in many cases should be at right angles to those previously run, and looking out sharp with the eye as well.

In calm weather, when there is a tide, a sharp eye may detect a pinnacle rock by the ripple it may form.

It is in looking out for and utilising such small indications that the genius of the true surveyor displays itself, and many are the rocks that have been missed for want of such sharp intelligence.

Close Ex-
amination
of a Shoal.

A very distant object in transit with objects situated on the shore at close and regular intervals, affords the most satisfactory means of running lines of soundings across a shoal at distances apart much closer than can be depicted on a chart on any reasonable scale. Cross-transit lines to cut off the lines of soundings at each end will keep the work within prescribed limits. The ground can then be examined in the minutest manner possible with great facility. Fixing is necessary only when a shoal cast is obtained, when a buoy should be dropped at once, and the summit of the obstruction felt for by the lead. The further removed the distant object, the more nearly parallel to each other become the lines of soundings, and therefore the more effectively is the ground covered. The nearer the front object, the more sensitive is the transit line. If cross-transits are not available, it is frequently advisable to mark by buoys the four corners of the area to be examined.

The examination is more thoroughly carried out by sounding on transit lines crossing the shoal in two or more directions, cutting one another at broad angles.

Small
Buoy.

A small nun buoy, with light chain and a weight to anchor it by, is useful in the sounding-boat to drop over on a shoal spot, so as to guide a boat working round and round while trying for still shoaler water.

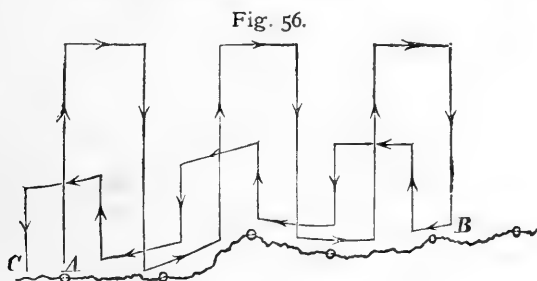
Doubling
the
Shoaler
Lines.

In many cases it is convenient to run double the number of lines in shoal water—say, out to 7 fathoms—that are required in greater depths. In this case one set of lines will

be run first, and when the boat gets to the end of her allotted space, she will return in the opposite direction, and run intermediate lines.

See Fig. 56, where we suppose the boat to start at A, work along the long lines to B, and then return to C along the intermediate lines, crossing the old work at every line, and thereby getting a check on it.

A lead-line of 7-stranded sounding-wire or cod-line, with 7 pounds lead, will enable soundings in shoal water to be obtained at intervals of a few feet merely by lifting the lead off the bottom and letting it go again for a fresh cast without hauling it up and heaving it in the ordinary manner. The boat must be kept accurately on pre-arranged transit lines, and move very slowly. A skiff is most convenient for this sort of work, and the leadsmen should be close to the observer.



In sounding a harbour channel on a large scale, it is often convenient to stretch a lead-line across from side to side, and sound at regular distances apart by this line, shifting it for each section required.

Sweeping for a reported pinnacle rock is resorted to when sounding fails to discover it. Two or more boats, pulling abreast, tow a lead-line between them, well weighted under the stern of each boat. If one weight in the centre is used, the rock may very likely be missed. The size of the boats will govern the length of line between them. An iron bar is still better. It is by no means an easy thing to do efficiently, so that all the ground shall be traversed without unnecessarily going over it again and again. If steam-cutters are used, care must be taken not to go too fast for the weights attached, or the bight of line will be towed nearer the surface than is intended.

Sweeping
over a
Large
Area on a
Broad
Front.

An effective method of sweeping for a rock on a broad front is much to be desired. Having sounded out a rocky channel in the usual manner, it would be a great satisfaction to be able to sweep over it rapidly to a depth of about 6 fathoms, to ensure no danger being left undetected.

It is possible that a solution of the difficulty may be found by towing from two steamboats submarine sentries set to a depth of 6 fathoms, and connected by a piece of 7-stranded wire 100 yards long. The boats should steam on parallel courses, and maintain their proper distance apart by means of a distant line. Lieutenant Helby, R.N., in a lecture delivered at the United Service Institution, has proposed the use of the Otter Trawl in connection with submarine sentries.

Experiments on these lines are much required.

Sweeping
over a
Limited
Area.

A long heavy iron bar suspended horizontally at the required depth under the boat or barge will indicate boulders lying on the bottom that might be missed by the lead. This procedure is particularly useful when sounding a rocky area that has been dredged.

An effective sweep is formed by a couple of railway irons, each 15 feet long, placed end to end, and joined by a third railway iron in the centre.

A 30-foot spar should be placed across the gunwale of the boat, with distant lines marked in feet, run through a block at each end, and connected with the ends of the iron sweep, by means of which the required depth is ensured throughout the length of the sweep. Any deviation from the horizontal due to the boat heeling over on one side is detected by the mark on the distant line dipping below the water, and the corresponding mark on the other distant line being raised by an equal amount.

The operation of sweeping is most conveniently carried out by allowing the boat to drop down slowly on transit lines running in the direction of the stream, and controlling her by lines secured to buoys suitably placed on each bow and quarter.

An examination of the bottom by divers is sometimes desirable.

Anchor-
ages.

An anchor should not be placed on a chart unless the officer in charge is assured that sufficient examination has been made

to warrant it. On a small-scale coast survey, if no plan of the anchorage has been considered necessary, the soundings should at least have been thickened up to ensure, as far as the scale will admit, that no hidden danger remains. Fishermen should be interrogated, and their proceedings watched.

Leading marks should invariably be run by the ship herself before recommending them. An original chart should indicate such marks, wherever they can be found, and are likely to be serviceable. Leading Marks.

Clearing marks are particularly useful to the navigator, and wherever they exist they should be noted on the chart and in the sailing directions, together with the best marks for fixing the position of the ship. Clearing Marks.

Shoal-banks, out of sight of land, or too far off to use marks, can be sounded by starring round the ship, at anchor on it, or off its edge. For these, compass bearings of the ship taken from the boat, with distance measured by the mast-head angle, will probably suffice in accuracy, the boats sounding in lines radiating from the ship in all directions. Sounding Shoals out of Sight of Land.

It should be noted that this method of sounding covers the ground very unequally, and is objectionable on that account.

A large canvas ball or cylinder, on a light framework of iron, and painted black, will be found very useful at the mast-head when taking the angle for this purpose, as it will clearly define the mast-head, and also indicates "Ship in position."

Boats or beacons can be moored in convenient positions, and fixed by angles to one another and to and from the ship, also at anchor, and the base obtained by mast-head angle, if it is necessary to sound a bank a little more accurately. These will then be used as marks, and the soundings fixed by angles in the ordinary way.

Having starred round the ship at anchor on a shoal, it is sometimes advisable to leave a boat at the anchorage to mark the position while the ship weighs and proceeds to search for another shoal patch, which, when found, is connected with the boat's position by her bearing from the ship and mast-head angle taken from the boat.

On suddenly striking shoal-water out of sight of land, the ship may be anchored at once, and the shoal sounded out by

Unex-
pectedly
striking
Shoal
Sound-
ings.

boats starrng round her ; or, if the shoal is of large extent and may be prudently crossed in the ship, it is a good plan to lay down two beacons on a bearing and patent log distance of 4 or 5 miles. With another beacon (or mark-boat, carrying a large black flag) fixed by means of this base, and forming an equilateral triangle with it, the ship being anchored as a fourth point, soundings may be carried out by the boats, and fixed by station pointer. The ship's position should be determined by observations of twilight stars.

A convenient scale for work of this sort is $\frac{1}{2}$ inch or 1 inch to the mile.

Large
Banks out
of Sight
of Land.

When surveying a large bank, where accuracy is desired, the beacons should be placed on a regular plan, and nothing is better from every point of view than anchoring them in two lines, so as to form equilateral triangles and a series of parallelograms, the beacons being about 5 miles apart. This distance permits the corners of the parallelograms being seen from one another. Bases are obtained by patent log, and astronomical observations fix the extreme points.

If sounding out the triangles by boats, a mark-boat, flying a flag from a bamboo lashed to the mast, can be moored half-way between the lines to aid fixing.

Eight or nine beacons are desirable to carry out a satisfactory floating triangulation of large extent. The beacons should carry flags distinguishable from each other, and the vertical height of each should be known, in order that the necessary correction for false stations may be calculated and applied to the angles taken for fixing the beacons.

Beacons are visible from boats to a distance of about 6 or 7 miles, and from the ship up to 10 or 12 miles, but they are very difficult to observe at that distance. The bearing of a distant beacon can be observed with a theodolite used as a bearing plate when the beacon is too faint to obtain its direct bearing by standard compass. For this purpose the theodolite, when set to zero, should point to a fixed mark in the ship in a direction exactly parallel to the fore and aft line. This should be verified by direct observation in connection with the lubber line of the standard compass when at a quiet anchorage. The first beacon that is laid down should be left in position as a point of reference to be eventually connected

by meridian distance with the farthest beacon at the other extreme of the survey. The other beacons are weighed and placed in new positions as the survey progresses.

Astronomical observations by twilight stars should be obtained at beacons at intervals of about 20 miles, and the intermediate triangulation, with the soundings dependent upon it, is squared in between those positions, due consideration being given, however, to the orientation derived from the chain of compass bearings, which should not lightly be thrown over.

Sectional lines off coral reefs are sometimes now required to show the exact slopes for scientific purposes, or for cables. Reef
Sections.

It is not an easy operation, and cannot be hurried.

The soundings must be close, to show the exact slope when it is, as in many cases, steep.

The section must be run on a transit line, and there are many ways of fixing the distance.

A boat anchored on the edge of the reef for the outer transit mark, with a long bamboo or other light spar stepped, will, by means of vertical angles, afford a means of ascertaining the distance up to perhaps half a mile, but beyond it will be necessary to have another boat or mark on the reef at a fixed distance from the transit-line, to which horizontal angles can be taken, making, in fact, an exaggerated ten-foot pole. Other methods will suggest themselves to the surveyor.

The diagram should be drawn on a true scale—*i.e.*, the vertical and horizontal scales equal—an inch to 30 fathoms, and *the slope to the left*, so as to facilitate comparisons with other diagrams.

In all sounding the lead-lines should be measured on return on board, and a note made in the book, “Lead-line correct,” or so much out. When the line has not been used for some time, it should be measured before leaving in the morning also; but if it has been examined the evening before, this will not be necessary. Measur-
ing
Lead-
Lines.

While on this subject, it may be noted that new lead-line should never be used for boats’ soundings. At the beginning of the commission it may be necessary to do so, but afterwards make lead-lines out of old well-stretched stuff that has been used for deep lines for ships’ sounding, and measure and mark them when wet.

Necessity
for
Fractions.

The soundings must be put into the book to the exact depth obtained, but it will depend upon the scale, the general accuracy of the chart, and the thickness of the soundings, how far halves and quarters will be placed on the sheet. As a rule, fractions should be retained up to 6 fathoms; and over that depth only the even fathom, taking, of course, the fathom under the depth. Thus a sounding which, when reduced to low-water, is $9\frac{3}{4}$, will appear as 9 fathoms.

Reducing
Sound-
ings.

The necessity for accuracy in reducing soundings to low-water will also very much depend on the scale of the chart and the depths. It is evident that with soundings of over 6 fathoms at low-water, if we are using a small scale, where the size of the figure placed on the chart will, in reality, cover ground on which we have taken five or six soundings, any nicety of reduction is an absurdity, and labour thrown away; but in *shallow* water the reduction will be just as necessary in a small scale as a large, as a sounding of 5 fathoms will be a danger or not, according to what amount of reduction we apply.

Calling
Sound-
ings.

It is usual in surveying vessels to depart from the time-honoured habit of calling soundings, and to call simply "Six and three-quarters," "Five and a half," and so on. This is simpler, and saves time. The men should also be trained to call out sharply, and on no account allowed to drawl.

There are, however, two exceptions to this. "Seven" and "Eleven" have a great similarity when called from the chains, and to prevent mistakes "Deep eleven" should be called. Similarly, "Nine" and "Five" sound much alike, and "Deep nine" should be given. "Five" and "Seven" are given simply.

"Shoal
Water."

On all occasions, whether in ship or boats, when the leadsmen suddenly gets a shoaler cast than expected from his previous soundings, he should call out "Shoal water," without waiting to complete his usually fruitless endeavours to gather in the slack line and find out the depth. The author has been on shore from the neglect of this, the leadsmen being foolish enough to wait until he had repeated his cast, so as to give the correct depth, and gave no warning to the officer on the bridge until too late.

Belcher proposes a plan for ascertaining the depth on a bar which it is desired to cross, without risking a capsizing, which

may be quoted, though we have no knowledge of its having been practically tried. He suggests anchoring the boat as close to the bar as is safe, with the tide at flood, and veering away a barricoe with a grapnel hanging at a given length of rope. The barricoe is permitted to drift freely over the bar, when the anchor catching, will give a shock to the barricoe that will be seen by the watcher in the boat, and will indicate that a less depth than the length of the cable allowed to the anchor is on that part of the bar.

Belcher's
"Sound-
ing a
Bar."

The line attached to the barricoe, with presumably a tripping connection with the grapnel, will bring the apparatus back to the boat, when she can test another part of the bar in the same manner.

SHIP SOUNDING.

The soundings over a certain depth—about 20 fathoms—can generally be most advantageously done from the ship.

Where a steam-winch is fitted, soundings can be got with great rapidity; and by dropping the lead from forward and heaving it up to a davit fitted on the taffrail, up-and-down casts can be got in 40 fathoms at a speed of about $4\frac{1}{2}$ knots without stopping with a 100-pound lead.

Usual
Plan.

If a long spar be fitted as a derrick aft, soundings can be obtained in water up to 20 fathoms, by merely swinging the lead and letting it go without heaving forward.

A large field-board, about 42 inches by 36 inches, fitted with legs to form a table, is necessary for ship's sounding.

Ship's
Sounding-
Board.

A scale showing the distance on the sounding-board passed over per minute of time at the average speed at which the ship is travelling, is useful to ensure turning at the right moment, and to avoid overshooting a certain spot, as when proceeding from one line to another. The time must be noted at each fix.

For deeper water a variety of methods have been devised for getting the lead forward and dropping it rapidly.

Arrange-
ment for
Expe-
ditious
Sounding.

The following is now generally used :

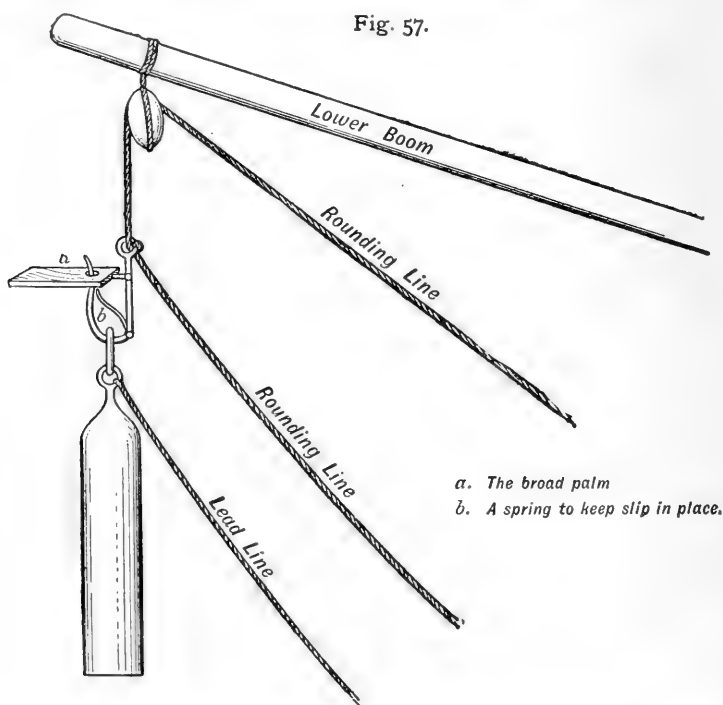
The lower boom is got out and topped to an angle of about 40° .

An endless rounding-line of lead-line is carried through a block at the end of the boom, by leading blocks to the steam-

winch, and to the derrick or davit aft. A slip is attached to this with a broad projecting palm of sheet-iron to the catch (see Fig. 57).

To this the lead is attached, and hove forward by the winch. When up to the boom, the rounding-line is let go, and on striking the water the palm releases the catch, and the lead falls free to the bottom.

The rounding-line is at once rounded aft again, ready for the slip to be again attached when the lead comes up.



There are varieties in the detail of the fittings, according to different ideas.

By dropping the lead well away from the ship, the chances of the lead-line fouling the screw, if the helm is over, are much lessened.

Alterna-
tive
Method.

Another plan has recently been tried with successful results. By its means the lead can be slipped from any desired position as it travels forward, and it has the advantage

of keeping the lead-line clear of all projections on the ship's side.

The lower boom being got out and drooped as far as the rolling of the ship will permit without dipping it, a wire jackstay connecting the end of the boom with the sounding davit, is set taut by means of the foreguy.

The slip holding the lead is attached to a carrier travelling on rollers on the wire jackstay, and is hauled forward by the rounding-line. The lead is slipped by the tripping-line, which also hauls the slip back again to the sounding-davit in readiness for the next sounding. The wire jackstay being inclined downwards, the lead runs forward freely if the jackstay is kept taut.

Fig. 58 shows the arrangement, which has been found to work satisfactorily, and is more expeditious than the former plan.

A variety of instruments has been invented for giving the accurate depth when the line cannot be got up and down ; some depending on a fan which works a series of cogged wheels, as Massey's ; others on pressure at different depths. These are all useful up to a certain point, and when their errors have been obtained, may sometimes be attached to the lead with advantage.

Instruments for recording Depth.

Recorders, however, of great value to navigation are of no use in surveying operations, and the majority of these navigational inventions are liable to small errors, which we must not have in depths which are to be placed on charts.

Burt's bag and nipper are useful when the ship drifts away from the vertical position over the lead, and one should always be handy when sounding, but great care is necessary that it bites the line.

It is evident that a *perfect* machine is more trustworthy than the record of an up-and-down cast with the ship in motion, as given by a fallible man ; and when such perfect machine is invented it will be gladly adopted by surveyors ; but up to the present time the machines are more liable to error than a trained man under most circumstances.

Perfect Machine yet to be Made.

A wire sounding machine on the poop is very useful for depths over 60 fathoms. Under that depth the hemp sounding-line is more expeditious. Using an ordinary 28-pound deep-sea lead connected with the end of the wire

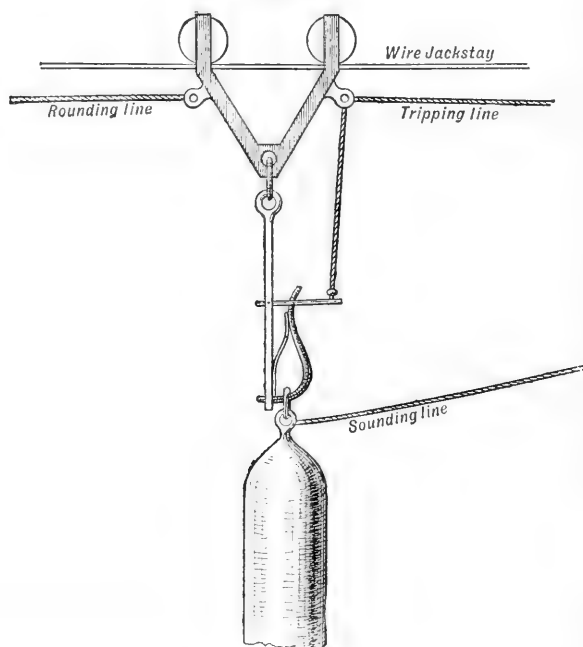
Wire Sounding Machine on the Poop.

by a couple of fathoms of hemp, the ship may be started ahead directly bottom is struck, even at a depth of 1,000 fathoms, speed being gradually increased. Soundings in great depths may be successfully obtained from aft in circumstances when the Lucas machine on the forecastle could not be used owing to heavy seas washing over it.

Long
Lines of
Soundings
off Shore.

In localities where currents are prevalent and vary, when we are running long lines of soundings in the ship off-shore, out of sight of land, it is very important to get on the return

Fig. 58.



line towards the shore a fix as soon as possible. The soundings we are obtaining may be hereafter used, especially where fogs are frequent—as, *e.g.*, British Channel, Bay of Fundy—to give vessels a notion of their position, and we must therefore use every dodge to get our true position at the earliest opportunity, so as to depend upon dead reckoning as little as we can.

Two theodolite stations, from which a large flag at the mast-

head can be observed as soon as it appears above the horizon, is a plan sometimes employed. These need not see one another. As long as their relative positions on the chart are known, and the true bearing of the zero employed has been established, the angles to the ship can be plotted. The smoke from the ship when she herself is below the horizon will often enable a valuable angle to be observed. A single theodolite line is often of great value, as it can be utilised in conjunction with angles, bearings, or observations from the ship herself.

It is scarcely necessary to say that an observer will be on the topgallant yard of the ship, as he may, from atmospheric or local causes, be able to see something on the land before the theodolite observers catch sight of the ship. Surveyor Aloft.

True bearings come in useful again. The angular distance between the sun and the mountain or other object seen from aloft, will be taken by the observer aloft, while the sun's altitude is taken from deck for the azimuth.

Another method is to have one or two ships anchored as far from the land as they can fix, which observe and are observed from the sounding-ship, as she runs in and out on her lines. A ship can easily in light winds anchor in 100 fathoms, and even in deeper water. Tenders

When land stations are employed, heliostats are useful, as informing the running ship that she is seen from the station. A flash will tell the officer aloft that a sounding can be taken, with the certainty of an angle being got to the ship, for which, perhaps, she has been waiting. Use of Heliostat

If not able to return and pick up the land before nightfall, blue lights and rockets are useful, both from stations and moving ship. Position after Dark.

A true bearing of a light or mountain, if visible, as it often is a great distance on moonlight nights, can be obtained in the Northern Hemisphere very conveniently by the angular distance from the Pole Star, as described on p. 360. This angular distance can again be taken from aloft; but in the case of Polaris we require no altitude. Use of Stars.

If Polaris is not available, a time azimuth of a star near the prime vertical will give a good result. Should the resulting longitude differ much from the assumed, it may be

necessary to recalculate. Altitude azimuths cannot be much trusted in at night.

Compass
Bearings.

When objects are visible from deck at night, and we can rely on the compass, very good bearings can be taken with the standard, if the lighting arrangements are properly fitted.

Deck-
Book.

A ruled "deck-book," as now supplied, is convenient for ship's sounding. In this everything taken from the ship should be recorded, as rounds of angles when the ship is used as a station in main triangulation; elevations with sextant, and the corresponding fix; sketches of little bits of coast, etc.

SEARCHING FOR VIGIAS.

Difficulty
of Dis-
proof.

In searching for a vigia, it is difficult to say when its existence is to be considered as disproved. This is especially the case in comparatively shallow waters, when nothing short of a detailed examination of the ground by the aid of floating beacons, if shore objects are not in sight, is of any avail.

Although experience shows that nine out of ten of these bugbears and blots on the oceanic charts have been mistakenly placed there, from reports of floating whales, wrecks, and patches of *confervæ* taken for discoloured water near a bank, etc., still, we must be careful not to assume from a hasty search that no shoal-water exists near a given locality.

Area of
Search.

The area over which to search must always be large, as the reckoning of the reporting ship, especially as regards longitude, may often be considerably in error. In the vicinity of such reefs, also, currents are generally accelerated, and altogether we must allow a large margin in undertaking to search for a danger reported in a particular spot.

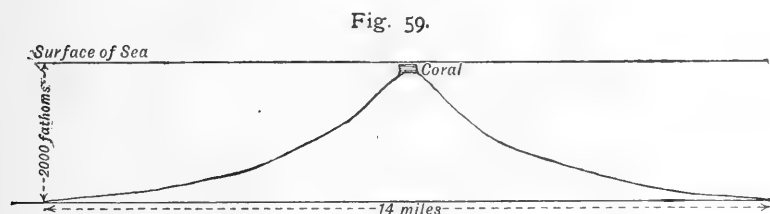
Small
Area very
Possible
without
much
Indica-
tion.

In bright, clear weather coral banks will show some miles with the sun in the right direction; but under other circumstances it is quite possible for a ship to pass within a mile, or less, of a bank with as little water as 3 fathoms on it, without its being detected from the mast-head. Soundings are the only effective means of verifying or disproving the existence

of a reported danger. As a general rule, the shallower the water, the more troublesome the task, and the greater the necessity to use floating beacons to enable a close search to be carried out effectively.

The well-known instances of the Avocet Rock in the Red Sea, and the Quetta Rock near Torres Strait, rising from depths of 33 to 39 fathoms and 10 to 12 fathoms respectively, are cases in point.

Submarine banks rising from oceanic depths must necessarily stand on bases many square miles in area. Of recent years our knowledge of the angle of slope that may be expected to occur at different depths has been much extended. Assuming that coral reefs are built on submerged mountain-peaks, the annexed sketch (Fig. 59) will show approximately the slope



usually found in great depths. It will be noted that as the depth decreases the slope rapidly becomes steeper. Positive soundings of considerable depth will cover a comparatively large area, but soundings of 100 fathoms, no bottom, do not assure us of anything to a certainty, except that the reef does not exist within a few hundred yards of that cast.

Banks can frequently be diagnosed from a sounding, though deep, a little less than others around, and the only way to make certain that a bank does not exist is to follow up the direction of the slope with positive soundings. No bottoms are of little value.

From depths of upwards of 2,000 fathoms the slope is so gradual that a bank rarely, if ever, approaches the surface within 7 miles of such a sounding; therefore anywhere within a circle of 7 miles' radius, embracing an area of 150 square miles round a bank rising from these depths, a sounding should show some decided indication of a rise in the bottom.

Experience has shown that in coral waters the edge of a bank is a favourite spot for the growth of a small danger. This part of a bank should therefore be closely searched.

Spacing of
Sound-
ings.

In depths of 2,000 fathoms soundings at intervals of 7 miles, run in parallel lines 7 miles apart, enclosing areas of 50 square miles between any four adjacent soundings, should effectually clear up the ground and lead to the discovery of any shoal. In depths over 2,500 fathoms the soundings might be more widely spaced.

In depths of 1,500 and 1,000 fathoms shoals are very unlikely to occur within a radius of $3\frac{1}{2}$ miles and 2 miles respectively from such soundings; but as the depth decreases the angle of slope rapidly increases, and a shoal might occur within three-quarters of a mile or even half a mile from a sounding of 500 fathoms.

Shoals are not usually equally steep on all sides. In some directions the slope is generally more gentle, and thus a better chance is afforded of picking them up by sounding systematically.

An appreciation of these facts will indicate the distance apart at which it is proper to place soundings in squares suitable to the general depth of water.

Use of
Contour
Lines.

Contour lines will soon show in which direction to prosecute the search if any irregularity of depth is manifested. When once a decided indication is found, it is not difficult to follow it up by paying attention to the contour lines as developed by successive soundings.

Sub-
marine
Sentry
Valuable.

The submarine sentry, now supplied to all surveying ships, towed at a depth of 40 fathoms, is here invaluable, and may save hours of hunting. Discoloured water, rippings, fish jumping, or birds hovering about may assist in locating a shoal. A handsome reward to fishermen on the spot is often effectual.

Many banks in unsuspected positions have been discovered by means of the submarine sentry, which should be constantly towed, set to about 30 fathoms, when on passage. It sometimes fails to strike on a very soft bottom, but it is liable more frequently to give false alarms, especially if the ship is pitching heavily and going at slow speed.

With kites fitted to float with the ship stopped, the chances of fouling the screw are much diminished. As the wire generally

carries away close to the kite, it saves loss if a preventer of slack wire, secured to the tail of the kite, be spliced into the main wire about 4 fathoms up it.

A report being more liable to errors of longitude than of latitude, a greater margin is necessary in that direction. Long parallel lines east and west are preferable, but the necessity of turning the ship more or less head to wind at every sounding makes it desirable to run the lines with the wind abeam, as this tends to disturb the reckoning least. Method of Search.

The difficulty of fixing the position at sea to within a couple of miles or so adds another element of uncertainty to our search, so that it is only by running the lines of soundings closer than would otherwise be needful that we can at length say positively nothing is there. Doubt as to our own Position.

A good idea of the current may be obtained from the general direction of the ship's head whilst sounding, considered with reference to the strength and direction of the wind, and it should be allowed for in shaping the course to preserve the parallelism of the lines, but the less frequently the course is altered the better. Estimation of Current.

It is convenient to adopt a scale of $\frac{1}{4}$ inch to the mile where sea observations are concerned, and to graduate the sheet on Mercator's projection. Scale.

A good position in the morning twilight should be obtained by two pairs of stars on bearings perpendicular to each other, the lines of position of one pair thus cutting those of the other nearly at right angles. Astronomical Positions.

During the daytime the dead reckoning should be checked by lines of position from observations of the sun about every two hours throughout the day, taken preferably whilst a sounding is being obtained and the ship stationary. Evening twilight stars give another position.

During the night it is the safest plan to steam slowly head to wind through the area already examined ; the accuracy of the dead reckoning is then preserved better than if lying-to. Procedure at Night.

The interval of darkness may occasionally be used to run an additional line of soundings head to wind outside the area examined during the day, and thus add to the ground cleared up. If there is a bright moon and good horizon, positions by stars should be obtained at intervals.

Lying-to
near
Vigias at
Night.

It is rather unpleasant to be drifting about at night with reported reef in the vicinity, and by no means a bad precaution is to ease a kedge anchor down to 100 fathoms or so, which may bring the ship up, or, at any rate, show, by drawing ahead, that bottom is reached before she strikes on the reef. Constant use of the wire sounding machine is above all things necessary in such circumstances.

Credence
in Re-
ports.

If the reported danger is out of the usual track of ships, there is nothing improbable in its having escaped notice up to that time; but where the locality is frequently passed over there is more *prima facie* reason for doubting the report, and in many instances a cross-examination of the person making the report will show how very slight is the ground for it. An actual cast of the lead seems a feat impossible to make a mistake about, but instances have occurred where this has been proved, even with so-called "bottom" brought up. In cases, however, where a sounding *has* been obtained we must conclude the report to be true, and a rigorous search must be made before the vigia can be obliterated. There are numerous instances of banks having been erroneously reported by ships using the Thomson machine, and failing to stop and verify the sounding by an up-and-down cast.

Prelimi-
nary
Search.

As a general rule, for the commencement it is best to run lines east and west in or near the latitude reported, as this is more likely to be near the truth than the longitude.

When going to make an exhaustive search, the first day is perhaps best spent in doing this without getting more than may be one positive sounding, as we can cover more ground, and if the danger exists we have a good chance of finding it by sight, or by the soundings taken when the ship is running, as, of course, the deep-sea lead will be kept going constantly.

Decision
rests with
Hydro-
grapher.

In every case, of course, the surveyor transmits home a plan of his track and soundings, as it is at headquarters only that a decision on the matter can be arrived at.

Under the head of "Sea Observations" will be found hints as to early ascertaining of the ship's position, a most important matter on each morning.

CHAPTER IX

TIDES

ALL soundings in published charts are given for low-water at ordinary spring-tides; we therefore want all the information we can get about the tides, and the very first thing to be done on arriving on the surveying ground is to commence observations on them.

Tidal
Observa-
tions com-
menced at
once.

There are very few parts of the world in which we have absolutely no knowledge of the tidal movement, so we have generally something to commence upon—that is, we usually know within an hour or two the time of high-water at full and new moon, called H. W. F. and C. on the charts, as except in estuaries or peculiarly shaped coasts, this will not differ greatly from places near at hand, and the same may be said for the range of the tide.

It will altogether depend upon our length of stay in any locality as to what we can hope to find out about the tides. To get full information requires observation during months in succession, as in many parts the tides vary considerably at different times of year. The number of high and low tides in a day in certain places departs from the normal phase of from six to seven hours for each rise or fall; in others the tide will take longer to rise or fall than *vice versa*, etc. A long series of this kind is therefore very valuable, as the tidal theories are at present far from fulfilling all the requirements of observation all over the world, and good data are much wanted; but it is not often that the surveyor can obtain such a series.

Different
Observa-
tions for
Different
Require-
ments.

It will be seen, then, that tidal observations for the practical reduction of soundings for purposes of navigation are one

thing, and those for obtaining additional data for scientific investigation are another.

We shall mainly concern ourselves with the former, where much rougher observations are usually admissible ; but here, again, it must depend upon the scale and nature of our chart what degree of nicety is requisite.

A few words on the Theory of the Tides will be given at the end of the chapter.

Tide
Tables.

The reader is referred to Dr. Whewell's treatise on the tides, published in the preliminary part of the Admiralty Tide Tables, for much information respecting their movement.

Local
Circum-
stances.

A regular series of observations, even for our practical work, should be taken if possible ; but in many cases the necessity for leaving tide-watchers encamped is inconvenient, and may be unhealthy, and we may have to be satisfied by obtaining what will be sufficient to enable us to construct the chart, which is our immediate business.

In other cases we may only be staying a few days at a place, as when making a plan of a small isolated harbour.

Observa-
tion Indis-
pensable.

What we absolutely require in making a chart is to know the height of the water whilst sounding is going on, above the level of low-water springs, which is called the "datum for reduction."

We shall also wish to ascertain, if possible, the "establishment," which is the time of high-water at full and new moon, called in the charts "High-water at full and change"; the rise of spring-tides above our datum ; and the range of the tides at neaps, and the time occupied by the rise and the fall of each tide, as these will give valuable information to the navigator.

We may here give definitions of some of the terms used in speaking of the tides.

Defini-
tions.

"Rise" of a tide is the height of the high-water level above the low spring datum.

"Spring rise," as given on the Admiralty chart, implies the height to which ordinary spring-tides rise above the datum to which the soundings are reduced. This in all cases should be L. W. O. S.

The height to which extraordinary spring-tides rise above

the datum should be stated in the title of the chart, together with the amount to which they fall below the datum.

“Range” is the difference between the height of high and that of the low water which immediately follows it, without any reference to the datum.

The “semimenstrual inequality of heights” is the difference between the heights of spring and neap tides above mean water-level.

The “diurnal inequality of heights” is, in irregular tides, the difference between the height of high-water of each successive tide.

The “age of the tide” is the interval between the time of new or full moon and the time of the next spring-tide, and varies from one and a half to three days.

The “lunitidal interval” is the time that elapses each day, between the transit of the moon over the meridian and high-water following.

The “establishment” may be also defined as the lunitidal interval when the time of the moon’s meridian passage is $0^h. 0^m$. or $12^h. 00^m$. This is called the “vulgar establishment.”

The “mean establishment” is the mean of all the lunitidal intervals in a semilunation, and may differ considerably from the vulgar establishment. The latter is the high-water full and change given in the charts.

The “semimenstrual inequality of time” is the difference between the greatest and smallest lunitidal interval.

The “diurnal inequality of time” is, in irregular tides, the difference between the lunitidal intervals of each successive tide.

The time and height of the tide is ever changing, caused by the relative positions of the sun and moon, and these more or less regular variations are further affected by winds and by the height of the barometer. The difference in level due to the latter may be taken roughly as a foot for every inch of the barometer above or below the mean barometer, high barometer causing lower tides.

Cause of
Varieties
in Tides.

The time of the moon’s transit over the meridian gives us a rough measurement of the relative position of sun and moon in right ascension, and it is therefore to this meridian passage of the moon that we refer all calculations of the tides.

Observa-
tions re-
ferred to
Moon’s
Transit.

If the tides are regular, we shall find that on days on which the moon passes the meridian at the same time, the times and heights of high and low water will be the same.

This knowledge is very valuable in many surveys where from local causes we cannot always have a tide-pole going, as from previous observation we can, when the tides have been found to be regular, construct a table founded on moon's meridian passage, from which we can take out a reduction for soundings when working on a small scale.

When we arrive on our surveying ground, then, one of the first things to do will be to set up a tide-pole, whatever is going to be the character of our observations.

Position
for Tide-
Pole.

For this we want a sheltered spot, if we can find one, and also firm ground on which to place it, as nothing is more annoying than to find the pole down, especially when out of sight of the ship, when the tide-watchers, unassisted, generally succeed in putting it up again in a different position.

If a pier is available, there is nothing so simple and satisfactory as a plank secured to it, marked in feet and inches, the former being painted red, white, and blue alternately, with bold black figures.

Tide-
Poles.

If we have no pier, an ordinary spar, shod with an iron spike and painted as above, driven as far into the ground as possible and well stayed to heavy weights, anchors, rocks, or whatever we can get, will stand well, and generally answers our practical purposes. This may sometimes be so placed as to be read from the ship with a glass.

If, however, there is no shelter and much wash of the sea, and accurate observations are required, we must use a tube of some kind.

A square one of deals can be knocked up on board ; but it must not be too small, as we shall want a slit down one side, through which an indicator fixed to a rod carried by a float inside may work, and the water washing in by this slit will destroy the value of the tube, unless the area of it be large enough to make the water thus admitted too insignificant in quantity to disturb practically the surface of the water inside. Where there is not much range of tide, the slit can be dispensed with, and the rise and fall marked by an indicator protruding from the top of the tube (which in this case could be a boiler

tube), and marking on a scale lashed so as to project above the tube. The water would be admitted by holes bored near the bottom of the tube if it is to be placed on muddy ground.

The zero of the tide-pole must be frequently compared with the fixed datum mark to detect any sinkage of the pole. If stepped on a wooden framework, there is less liability of this occurring on soft ground.

Many attempts have been made to devise an automatic tide-gauge suitable for use in the conditions ordinarily met with by surveying ships. Auto-
matic
Tide-
Gauge.

At present the most promising apparatus is a pneumatic arrangement consisting, in its simplest form, of an india-rubber tube, one end of which is secured to a weight, lowered to the bottom in a depth of about a fathom at low-water, and the other end connected to an air-pump placed in a tent on the beach, a Bourdon pressure-gauge being introduced close to the air-pump. Air is pumped in until it attains sufficient pressure to escape by expelling the water in the tubing, which will be indicated by the Bourdon pressure-gauge ceasing to rise.

A scale of pressure corresponding to different depths is formed by noting the readings of the pressure-gauge as the tide rises on the tide-pole. It may be more readily determined by lowering the end of the tubing with the weight attached in deep water to different depths, and noting the corresponding readings of the pressure-gauge.

The next development was to admit compressed air from a reservoir by turning a cock when required to make an observation, instead of using a pump. Subsequently a continuous stream of compressed air was allowed to escape slowly through the tubing, the supply being drawn from a reservoir of three cylinders connected together, and charged by a powerful hand-pump to a pressure of 100 to 150 pounds per square inch, thus giving a continuous indication of the height of the tide by reading the pressure-gauge. The three cylinders contained sufficient air to last from ten days to a fortnight.

The instrument in its final stage is now under trial, an attempt having been made to replace the Bourdon gauge by something more sensitive and efficient. This has been attained by means of a mercury recorder, as shown in the accompanying

figure (Fig. 60). The pressure of air being sufficient to expel the water from the lower end of the flexible tubing at the maximum height to which the tide rises, a continuous stream of air slowly escapes into the water whilst the apparatus is in action.

Bubbles of air will then be observed rising to the surface of the water in the Klinger's water-gauge glass, indicating thereby that the air is escaping freely through the tubing.

The bight of a short length of tubing between the Klinger's water-gauge and the closed vessel containing the mercury is carried well above the level of the water-gauge, in order to prevent the possibility of water being carried up if the air-pressure should be turned on suddenly, or with too much force.

The iron float in the vertical pipe projecting from the mercury vessel is of considerable weight, and rises or falls in accordance with the air-pressure that is acting at the moment on the surface of the mercury.

The movement of the iron float is recorded by a pencil on a cylinder revolved by a clock. The pencil is secured to a substantial metal block working freely in guides.

The air escapes through a hole about $\frac{1}{16}$ inch in diameter in a metal plug screwed into the end of the flexible tubing.

This metal plug is secured to a sinker in such a manner as to keep it off the bottom, and clear of mud or weeds which might choke the orifice, and is dropped in a depth of from 3 to 6 feet at low-water.

It has been found by experiment that the daily loss of air-pressure from the reservoir is rather large at first, but steadily diminishes. The loss is always roughly one-tenth of the total pressure remaining day by day. Thus, starting with 200-pound pressure, the first day it loses 20 pounds; by the time the pressure gets down to 100 pounds it will be losing 10 pounds a day, and at 50 pounds only 5 pounds a day. When the reservoir gets below 50 pounds' pressure, it should be recharged, as it is found that with pressures as low as 30 or 40 pounds the readings were always too small near high-water. This minimum probably varies with the actual depth at high-water over the sinker; in the case referred to this depth was 27 feet.

The scale of height on the recording cylinder is obtained from actual experiment, and when once determined it should be constant.

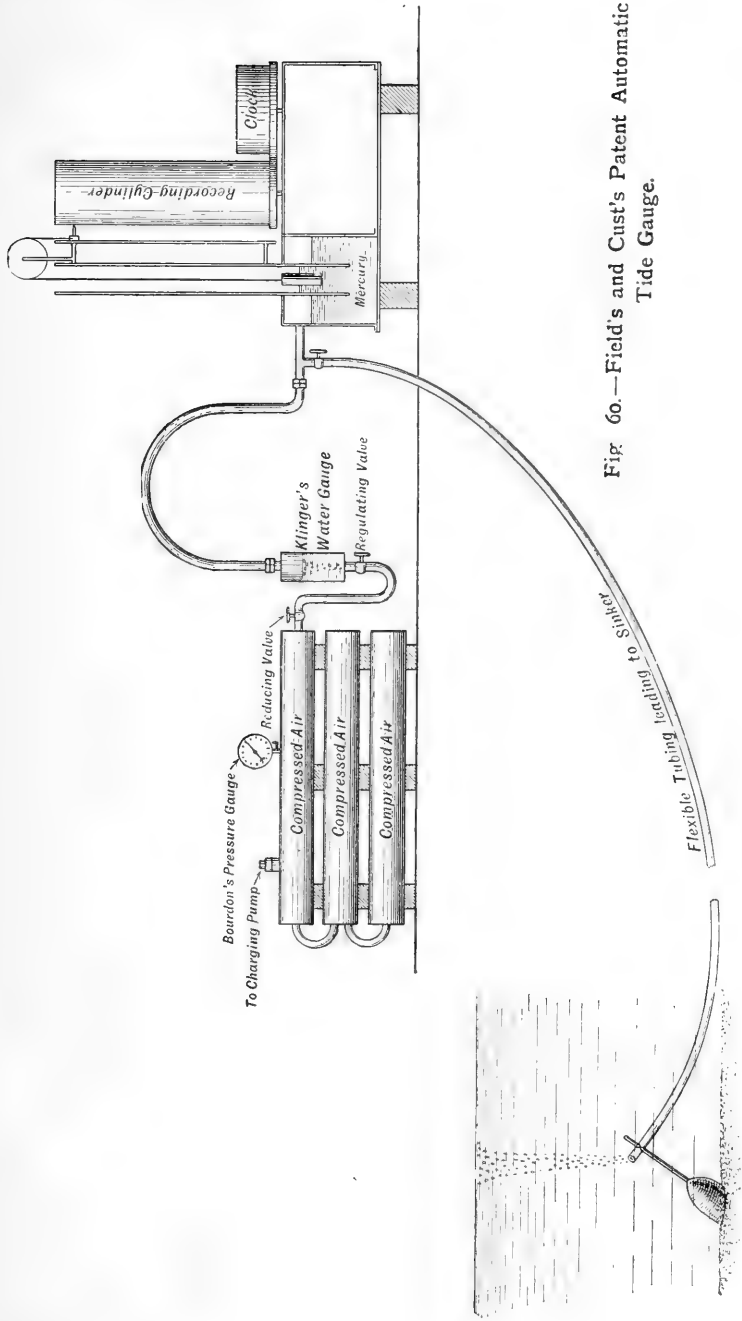


Fig. 60.—Field's and Cust's Patent Automatic Tide Gauge.

By cutting out the automatic recording arrangement and using in its place a Bourdon gauge graduated to 300 pounds' pressure, the apparatus has been used on the ship at anchor in 35 fathoms, the rise and fall of tide being accurately determined by the difference of pressure recorded on the gauge. Half-hourly observations were made during several days; the results when plotted on squared paper practically agreed with the curve shown by observations on a tide-pole on shore.

Fixed
Mark for
Reference.

Whenever it can be done, a mark should be made on some fixed object near the tide-pole, corresponding to some mark on the pole, which can then be replaced in the same position if it accidentally gets displaced.

Levels should also be carried to some permanent mark in the vicinity, and the difference of level between this mark and the datum given in the chart, with the object of enabling future surveys to be reduced to the same datum level. This is most important. When, as is the case of many civilised countries, there is a fixed plane of reference for land surveys, and bench marks are available, the tidal datum should always be connected with such fixed plane.

Time of
Observa-
tions.

The level of the water on the tide-gauge should be noted every hour, if we are going to make a regular series, both night and day. If simply to get a datum for soundings, day observations only are necessary, except at springs, when it is as well to get the high and low water at night also, as night tides in some places and at some seasons are lower or higher than the day ones. The tide-watch should be set to mean time of place.

It is not amiss in any case, when nothing is known of the tides, to observe for twenty-four hours, at half-hour intervals, as a commencement, as this will tell us whether the tides are regular or not, and we can take observations accordingly.

High and
Low
Water
Observa-
tions.

To get the time and height of high and low water accurately, observe every ten minutes, for half an hour or so, before and after high and low water, and calculate from these records the exact time and height required.

Graphic
Method.

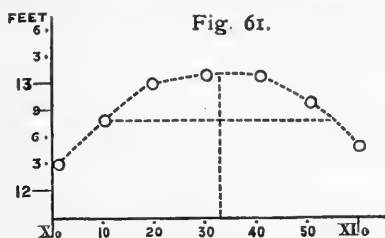
This is best done by projecting graphically thus: Divide a line into equal parts to represent hours and minutes, and from this, at the corresponding time, set off at right angles distances, on any chosen scale, to represent the height of tide registered at that time. These spots, joined by a curve, will enable the

time and height of high or low water to be arrived at much nearer than by simple observation.

Thus, suppose we have noted—

| | | | | | ft. | in. |
|-------|---------|----|----|----|-----|-----|
| X. | 00 A.M. | .. | .. | .. | 12 | 3 |
| | 10 | .. | .. | .. | 12 | 8 |
| | 20 | .. | .. | .. | 13 | 0 |
| | 30 | .. | .. | .. | 13 | 1 |
| | 40 | .. | .. | .. | 13 | 1 |
| | 50 | .. | .. | .. | 12 | 10 |
| ..XI. | 00 | .. | .. | .. | 12 | 5 |

We project these as in accompanying Fig. 61, and by drawing a horizontal line from the X. 10 position to the opposite side of the curve, bisecting it, and letting fall a perpendicular to the line of time, we find X. 32 as the time of high water. The compass, measuring the highest point of the curve, gives a little over 13 feet 1 inch as the height marked on the pole.



If we are at the place during the spring-tides, we can get a fair low-water datum by observation, and all soundings will be reduced to that, by the height marked on the pole above this datum, at the time the soundings were taken each day, being subtracted from them. But it may happen that we arrive at the place a few days after a spring-tide, and leave again before the next one. The only thing to do is to note the high-water mark on the shore, and ascertain by measurement how far it is above the high tide of the day as marked also on the shore, subtract the same quantity from the low-water mark on the pole of that day, and call that the low-water spring datum, subtracting perhaps a foot or two extra to be on the safe side.

Thus, suppose at high water our pole marks 13 feet 1 inch, and the high-water mark on the beach is 2 feet 6 inches above the level of the sea at that time ; at low water the pole marks 5 feet 8 inches. This will give us 3 feet 2 inches as the probable

low-water spring mark. If we reduce our soundings 2 feet below this to the 1-foot mark, we shall be pretty certain not to give too much water on shoal spots.

An approximation of this kind would be, of course, noted on the chart when sent home, and also the manner in which the rise of spring-tides, which would be given as 14 feet, has been obtained.

Rougher
Approximation of
Datum.

In still rougher work, an approximation of the rise of the tide may be got by having a marked boat-hook held upright at the water-line at time of low water ; the observer then places his eye at the high-water mark on the beach, and reads the mark on the boat-hook, where the horizon line cuts the latter, which will be the fall of the tide that day below high-water mark. If it is the high-water mark of the day that is so used, the result is the range of the tide for the day ; and if the distance that the spring's mark is above the day high-tide mark can be measured, we can arrive at the full rise and fall, as in the last article.

This may be very useful in making a hurried plan of a bay, and thus the height of the water can be got by the officer putting in the coast-line from time to time during the day, without delaying him much, and to the great advantage of the correctness of the soundings being taken at the time.

Estima-
tion of
Establish-
ment.

The "vulgar establishment" is an exceedingly loose term, as given on the charts. As it is strictly only on days when the moon's mer. pass. is 12^h . or 0^h ., that it can be directly observed, the surveyor is obliged to approximate to it in most cases. This perhaps matters the less from the fact that the establishment, even when correctly obtained, is seldom invariable.

The best way to approximate is to project the line of lunital intervals, and measure the length of the abscissæ from XII^h . and 0^h . for the vulgar establishment, meaning them if we get more than one.

If the tides are regular, especially as regards the semi-menstrual inequality, the establishment may be roughly determined by a method given in the article in the Tide Tables, from an observation of the tide at any period of the moon's transit, but which we shall not further discuss, as, in a case where it would be required, we should not know whether the

tides are regular or not, and any assumption would probably end in very erroneous results. Interpolating
Height of
Tide.

In a case where only the high and low waters on any day are obtained, the height of the tide, at times between, is best to be got at by drawing a curve in the imagined course of the tide, after the manner of Fig. 61; the height on the pole at any time can then be taken from this, and a table of reduction formed for the day.

Thus, in Fig 62, where we have only got the times and heights at high and low water on that day, viz., H. W. at VI. 50, mark on pole 15 feet 7 inches; L. W. at I. 10, mark on pole 8 feet.

On a piece of paper, either ruled in squares for the purpose, or on ready-printed squared paper, which is very useful to

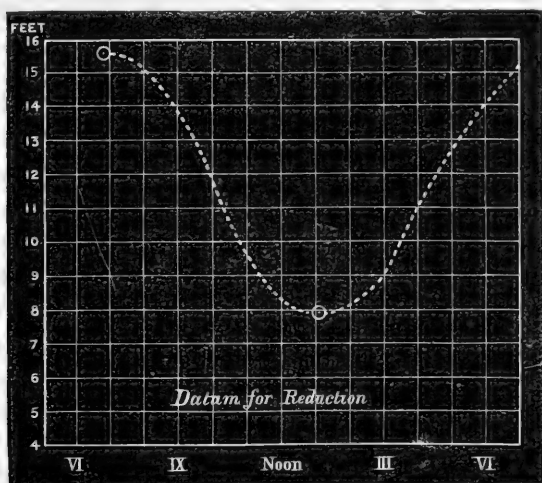


Fig. 62.

have by one for these like occasions, we draw a curve after the fashion shown. Then, supposing our datum for reduction to have been settled as 4 feet on the pole, our table of reduction for the day will stand as follows :

| | | | |
|------|--------------------------------------|-----|---------------------------------------|
| VI | 11 ft. or 2 fms. | I | 4 ft. or $\frac{3}{4}$ fm. |
| VII | $11\frac{1}{2}$ " or 2 " | II | $4\frac{1}{2}$ " or $\frac{3}{4}$ " |
| VIII | 11 " or 2 " | III | $5\frac{1}{2}$ " or 1 " |
| IX | $9\frac{3}{4}$ " or $1\frac{3}{4}$ " | IV | 7 " or $1\frac{1}{4}$ fms. |
| X | 8 " or $1\frac{1}{2}$ " | V | $8\frac{1}{2}$ " or $1\frac{1}{2}$ " |
| XI | 6 " or 1 fm. | VI | $10\frac{1}{2}$ " or $1\frac{3}{4}$ " |
| Noon | $4\frac{1}{2}$ " or $\frac{3}{4}$ " | VII | $11\frac{1}{2}$ " or 2 " |

This will of course be, as all attempts at arriving at anything with insufficient data, only an approximation, but will probably be near enough for the purposes we want. If we intend great accuracy, we shall make arrangements to have the tide carefully observed throughout the day, whenever sounding is going on, and take every precaution not to be reduced to these straits.

Table of
Reduction
under
Certain
Circum-
stances.

When tides are found to be regular, a table of reduction may be formed from the observations during one or more complete lunations, by tabulating the tides according to the time of moon's upper transit. Such a table may be very useful when the *scale of the chart is small*; and sounding can be carried on under these circumstances, viz., regular tides, and scale of chart small, when no direct observations can be got.

In many places external circumstances control our wishes. For instance, it was found on the East Coast of Africa that if men were landed to make a regular series of day and night observations on the tides, fever generally ensued, and consequently the record was restricted to day tides. Again, the tide-pole may have to be so placed on a shelving shore, or among reefs, that a boat would have to be used to go out at high water to read it, and this may not be convenient. Often, for considerable tracts among reefs, observations may be impossible, and a table deduced, as above suggested, from former observations may be then used.

Graphic
Projection
of Tidal
Move-
ment.

In forming such a table, it is best to project the tidal curves as shown in Fig. 62, but by the hourly observations. It will then be seen whether the tides are regular, and days of similar time of upper transit of the moon can be compared, to see whether a table will give us the reduction near enough for practical purposes.

Surveying ships are now supplied with abstract forms for the projection of high and low waters in such a manner that the regularity or otherwise of a tide can at once be seen.

They provide for the record of the lunitidal interval, the moon's meridian passage: the declination of the sun and moon, apogee and perigee, and the mean time of the high water following the superior transit, and of the highest tide in the twenty-four hours.

A horizontal line in the upper part of the diagram is divided

into hours with vertical lines drawn through them. These hours are those of the moon's superior transit. Directly under the position on the line of the local mean time of the moon's superior transit is plotted, as a dot, the high water next following that transit, at its proper position for height of the tide, as measured by the side scale in feet. When the next day's similar high water has been similarly plotted, the intermediate high water is plotted half-way between them at its proper height and the two low waters similarly interpolated.

The high waters following the superior transit are joined by a red line, and the intermediate high waters by a blue line.

Similarly with the low waters, taking care to note which are the low waters next following the superior transit. Each dot is joined to its next successive high or low water by a black line.

The lunitidal interval for each high water is plotted in the place provided, and the dots joined. By adding the moon's superior transit to the lunitidal interval as plotted, the mean time of each high water can, if necessary, be ascertained.

When the curves of the sun's and moon's declination, and of the moon's parallax, are plotted, the general movement of the tide, and its relation to the positions of the sun and moon in declination, can at once be seen.

The mean between each high and low water height will give roughly the mean water-level.

To obtain the true mean water-level in a few days, other observations must be taken, and we subjoin an extract from the "Instructions to H.M.S. *Challenger*," which contains full directions, only adding that these observations must be made when the tide is moving normally, that is, when there are no strong winds to raise or depress the water-level.

True
Mean
Water-
level.

"A good determination of the mean-level sea by the simple operation of taking means may be made, in less than two days, with even a moderate number of observations *properly distributed so as to subdivide both solar and lunar days into not less than three equal parts*. Suppose, for example, we choose eight-hour intervals, both solar and lunar. Take a lunar day at twenty-four hours forty-eight minutes solar time, which is near enough, and is convenient for division; and choosing any convenient hour for commencement, let the height of the

water be observed at the following times, reckoned from the commencement :

| h. m. | h. m. | h. m. |
|-------|-------|-------|
| 0 0 | 8 0 | 16 0 |
| 8 16 | 16 16 | 24 16 |
| 16 32 | 24 32 | 32 32 |

“ The observations may be regarded as forming three groups of three each, the member of each group being separated by eight hours solar or lunar, while one group is separated from the next by eight hours lunar or solar. In the mean of the nine results the lunar and solar semidiurnal and diurnal inequalities are all four eliminated.

“ Nine is the smallest number of observations which can form a complete series. If the solar day be divided into m and the lunar into n equal parts, where m and n must both be greater than 2, there will be m^n observations in the series ; and if either m or n be a multiple of 3, or of a larger number, the whole series may be divided into two or more series having no observation in common, and each complete in itself. The accuracy of the method can thus be tested, by comparing the means obtained from the separate sub-series of which the whole is made up.

“ Should the ship's stay not permit of the employment of the above method, a very fair determination may be made in less than a day, by taking the mean of n observations taken at intervals of the n th part of a lunar day, n being greater than 2. Thus if $n=3$, these observations require a total interval of time amounting to only sixteen hours thirty-two minutes. The theoretical error of this method is very small, and the result thus obtained is decidedly to be preferred to the mere mean of the heights at high and low water.

“ The mean level thus determined is subject to meteorological influences, and it would be desirable, should there be an opportunity, to redetermine it at the same place at a different time of year. Should a regular series of observations for a fortnight be instituted, it would be superfluous to make an independent determination of the mean sea-level by either of the above methods at the same time.”

Owing to diurnal inequality and other causes, principally perturbations of atmospheric pressure and the effect of wind,

mean water-level, as found by meaning the height of one high water with the following low water, will differ from that found by meaning the same low water with the following high water.

A fair approximation to the true mean water-level may be obtained by taking the mean of the results for successive tides obtained as above for a period extending over a considerable interval, especially if the barometric height be allowed for; the longer the period the better.

When mean water-level has been found, it should be referred to the Ordnance Datum in the United Kingdom, or abroad to some fixed plane of reference; and for this purpose natural marks should be preferred, such as rocks which cover or uncover at certain conditions of tide. If no natural marks are available, artificial marks should be created. The mark adopted should be stated in the memoir of the chart.

In some cases the mean level of the water may be made use of as a temporary datum for reducing the soundings.

Mean
Level as
Datum.

If, for instance, we commence soundings in a place where we do not yet know the spring's range, but intend to get it accurately after some months' observations, we may find it convenient to reduce all soundings to the mean level, as found by meaning each day's high and low water. Then, when we have ascertained the level of low springs below this mean level, one uniform quantity will have to be subtracted from every sounding, which will save a good deal of complication and waiting, as the soundings may all be plotted without fear of mistakes in reducing them afterwards.

This mode will be mostly used for shallow channels, where a difference of a foot or two is an important matter, but it is liable to the error caused by variation in the height of the mean tide-level.

When a long series of observations is plotted on the abstract form provided for the purpose, any peculiarity in the day and night tides should be noted in connection with the sun and moon's declination.

Analysing
a Tidal
Diagram

Other peculiarities connected with the moon's declination should also be noted if they exist, such as that of the diurnal inequality of height of low water being greater than the diurnal inequality of high water, or cases in which the diurnal inequality in the times becomes so large that there is only one

tide in twenty-four hours. This generally happens only for a few days in each semi-lunation ; at other times there are two tides, as usual.

In some places the tides rise and fall four times in the twenty-four hours ; these may be called *double half-day tides*. They do not commonly extend over any considerable length of coast.

A notable feature is the sequence in which the higher and lower low waters occur relatively to the higher and lower high waters. The connection between the moon's parallax and the range of tide is usually very characteristic, especially in the Pacific amongst the islands. The highest tides generally occur at equinoctial springs, when perigee occurs towards new or full moon. It sometimes happens, as in the Bay of Fundy, that the variation in range from perigee to apogee is greater than the difference in range at mean springs and mean neaps. All over the Pacific the leading feature of the tides is a pronounced diurnal inequality in time and height, which accords with the declination of the moon ; and it is also subject to an annual variation with the change in the declination of the sun. When the moon is farthest north or south of the Equator, the inequality between the two tides of the day is greatest. The extreme tides of the year necessarily occur at the nearest point to the solstices at which the moon reaches its maximum declination. On the other hand, the tides become equal when both the sun and moon are on the Equator, or when they are on opposite sides of the Equator at distances north and south which are proportional to their respective effects.

Determination of the Establishment in the Case of Large Diurnal Inequality in the Times.

When the diurnal inequality in the times is very perceptible, allowance is made for it by drawing a curve, cutting off equal portions above and below the zigzags formed by joining the points denoting the lunitidal intervals for successive high waters. This *mean line* will be of a wavy form in consequence of the semi-diurnal inequality ; and the ordinate corresponding to the new or full moon, or to the hours 0 or 12 of moon's transit, will give the establishment. But if we apply this establishment to predict the time of tide on any day, we must remember that the diurnal inequality will affect it.

Approximate Prediction of Tides by Means of Tidal Diagram.

To predict the tide for any particular day, if the diagram extends over a considerable period, conditions as similar as possible to those existing on that day as regards sun and

moon's declination and moon's parallax, should be sought for on the diagram.

The lunitidal interval and range of tide corresponding to the preceding transit of the moon for the day on that section of the diagram where the above-mentioned conditions are most nearly satisfied will give the approximate H. W. F. and C., and range of tide for that day. The reduction to low water for each hour of the tide can then be deduced by the method described by Captain T. H. Tizard, R.N., C.B., F.R.S., on p. 149 of the Admiralty Tide Tables.

If the diagram does not include the particular combination of conditions required, the predicted results may be considerably in error; but it is sometimes possible to apply rough corrections for the differences between the astronomical conditions for the day and those of the diagram by studying the effect on the lunitidal interval and range of tide produced by changes in the astronomical elements during the various lunations shown on the diagram, if there are a sufficient number to enable conclusions to be drawn.

As a river is ascended, the range of tide tends to first increase to a maximum, and then to decrease, until the undulation disappears entirely. Tide-poles for the reduction of soundings should therefore be erected at suitable intervals, and compared with each other, and a note should be made of the spot where the rise and fall ceases.

Range of Tide at Different Points in a Tidal River.

Comparisons should be effected by noting the high and low waters on all the tide-poles on the same days; the more observations that can be obtained, the better. The results will show the level of the river both at high and low water, especially if time admits of a series of levels being taken from the lower to the upper pole.

Comparison of Tide-Poles.

When the range of tide is very large, it is often convenient to use two or more tide-poles, placed near each other, but at different levels, so that when one tide-pole is nearly covered the other will show only 1 or 2 feet. Both tide-poles should be read and recorded in the interval, during which they may be compared.

Double Tide-Poles.

The indications of a tide-pole placed in river estuaries cannot be relied on to show the rise and fall of tide at any considerable distance on either side along the coast.

Tide-Pole in River Estuaries

**Effect of
Mud-
Flats.**

In a harbour such as Portsmouth, where extensive mud-flats cover or uncover at about the same time when the water reaches one particular level, its rise or fall will be arrested for a short interval whilst the water is overflowing or flowing off the flats.

**Directions
for reduc-
ing Tidal
Observa-
tions.**

The following method, taken from a pamphlet issued by the Hydrographic Department, is that adopted at the Admiralty in discussing the observations, and deducing therefrom the tide hour or establishment, marked on the chart as H. W. F. and C. Also the mode of determining the mean spring range or rise, the mean neap range, and the mean neap rise :

1. The time and height of high and low water should be observed successively both by day and night ; this is absolutely necessary to make the observations valuable ; and the watch or clock by which the times are taken should always show *mean time at the place*.

2. The greater the number of daily results thus obtained, the more accurate will be the value deduced from them ; but six months' observations under normal conditions are absolutely necessary for a definite determination.

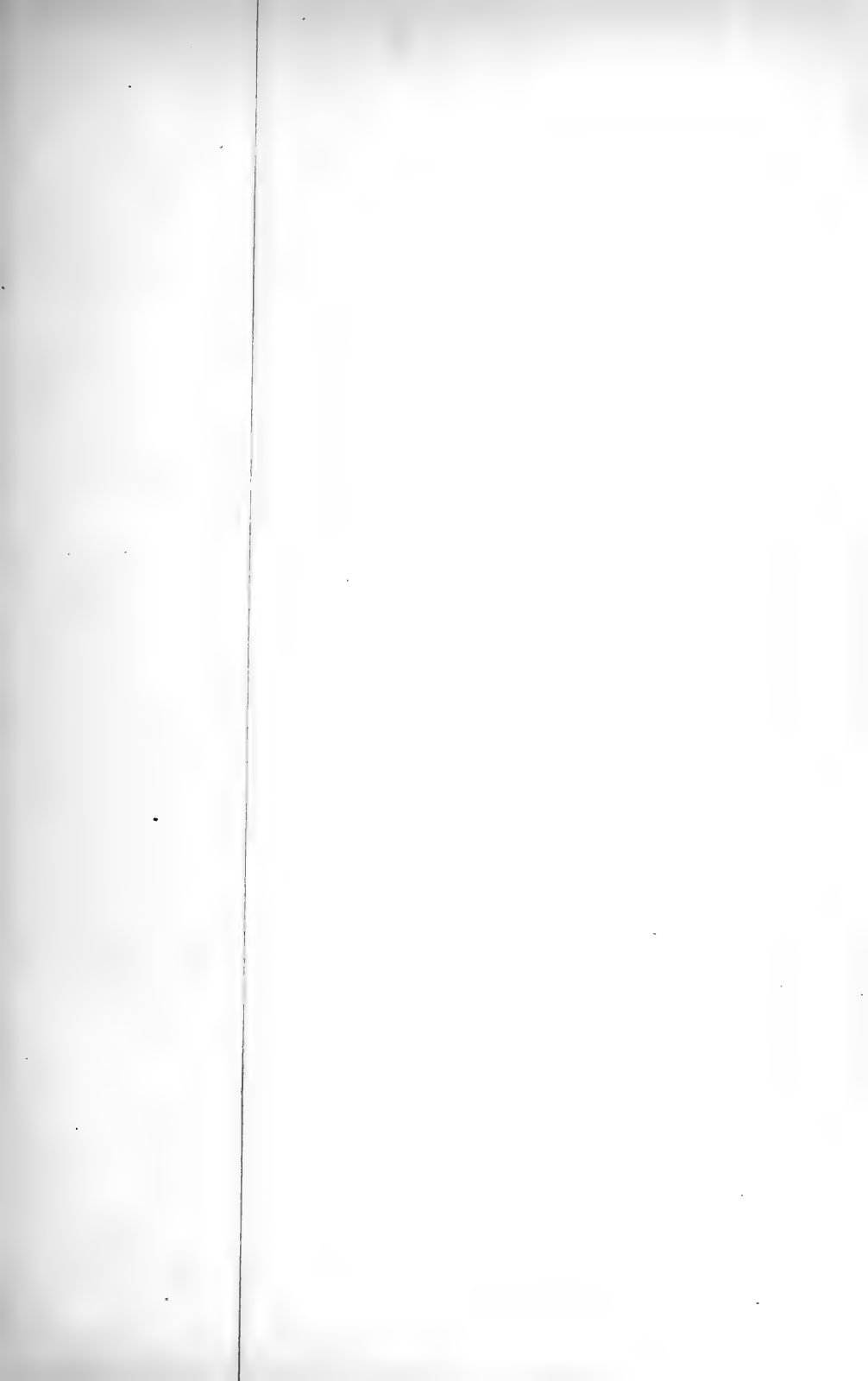
Where the conditions are varied at certain seasons, such as obtains in different monsoons at different periods, a year's observations are required.

If, however, circumstances will not admit of a continued series of tidal observations being made, the high and low waters that take place on those days when the moon's transit occurs between 11^h and 2^h and 6^h and 8^h should be preferred. From the former (11^h and 2^h) the tide hour or establishment and the mean spring range are obtained, and from the latter (6^h and 8^h) the mean neap range is known.*

3. The times and heights of high and low water must be compared with the moon's upper and lower transit, in order to obtain the mean intervals and the mean heights (see Table I.). The transit in the Nautical Almanack being given for the meridian of Greenwich, the transit for any other meridian must be corrected for longitude, a table for which is given in most works on navigation.

4. The times and heights are to be classified according to the moon's transit, beginning with the high and low water

* With regard to this, it is considered that the remark only applies to the abnormal tides of Europe, and not to the Pacific tides, etc.



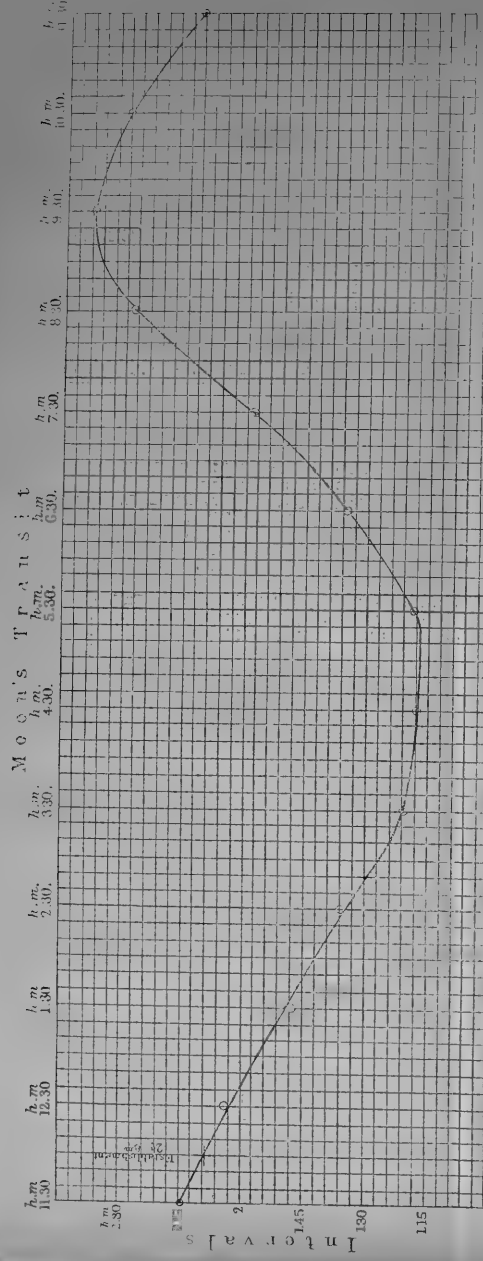


Diagram 1

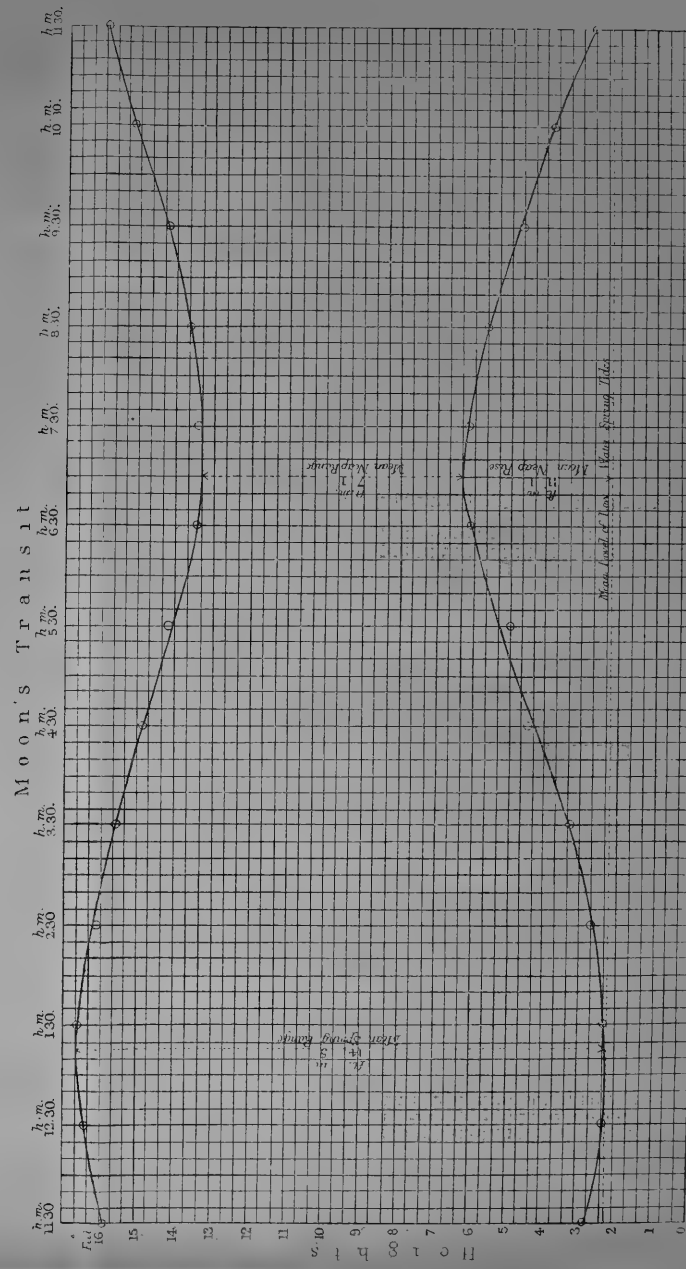


Diagram II.

immediately following the transit that occurs between 11^h and 12^h , then between 12^h and 1^h , then 1^h and 2^h , and so on to 10^h and 11^h . The mean of each class of transits being taken from the mean of the high water following that transit, the remainder shows the mean interval after that mean transit (see Table I.).

5. Having thus obtained the twelve mean intervals, then, on a sheet of squared paper (the horizontal lines being considered ordinates, and the vertical lines the abscissæ), set off the successive mean intervals as ordinates, the moon's transit being the abscissæ.

Through the extremity of these ordinates draw a curve, and the ordinate of this curve that corresponds to the moon's transit 0^h or 12^h is the time of H. W. F. and C. given on the Admiralty charts (see Diagram I.). This is the vulgar establishment, the mean establishment being the mean of all the intervals after the moon's transit.

6. The *heights* of high and low water as ordinates are laid off in a similar way to the intervals, the time of the moon's transit, as before, being the abscissæ. Through the extremity of these ordinates draw a curve, and the difference between the *minimum*, or least height of low water, and the *maximum*, or greatest height of high water, shows the mean spring range or rise; the difference between the *maximum* height of low water and the *minimum* height of high water gives the mean neap range; and the difference between the *minimum* height of low water and the *minimum* height of high water the mean neap rise (see Diagram II.). Where the diurnal irregularity is great, the results should be given both for the maximum and minimum tides.

7. The *minimum* height of low water is assumed as the zero or mean low-water level at springs, being the level to which the soundings on the Admiralty charts are reduced, and also the zero or standard level from which the heights given in the Admiralty Tide Tables are calculated. The zero of the tide-scale (as in Diagram II.) may be some feet below the mean low water or zero here assumed, in order to embrace the range of the equinoctial tides.

As an example, Table I. shows the procedure described above, the times and heights of high and low water being taken from a series of tidal observations taken at Dunbar.

TABLE I.

SHOWING THE MEAN INTERVAL BETWEEN THE TIME OF HIGH WATER AND THE MOON'S TRANSIT PRECEDING HIGH WATER, AND THE MEAN HEIGHTS OF HIGH AND LOW WATER CORRESPONDING TO THAT TRANSIT.

| | | High Water. | | | | Low Water. | |
|----------------|-----------------|--------------------------------|---------|----------------|-----------------|--------------------------------|---------|
| Date. | Moon's Transit. | Following a Preceding Transit. | | Date. | Moon's Transit. | Following a Preceding Transit. | |
| | | Time. | Height. | | | Time. | Height. |
| 1856. | h. m. | h. m. | ft. in. | 1856. | h. m. | h. m. | ft. in. |
| June 2, p.m. | 11 27 | 1 27 | 16 7 | June 2, p.m. | 11 27 | 7 50 | 2 8 |
| 3, a.m. | 56 | 1 50 | 16 7 | 3, a.m. | 56 | 8 10 | 2 8 |
| 17, p.m. | 4 | 1 35 | 15 3 | 17, p.m. | 4 | 7 27 | 4 0 |
| 18, a.m. | 32 | 1 40 | 15 7 | 18, a.m. | 32 | 7 40 | 3 9 |
| July 1, p.m. | 13 | 1 5 | 15 5 | July 1, p.m. | 13 | 7 40 | 3 6 |
| 2, a.m. | 42 | 1 55 | 15 8 | 2, a.m. | 42 | 8 5 | 2 10 |
| 17, a.m. | 16 | 1 20 | 15 2 | 17, a.m. | 16 | 7 40 | 3 8 |
| 17, p.m. | 46 | 1 50 | 15 6 | 17, p.m. | 46 | 7 55 | 3 8 |
| 31, a.m. | 26 | 1 42 | 15 3 | 31, a.m. | 26 | 7 50 | 2 6 |
| 31, p.m. | 53 | 2 12 | 15 3 | 31, p.m. | 53 | 8 5 | 3 10 |
| Aug. 15, p.m. | 27 | 1 42 | 16 4 | Aug. 15, p.m. | 27 | 7 40 | 4 1 |
| 16, a.m. | 56 | 1 42 | 15 8 | 16, a.m. | 56 | 8 0 | 2 0 |
| 29, a.m. | 1 | 1 35 | 15 9 | 29, a.m. | 1 | 7 57 | 3 4 |
| 29, p.m. | 24 | 2 5 | 16 2 | 29, p.m. | 24 | 8 5 | 4 8 |
| 30, a.m. | 45 | 2 5 | 15 10 | 30, a.m. | 45 | 8 27 | 2 9 |
| Sept. 13, p.m. | 1 | 1 20 | 16 2 | Sept. 13, p.m. | 1 | 7 27 | 2 4 |
| 14, a.m. | 28 | 1 40 | 16 8 | 14, a.m. | 28 | 7 57 | 0 4 |
| 14, p.m. | 54 | 2 10 | 16 10 | 14, p.m. | 54 | 8 10 | 2 4 |
| Oct. 13, p.m. | 21 | 1 42 | 17 2 | Oct. 14, a.m. | 47 | 7 57 | 0-6 |
| 28, p.m. | 24 | 1 57 | 15 6 | 28, a.m. | 3 | 7 40 | 1 11 |
| | | | | 9, a.m. | 45 | 8 27 | 2 0 |
| 20 days | 11 29'8 | 1 43'7 2 13'9 | 15 11'0 | 21 days | 11 30'8 | | 2 9'3 |

| | | | | | | | |
|----------------|--------|-----------------|--------|----------------|--------|------|-------|
| June 3, p.m. | 0 26 | 2 20 | 16 10 | June 3, p.m. | 0 26 | 8 27 | 2 5 |
| 4, a.m. | 56 | 2 45 | 16 7 | 4, a.m. | 56 | 8 55 | 2 0 |
| 18, p.m. | 1 | 2 5 | 15 3 | 18, p.m. | 1 | 7 57 | 3 5 |
| 19, a.m. | 30 | 2 30 | 15 9 | 19, a.m. | 30 | 8 35 | 3 2 |
| July 2, p.m. | 12 | 2 10 | 15 9 | July 2, p.m. | 12 | 8 27 | 3 6 |
| 3, a.m. | 40 | 2 50 | 15 10 | 3, a.m. | 40 | 9 0 | 2 6 |
| 18, a.m. | 16 | 2 0 | 15 10 | 18, a.m. | 16 | 8 20 | 2 9 |
| 18, p.m. | 45 | 2 42 | 16 2 | 18, p.m. | 45 | 8 27 | 3 5 |
| Aug. 1, a.m. | 17 | 2 27 | 15 9 | Aug. 1, a.m. | 17 | 8 30 | 2 1 |
| 1, p.m. | 42 | 2 57 | 15 4 | 1, p.m. | 42 | 8 50 | 3 8 |
| 16, p.m. | 23 | 2 25 | 16 11 | 16, p.m. | 23 | 8 30 | 2 6 |
| 17, a.m. | 51 | 2 30 | 17 4 | 17, a.m. | 51 | 8 55 | 0 9 |
| 30, p.m. | 7 | 2 42 | 15 4 | 30, p.m. | 7 | 8 45 | 3 5 |
| 31, a.m. | 27 | 2 57 | 16 7 | 31, a.m. | 27 | 9 10 | 2 1 |
| 31, p.m. | 47 | 3 0 | 16 4 | 31, p.m. | 47 | 9 25 | 3 6 |
| Sept. 15, a.m. | 20 | 1 55 | 17 9 | Sept. 15, a.m. | 20 | 8 27 | 0-1 |
| 15, p.m. | 45 | 2 42 | 17 8 | 15, p.m. | 45 | 8 55 | 1 3 |
| Oct. 14, p.m. | 14 | 2 15 | 17 7 | Oct. 15, a.m. | 41 | 8 30 | 0 3 |
| 29, p.m. | 6 | 2 40 | 15 11 | 30, a.m. | 28 | 8 42 | 2 0 |
| 30, p.m. | 51 | 3 12 | 15 11 | | | | |
| 20 days | 0 28'8 | 2 33'2 2 4'4 | 16 3'8 | 19 days | 0 30'2 | | 2 4'2 |

TABLE I.—*continued.*

SHOWING THE MEAN INTERVAL BETWEEN THE TIME OF HIGH WATER AND THE MOON'S TRANSIT PRECEDING HIGH WATER, AND THE MEAN HEIGHTS OF HIGH AND LOW WATER CORRESPONDING TO THAT TRANSIT.

| Date. | Moon's Transit. | High Water. | | Date. | Moon's Transit. | Low Water. | |
|---------------|--------------------|--------------------------------------|---------|---------------|--------------------|--------------------------------------|---------|
| | | Following a Preceding Transit. | | | | Following a Preceding Transit. | |
| | | Time. | Height. | | | Time. | Height. |
| 1856. | h. m. | h. m. | ft. in. | 1856. | h. m. | h. m. | ft. in. |
| June 4, p.m. | 1 26 | 3 5 | 16 4 | June 4, p.m. | 1 26 | 9 15 | 3 2 |
| 5, a.m. | 55 | 3 30 | 16 0 | 5, a.m. | 55 | 9 30 | 2 1 |
| 19, p.m. | 0 | 3 0 | 15 10 | 19, p.m. | 0 | 8 50 | 3 8 |
| 20, a.m. | 30 | 3 0 | 16 0 | 20, a.m. | 30 | 9 20 | 2 9 |
| 20, p.m. | 59 | 3 35 | 15 9 | 20, p.m. | 59 | 9 40 | 3 1 |
| July 3, p.m. | 8 | 3 10 | 15 10 | July 3, p.m. | 8 | 8 55 | 3 9 |
| 4, a.m. | 34 | 3 20 | 16 0 | 4, a.m. | 34 | 9 55 | 2 0 |
| 19, a.m. | 15 | 2 50 | 16 8 | 19, a.m. | 15 | 8 57 | 2 4 |
| 19, p.m. | 42 | 3 20 | 16 7 | 19, p.m. | 42 | 9 35 | 3 1 |
| Aug. 2, a.m. | 4 | 2 57 | 16 1 | Aug. 2, a.m. | 4 | 9 12 | 2 2 |
| 2, p.m. | 27 | 3 20 | 15 5 | 2, p.m. | 27 | 9 35 | 2 10 |
| 3, a.m. | 48 | 3 27 | 15 11 | 3, a.m. | 48 | 9 45 | 1 8 |
| 17, p.m. | 16 | 3 5 | 17 5 | 17, p.m. | 16 | 9 12 | 2 8 |
| 18, a.m. | 42 | 3 20 | 18 8 | 18, a.m. | 42 | 9 50 | 2 6 |
| Sept. 1, a.m. | 6 | 3 12 | 16 8 | Sept. 1, a.m. | 6 | 9 27 | 2 1 |
| 1, p.m. | 26 | 3 40 | 16 0 | 1, p.m. | 26 | 9 50 | 2 8 |
| 2, a.m. | 45 | 3 30 | 16 6 | 2, a.m. | 45 | 9 55 | 2 6 |
| 16, a.m. | 11 | 2 50 | 19 1 | 16, a.m. | 11 | 9 12 | 0 0 |
| 16, p.m. | 37 | 3 25 | 18 2 | 16, p.m. | 37 | 9 27 | 1 1 |
| Oct. 1, p.m. | 25 | 3 25 | 16 2 | Oct. 1, a.m. | 4 | 9 12 | 2 3 |
| 2, a.m. | 46 | 3 45 | 16 7 | 1, p.m. | 25 | 9 27 | 3 0 |
| 15, p.m. | 9 | 2 55 | 17 11 | 2, a.m. | 46 | 9 42 | 3 1 |
| 31, p.m. | 40 | 3 20 | 15 11 | 16, a.m. | 38 | 9 30 | 0 6 |
| | | | | 31, a.m. | 15 | 9 10 | 2 11 |
| 23 days | 1 28'3 | 3 15'7 1 47'4 | 16 7'0 | 24 days | 1 27'5 | | 2 4'9 |
| June 5, p.m. | 2 25 | 3 55 | 15 8 | June 5, p.m. | 2 25 | 9 50 | 3 3 |
| 6, a.m. | 52 | 4 15 | 15 3 | 6, a.m. | 52 | 10 30 | 1 11 |
| 21, a.m. | 28 | 3 50 | 15 6 | 21, a.m. | 28 | 9 57 | 2 0 |
| 21, p.m. | 56 | 4 35 | 15 6 | 21, p.m. | 56 | 10 20 | 3 6 |
| July 4, p.m. | 0 | 3 45 | 15 6 | July 4, p.m. | 0 | 9 35 | 3 10 |
| 5, a.m. | 24 | 3 40 | 15 9 | 5, a.m. | 24 | 9 50 | 2 7 |
| 5, p.m. | 48 | 4 27 | 15 4 | 5, p.m. | 48 | 10 12 | 4 0 |
| 20, a.m. | 10 | 3 40 | 16 10 | 20, a.m. | 10 | 9 42 | 1 8 |
| 20, p.m. | 35 | 4 20 | 16 8 | 20, p.m. | 35 | 10 20 | 2 11 |
| Aug. 3, p.m. | 9 | 3 50 | 15 0 | Aug. 3, p.m. | 9 | 9 57 | 3 8 |
| 4, a.m. | 29 | 4 5 | 15 7 | 4, a.m. | 29 | 10 10 | 2 0 |
| 5, p.m. | 49 | 4 35 | 14 8 | 4, p.m. | 49 | 10 35 | 3 9 |
| 18, p.m. | 7 | 4 20 | 17 3 | Sept. 2, p.m. | 5 | 9 50 | 3 0 |
| Sept. 2, p.m. | 5 | 4 0 | 15 7 | 3, a.m. | 25 | 10 12 | 2 4 |
| 3, a.m. | 25 | 4 0 | 15 10 | 3, p.m. | 45 | 10 25 | 3 2 |
| 3, p.m. | 45 | 4 27 | 14 9 | 17, a.m. | 3 | 9 42 | 0 4 |
| 17, a.m. | 3 | 3 40 | 18 3 | 17, p.m. | 30 | 10 5 | 2 6 |
| 17, p.m. | 30 | 4 0 | 18 8 | 18, a.m. | 57 | 10 27 | 1 7 |
| 18, a.m. | 57 | 4 10 | 18 5 | Oct. 2, p.m. | 8 | 9 45 | 4 1 |
| Oct. 2, p.m. | 8 | 3 57 | 16 2 | 3, a.m. | 31 | 10 12 | 3 2 |
| 3, a.m. | 31 | 4 0 | 16 0 | 3, p.m. | 54 | 10 5 | 4 3 |
| 3, p.m. | 54 | 4 25 | 15 3 | 17, a.m. | 38 | 10 5 | 1 6 |
| 16, p.m. | 8 | 3 50 | 17 3 | | | | |
| 23 days | 2 27'7 | 4 4'6 1 36'9 | 16 1'4 | 22 days | 2 30'0 | | 2 9'3 |

TABLE I.—*continued.*

SHOWING THE MEAN INTERVAL BETWEEN THE TIME OF HIGH WATER AND THE MOON'S TRANSIT PRECEDING HIGH WATER, AND THE MEAN HEIGHTS OF HIGH AND LOW WATER CORRESPONDING TO THAT TRANSIT.

| Date. | Moon's Transit. | High Water. | | Date. | Moon's Transit. | Low Water. | |
|---------------|--------------------|--|--------------------|---------------|--------------------|--------------------------------------|------------------|
| | | Following a Preceding Transit. | | | | Following a Preceding Transit. | |
| | | Time. | Height. | | | Time. | Height. |
| 1856. | h. m. | h. m. | ft. in. | 1856. | h. m. | h. m. | ft. in. |
| June 6, p.m. | 3 19 | 4 45 | 14 8 | June 6, p.m. | 3 19 | 10 42 | 4 1 |
| 7, a.m. | 44 | 4 50 | 14 9 | 7, a.m. | 44 | 11 5 | 3 2 |
| 22, a.m. | 24 | 4 35 | 15 9 | 22, a.m. | 24 | 10 40 | 2 7 |
| 22, p.m. | 50 | 5 5 | 15 7 | 22, p.m. | 50 | 11 5 | 4 1 |
| July 6, a.m. | 10 | 4 27 | 15 6 | July 6, a.m. | 10 | 10 50 | 3 3 |
| 6, p.m. | 32 | 5 5 | 15 0 | 6, p.m. | 32 | 10 50 | 4 7 |
| 7, a.m. | 52 | 5 5 | 15 5 | 7, a.m. | 52 | 11 20 | 3 6 |
| 21, a.m. | 1 | 4 12 | 16 8 | 21, a.m. | 1 | 10 40 | 1 0 |
| 21, p.m. | 26 | 5 5 | 16 4 | 21, p.m. | 26 | 10 50 | 3 0 |
| 22, a.m. | 51 | 5 5 | 16 8 | 22, a.m. | 51 | 11 12 | 1 5 |
| Aug. 5, a.m. | 8 | 4 27 | 15 1 | Aug. 5, a.m. | 8 | 10 50 | 2 3 |
| 5, p.m. | 28 | 5 5 | 14 3 | 5, p.m. | 28 | 11 5 | 3 9 |
| 6, a.m. | 47 | 5 20 | 14 10 | 6, a.m. | 47 | 11 35 | 3 1 |
| Sept. 4, a.m. | 5 | 4 40 | 15 8 | Sept. 4, a.m. | 5 | 10 30 | 3 4 |
| 4, p.m. | 26 | 4 57 | 15 0 | 4, p.m. | 26 | 10 55 | 4 0 |
| 5, a.m. | 48 | 5 20 | 14 11 | 5, a.m. | 48 | 11 5 | 3 11 |
| 18, p.m. | 25 | 4 35 | 17 2 | 18, p.m. | 25 | 10 55 | 2 11 |
| 19, p.m. | 54 | 4 50 | 17 2 | 19, a.m. | 54 | 11 5 | 3 6 |
| Oct. 4, a.m. | 18 | 4 35 | 15 3 | Oct. 4, a.m. | 18 | 10 35 | 4 6 |
| 4, p.m. | 43 | 4 45 | 15 3 | 4, p.m. | 43 | 10 50 | 4 6 |
| 17, p.m. | 9 | 4 35 | 16 10 | 18, a.m. | 40 | 10 50 | 3 3 |
| 21 days | 3 29 ⁵ | 4 49 ⁷ 1 20 ² | 15 7 ³ | 21 days | 3 31 ⁰ | | 3 2 ⁸ |
| June 7, p.m. | 4 9 | 5 50 | 14 3 | June 7, p.m. | 4 9 | 11 27 | 5 2 |
| 8, a.m. | 32 | 5 50 | 14 9 | 8, p.m. | 32 | 12 20 | 3 6 |
| 8, p.m. | 55 | 6 20 | 13 6 | 9, a.m. | 55 | 12 20 | 5 5 |
| 23, a.m. | 16 | 5 17 | 15 6 | 23, a.m. | 16 | 11 35 | 2 8 |
| 23, p.m. | 41 | 6 5 | 14 6 | 23, p.m. | 41 | 11 57 | 3 11 |
| July 7, p.m. | 13 | 5 42 | 14 10 | July 7, p.m. | 13 | 11 27 | 5 7 |
| 8, a.m. | 33 | 6 5 | 15 3 | 8, a.m. | 33 | 11 57 | 4 2 |
| 8, p.m. | 53 | 6 20 | 13 11 | 8, p.m. | 53 | 11 50 | 5 8 |
| 22, p.m. | 15 | 5 20 | 16 0 | 22, p.m. | 15 | 11 42 | 3 7 |
| 23, a.m. | 39 | 5 42 | 16 8 | 23, p.m. | 39 | 12 10 | 2 5 |
| Aug. 6, p.m. | 7 | 5 50 | 14 0 | Aug. 6, p.m. | 7 | 11 27 | 4 2 |
| 7, a.m. | 27 | 5 50 | 14 8 | 7, a.m. | 27 | 11 57 | 3 10 |
| 7, p.m. | 47 | 6 12 | 13 7 | 8, a.m. | 47 | 12 7 | 5 4 |
| 21, a.m. | 14 | 5 30 | 17 9 | Sept. 5, p.m. | 10 | 11 20 | 4 9 |
| 21, p.m. | 40 | 6 20 | 15 8 | 6, a.m. | 34 | 11 50 | 4 10 |
| Sept. 5, p.m. | 10 | 5 50 | 14 6 | 7, a.m. | 58 | 12 12 | 5 8 |
| 6, a.m. | 34 | 5 35 | 14 6 | 20, a.m. | 52 | 11 50 | 4 8 |
| 6, p.m. | 58 | 6 5 | 13 11 | Oct. 5, a.m. | 10 | 11 0 | 5 4 |
| Oct. 5, a.m. | 10 | 5 5 | 15 0 | 5, p.m. | 37 | 11 25 | 5 5 |
| 5, p.m. | 37 | 5 27 | 14 2 | | | | |
| 18, p.m. | 10 | 5 5 | 15 0 | | | | |
| 21 days | 4 28 ⁶ | 5 46 ⁷ 1 18 ¹ | 14 10 ² | 19 days | 4 30 ⁹ | | 4 6 ⁴ |

TABLE I.—*continued.*

SHOWING THE MEAN INTERVAL BETWEEN THE TIME OF HIGH WATER AND THE MOON'S TRANSIT PRECEDING HIGH WATER, AND THE MEAN HEIGHTS OF HIGH AND LOW WATER CORRESPONDING TO THAT TRANSIT.

| Date. | Moon's Transit. | High Water. | | Date. | Moon's Transit. | Low Water. | |
|---------------|--------------------|--------------------------------------|---------|---------------|--------------------|--------------------------------------|---------|
| | | Following a Preceding Transit. | | | | Following a Preceding Transit. | |
| | | Time. | Height. | | | Time. | Height. |
| 1856. | h. m. | h. m. | ft. in. | 1856. | h. m. | h. m. | ft. in. |
| June 9, a.m. | 5 16 | 6 30 | 13 9 | June 9, p.m. | 5 16 | 12 50 | 4 8 |
| 9, p.m. | 38 | 7 30 | 13 5 | 10, a.m. | 38 | 12 50 | 6 1 |
| 10, a.m. | 58 | 7 20 | 13 5 | 10, p.m. | 58 | 13 35 | 4 10 |
| 24, a.m. | 6 | 6 20 | 15 0 | 24, p.m. | 6 | 12 35 | 2 8 |
| 24, p.m. | 30 | 7 5 | 14 6 | 25, a.m. | 30 | 12 35 | 4 4 |
| 25, a.m. | 54 | 7 5 | 14 11 | 25, p.m. | 54 | 13 20 | 3 3 |
| July 9, a.m. | 12 | 6 27 | 14 2 | July 9, p.m. | 12 | 12 40 | 4 6 |
| 9, p.m. | 31 | 7 10 | 13 9 | 10, a.m. | 31 | 12 27 | 6 1 |
| 10, a.m. | 51 | 7 27 | 13 8 | 10, p.m. | 51 | 13 20 | 4 11 |
| 23, p.m. | 3 | 6 30 | 15 6 | 24, a.m. | 3 | 12 27 | 4 3 |
| 24, a.m. | 28 | 6 42 | 16 3 | 24, p.m. | 28 | 12 57 | 3 4 |
| 24, p.m. | 53 | 7 27 | 14 10 | 25, a.m. | 53 | 13 10 | 4 11 |
| Aug. 8, a.m. | 8 | 6 25 | 14 5 | Aug. 8, p.m. | 8 | 12 35 | 4 10 |
| 8, p.m. | 30 | 7 20 | 13 7 | 9, a.m. | 30 | 12 50 | 6 0 |
| 9, a.m. | 53 | 7 27 | 13 4 | 9, p.m. | 53 | 13 25 | 5 6 |
| 22, a.m. | 7 | 6 20 | 16 6 | 22, p.m. | 7 | 12 35 | 3 10 |
| 22, p.m. | 34 | 6 55 | 14 6 | Sept. 7, p.m. | 24 | 12 20 | 5 9 |
| Sept. 7, a.m. | 24 | 6 30 | 13 10 | 8, a.m. | 50 | 12 30 | 6 6 |
| 7, p.m. | 50 | 7 5 | 13 2 | 21, p.m. | 52 | 12 50 | 5 6 |
| 20, p.m. | 22 | 6 20 | 14 0 | Oct. 6, a.m. | 4 | 11 50 | 6 0 |
| Oct. 6, a.m. | 4 | 5 42 | 13 8 | 7, a.m. | 32 | 12 12 | 5 11 |
| 6, p.m. | 32 | 6 15 | 13 0 | 20, p.m. | 38 | 12 35 | 6 3 |
| 22 days | 5 29.1 | 6 48.7 1 19.6 | 14 2.8 | 22 days | 5 30.8 | | 5 0.0 |
| June 10, p.m. | 6 18 | 8 20 | 12 9 | June 11, a.m. | 6 18 | 1 50 | 6 6 |
| 11, a.m. | 37 | 8 35 | 12 9 | 11, p.m. | 37 | 2 20 | 5 0 |
| 11, p.m. | 57 | 9 27 | 13 7 | 12, a.m. | 57 | 2 50 | 7 0 |
| 25, p.m. | 17 | 8 5 | 14 4 | 26, a.m. | 17 | 1 50 | 5 4 |
| 26, a.m. | 41 | 8 20 | 14 9 | 26, p.m. | 41 | 2 20 | 3 6 |
| July 10, p.m. | 11 | 8 5 | 13 0 | July 11, a.m. | 11 | 1 42 | 6 6 |
| 11, a.m. | 32 | 8 20 | 13 5 | 11, p.m. | 32 | 2 20 | 5 6 |
| 11, p.m. | 53 | 9 5 | 13 1 | 12, a.m. | 53 | 2 50 | 6 8 |
| 25, a.m. | 18 | 7 40 | 15 9 | 25, p.m. | 18 | 1 57 | 4 3 |
| 25, p.m. | 44 | 8 27 | 14 2 | 26, a.m. | 44 | 2 20 | 5 6 |
| Aug. 9, p.m. | 16 | 7 57 | 12 10 | Aug. 1, a.m. | 16 | 1 35 | 6 7 |
| 10, a.m. | 41 | 8 20 | 12 10 | 10, p.m. | 41 | 2 12 | 6 2 |
| 23, a.m. | 2 | 7 27 | 14 8 | 23, p.m. | 2 | 1 25 | 4 7 |
| 23, p.m. | 30 | 7 55 | 13 5 | 24, a.m. | 30 | 1 50 | 5 2 |
| 24, a.m. | 59 | 8 35 | 14 5 | 24, p.m. | 59 | 2 40 | 6 9 |
| Sept. 8, a.m. | 17 | 7 30 | 12 11 | Sept. 8, p.m. | 17 | 1 15 | 6 7 |
| 8, p.m. | 45 | 8 15 | 12 6 | 9, a.m. | 45 | 2 27 | 6 7 |
| 21, p.m. | 21 | 7 35 | 13 10 | 22, a.m. | 21 | 1 27 | 6 3 |
| 22, a.m. | 50 | 8 12 | 14 0 | 22, p.m. | 50 | 2 20 | 7 9 |
| Oct. 7, a.m. | 0 | 6 50 | 12 3 | Oct. 7, p.m. | 0 | 0 42 | 6 6 |
| 7, p.m. | 29 | 7 45 | 12 6 | 8, a.m. | 29 | 1 57 | 6 6 |
| 8, a.m. | 57 | 8 20 | 12 6 | 8, p.m. | 57 | 2 27 | 7 5 |
| 21, a.m. | 31 | 7 35 | 13 0 | 21, p.m. | 31 | 2 5 | 7 6 |
| 23 days | 6 31.6 | 8 7.0 1 35.4 | 13 5.4 | 23 days | 6 31.6 | | 6 1.1 |

TABLE I.—*continued.*

SHOWING THE MEAN INTERVAL BETWEEN THE TIME OF HIGH WATER AND THE MOON'S TRANSIT PRECEDING HIGH WATER, AND THE MEAN HEIGHTS OF HIGH AND LOW WATER CORRESPONDING TO THAT TRANSIT.

| Date. | Moon's Transit. | High Water. | | Date. | Moon's Transit. | Low Water. | |
|----------------|--------------------|--------------------------------------|---------|----------------|--------------------|--------------------------------------|---------|
| | | Following a Preceding Transit. | | | | Following a Preceding Transit. | |
| | | Time. | Height. | | | Time. | Height. |
| 1856. | h. m. | h. m. | ft. in. | 1856. | h. m. | h. m. | ft. in. |
| June 12, a.m. | 7 16 | 9 35 | 13 6 | June 12, p.m | 7 16 | 3 35 | 5 7 |
| 12, p.m. | 36 | 10 12 | 13 6 | 13, a.m. | 36 | 4 5 | 6 11 |
| 13, a.m. | 56 | 10 20 | 13 6 | 13, p.m. | 56 | 4 35 | 5 5 |
| 26, p.m. | 5 | 9 5 | 13 10 | 27, a.m. | 5 | 2 50 | 5 3 |
| 27, a.m. | 30 | 9 12 | 15 2 | 27, p.m. | 30 | 3 35 | 4 0 |
| 27, p.m. | 55 | 10 12 | 14 6 | 28, a.m. | 55 | 4 5 | 5 3 |
| July 12, a.m. | 15 | 9 20 | 13 3 | July 12, p.m. | 15 | 3 27 | 5 10 |
| 12, p.m. | 37 | 9 57 | 13 3 | 13, a.m. | 37 | 3 57 | 6 9 |
| 26, a.m. | 11 | 8 42 | 14 9 | 26, p.m. | 11 | 3 0 | 4 11 |
| 26, p.m. | 36 | 9 42 | 13 11 | 27, a.m. | 39 | 3 35 | 5 7 |
| Aug. 10, p.m. | 6 | 8 42 | 12 6 | Aug. 11, a.m. | 6 | 2 50 | 7 1 |
| 11, a.m. | 23 | 9 20 | 12 3 | 11, p.m. | 33 | 3 55 | 6 0 |
| 24, p.m. | 28 | 9 15 | 13 9 | 25, a.m. | 28 | 3 27 | 6 5 |
| 25, a.m. | 57 | 9 50 | 13 10 | 25, p.m. | 57 | 4 20 | 6 6 |
| Sept. 9, a.m. | 14 | 9 5 | 12 6 | Sept. 9, p.m. | 14 | 3 12 | 7 1 |
| 9, p.m. | 43 | 9 27 | 12 7 | 10, a.m. | 43 | 4 20 | 6 7 |
| 22, p.m. | 17 | 8 55 | 13 6 | 23, a.m. | 17 | 3 40 | 6 9 |
| 23, a.m. | 45 | 9 42 | 13 7 | 23, p.m. | 45 | 4 20 | 7 8 |
| Oct. 8, p.m. | 25 | 8 55 | 12 7 | Oct. 9, a.m. | 25 | 3 35 | 6 0 |
| 9, a.m. | 52 | 10 10 | 13 0 | 9, p.m. | 52 | 4 25 | 6 4 |
| 22, a.m. | 20 | 9 20 | 12 8 | 22, p.m. | 20 | 3 40 | 7 7 |
| 21 days | 7 30.5 | 9 28.5 1 58.0 | 13 5.1 | 21 days | 7 30.5 | | 6 2.0 |
| June 13, p.m. | 8 16 | 11 5 | 13 10 | June 14, a.m. | 8 16 | 5 25 | 6 5 |
| 14, a.m. | 37 | 11 5 | 14 0 | 14, p.m. | 37 | 5 27 | 5 1 |
| 14, p.m. | 59 | 11 50 | 14 3 | 15, a.m. | 59 | 5 50 | 5 8 |
| 28, a.m. | 21 | 10 20 | 14 7 | 28, p.m. | 21 | 4 56 | 3 7 |
| 28, p.m. | 48 | 11 12 | 14 5 | 29, a.m. | 48 | 5 10 | 4 5 |
| July 13, a.m. | 1 | 10 20 | 13 2 | July 13, p.m. | 1 | 4 35 | 5 9 |
| 13, p.m. | 26 | 11 5 | 13 7 | 14, a.m. | 26 | 5 5 | 5 11 |
| 14, a.m. | 52 | 11 20 | 13 5 | 14, p.m. | 52 | 5 27 | 5 1 |
| 27, a.m. | 7 | 10 20 | 14 4 | 27, p.m. | 7 | 4 35 | 5 3 |
| 27, p.m. | 35 | 11 5 | 13 11 | 28, a.m. | 35 | 5 5 | 5 1 |
| Aug. 11, p.m. | 1 | 10 27 | 12 5 | Aug. 12, a.m. | 1 | 4 50 | 6 4 |
| 12, a.m. | 30 | 11 10 | 13 3 | 12, p.m. | 30 | 5 10 | 6 4 |
| 25, p.m. | 25 | 10 35 | 14 0 | 26, a.m. | 25 | 5 5 | 6 3 |
| 26, a.m. | 54 | 11 20 | 14 11 | 26, p.m. | 54 | 5 55 | 6 9 |
| Sept. 10, a.m. | 12 | 10 27 | 13 0 | Sept. 10, p.m. | 12 | 4 50 | 6 5 |
| 10, p.m. | 42 | 11 5 | 13 4 | 11, a.m. | 42 | 5 27 | 5 6 |
| 23, p.m. | 10 | 10 12 | 13 6 | 24, a.m. | 10 | 5 5 | 5 3 |
| 24, a.m. | 35 | 11 20 | 13 5 | 24, p.m. | 35 | 5 20 | 6 5 |
| 24, p.m. | 58 | 11 20 | 14 0 | 25, a.m. | 58 | 6 5 | 4 7 |
| Oct. 10, a.m. | 46 | 11 27 | 13 10 | Oct. 10, p.m. | 46 | 5 40 | 5 0 |
| 24, a.m. | 46 | 11 50 | 13 3 | 23, p.m. | 5 | 4 50 | 6 6 |
| 21 days | 8 31.5 | 10 59.5 2 28.0 | 13 8.8 | 21 days | 8 29.5 | | 5 7.2 |

TABLE I.—*continued.*

SHOWING THE MEAN INTERVAL BETWEEN THE TIME OF HIGH WATER AND THE MOON'S TRANSIT PRECEDING HIGH WATER, AND THE MEAN HEIGHTS OF HIGH AND LOW WATER CORRESPONDING TO THAT TRANSIT.

| Date. | Moon's Transit. | High Water. | | Date. | Moon's Transit. | Low Water. | |
|----------------|--------------------|--------------------------------------|---------|----------------|--------------------|--------------------------------------|---------|
| | | Following a Preceding Transit. | | | | Following a Preceding Transit. | |
| | | Time. | Height. | | | Time. | Height. |
| 1856. | h. m. | h. m. | ft. in. | 1856. | h. m. | h. m. | ft. in. |
| June 15, p.m. | 9 22 | 12 20 | 14 4 | June 15, p.m. | 9 22 | 6 20 | 4 0 |
| 16, a.m. | 46 | 12 40 | 14 0 | 16, a.m. | 46 | 6 20 | 4 4 |
| 29, a.m. | 16 | 11 27 | 14 9 | 29, p.m. | 16 | 5 50 | 3 8 |
| 30, a.m. | 44 | 12 10 | 14 7 | 30, a.m. | 44 | 6 12 | 3 11 |
| July 15, a.m. | 19 | 12 5 | 13 7 | July 15, a.m. | 19 | 5 50 | 4 11 |
| 15, p.m. | 47 | 12 20 | 14 1 | 15, p.m. | 47 | 6 10 | 5 0 |
| 28, a.m. | 4 | 11 27 | 14 3 | 28, p.m. | 4 | 5 40 | 5 0 |
| 29, a.m. | 33 | 12 5 | 14 2 | 29, a.m. | 33 | 6 10 | 4 3 |
| Aug. 12, p.m. | 0 | 11 25 | 13 8 | Aug. 13, a.m. | 0 | 5 50 | 5 6 |
| 13, a.m. | 30 | 11 57 | 14 0 | 13, p.m. | 30 | 6 10 | 5 3 |
| 27, a.m. | 21 | 12 5 | 14 4 | 27, a.m. | 21 | 6 20 | 4 8 |
| 27, p.m. | 48 | 12 35 | 14 6 | 27, p.m. | 48 | 6 35 | 5 8 |
| Sept. 11, a.m. | 10 | 11 50 | 13 8 | Sept. 11, p.m. | 10 | 5 50 | 5 2 |
| 12, a.m. | 39 | 12 5 | 14 0 | 12, a.m. | 39 | 6 20 | 3 3 |
| 25, p.m. | 22 | 12 20 | 14 10 | 25, p.m. | 22 | 6 5 | 6 3 |
| 26, a.m. | 44 | 12 20 | 15 3 | 26, a.m. | 44 | 6 45 | 4 4 |
| Oct. 11, p.m. | 38 | 12 20 | 15 2 | | | | |
| 25, p.m. | 26 | 12 35 | 14 3 | | | | |
| 18 days | 9 28'3 | 12 7'0 2 38'7 | 14 3'6 | 16 days | 9 27'8 | | 4 8'4 |
| June 1, a.m. | 10 2 | 12 40 | 16 3 | June 16, p.m. | 10 11 | 6 57 | 4 0 |
| 1, p.m. | 29 | 12 55 | 16 7 | 17, a.m. | 37 | 7 20 | 4 3 |
| 16, p.m. | 11 | 12 57 | 14 7 | 30, p.m. | 13 | 6 42 | 3 5 |
| 17, a.m. | 37 | 13 5 | 15 1 | July 1, a.m. | 43 | 7 12 | 3 5 |
| 30, p.m. | 13 | 12 20 | 15 0 | 16, a.m. | 16 | 6 50 | 4 9 |
| July 1, a.m. | 43 | 12 57 | 15 5 | 16, p.m. | 46 | 7 5 | 4 5 |
| 16, a.m. | 16 | 12 42 | 14 10 | 29, p.m. | 3 | 6 40 | 4 5 |
| 16, p.m. | 46 | 13 12 | 15 0 | 30, a.m. | 31 | 7 10 | 2 9 |
| 29, p.m. | 3 | 12 35 | 14 4 | 30, p.m. | 59 | 7 30 | 4 6 |
| 30, a.m. | 31 | 12 57 | 14 8 | Aug. 14, a.m. | 0 | 6 42 | 4 5 |
| 30, p.m. | 59 | 13 35 | 15 6 | 14, p.m. | 29 | 7 20 | 4 9 |
| Aug. 14, a.m. | 0 | 12 20 | 14 3 | 15, a.m. | 59 | 7 35 | 3 0 |
| 14, p.m. | 29 | 13 5 | 15 3 | 28, a.m. | 13 | 7 12 | 3 10 |
| 15, a.m. | 59 | 13 20 | 15 8 | 28, p.m. | 38 | 7 20 | 4 11 |
| 28, a.m. | 13 | 13 0 | 15 4 | Sept. 12, p.m. | 7 | 6 40 | 3 10 |
| 28, p.m. | 38 | 13 12 | 15 4 | 13, a.m. | 35 | 7 0 | 2 0 |
| Sept. 12, p.m. | 7 | 12 40 | 14 9 | 26, p.m. | 6 | 6 55 | 5 0 |
| 13, a.m. | 35 | 12 45 | 15 5 | 27, a.m. | 26 | 7 20 | 3 7 |
| 27, a.m. | 26 | 13 5 | 15 9 | Oct. 27, a.m. | 23 | 6 50 | 2 5 |
| Oct. 27, p.m. | 43 | 13 25 | 15 5 | | | | |
| 20 days | 10 27'0 | 12 56'3 2 29'3 | 15 2'6 | 19 days | 10 26'0 | | 3 10'5 |

Tidal
Streams
and Sur-
face Cur-
rents.

Current
Log.

The direction and rate of the tidal streams and other currents must be observed.

This is best done under ordinary circumstances from the ship at anchor by means of a current log, which is simply a very large log-ship, and is worked in the ordinary manner, but with a longer interval of time. The line, which is small, is marked at every 10 feet, and is permitted to run out for an even number of minutes, varying according to the velocity of the current.

Then the rate per hour of the current = number of feet run out, divided by one hundred times the number of minutes.

Thus, if the log-ship is permitted to run for three minutes, and 220 feet of line pass out,—

$$\text{Rate per hour} = \frac{220}{300} = 0.73 \text{ knot.}$$

This current log should be hove at stated times, whenever the ship is on her surveying ground, and at anchor, and an entry made in the current log whether there is anything recorded or not, as negative results are in some ways as valuable as positive ones. Where the tidal range is great, and streams change their direction, these observations will be made at comparatively short intervals, in order to ascertain the movement of the water at different time of the tide. Where streams are strong, and of importance in navigation, assistants will be sent to heave the current log from a boat at anchor in different positions.

The current log can be kept by quartermasters, with supervision. A watch or clock with a seconds-hand is a requisite. In the Current Book will be entered the position, time, direction of drift of the log-ship, number of minutes it was allowed to run out, and number of feet of line run out, wind and force. Blank columns for rate per hour, and time of tide, will be filled up afterwards by the officer discussing the currents.

When the tidal streams run with considerable force, the most convenient method of observing them is to follow a Kisbie life-buoy, having a weighted bucket slung about one fathom underneath it. Keeping the boat close to the life-buoy, and fixing at intervals by sextant angles and noting the time of each fix, the direction and rate of the stream is determined on plotting the fixes.

High water at a standard port, or on the neighbouring shore, should always be used as a reference for the direction and rate of the tidal streams at the different hours. Low water should not be referred to, for the reason that in many places it does not take place midway between the two high waters, and mistakes may arise. Thus the period of the stream should be referred to as so many hours *before* or so many hours *after* high water. The flood-stream is usually that which continues to run after high water, but it should not be spoken of as such, in order to avoid confusion.

The direction of the tidal stream will frequently change after high or low water, and when this occurs, we must endeavour to find out whether the change of stream occurs at a regular time of the tide, as this is an important point in the navigation of channels.

Time of
Change of
Direction
of Tidal
Streams

The streams inshore may also turn at a different time to the streams in the offing.

It is sometimes convenient to plot on squared paper the interval between high water and the turn of the stream as one ordinate, and the time of high water as the other; joining the points thus found, the irregularities that occur are shown graphically.

If the time of high water is not known, the interval between moon's transit and turn of the stream may be substituted for the first ordinate and the time of moon's transit for the second.

In channels connecting two open areas of sea, the general law is that the stream will run for some hours, often for three hours, after the tide, as indicated by the rise or fall on the shore, has turned. This makes it very confusing to speak of a stream as the flood or ebb stream, and the term east-going or south-going, or whatever the main direction of the stream may be, should always be used in preference; for in such a case the direction of a stream may be the same for the last three hours of the flood, and the first three hours of the ebb.

The Theory of the Tides is one of the most complicated subjects that can be considered.

Theory of
the Tides.

Recent investigations by Sir W. Thomson and Professor G. H. Darwin have shown that the tidal movement may be considered to be the resultant of as many as thirty-three separate tidal waves. Some are dependent upon the moon,

others on the sun. Some occur once in the day, or are diurnal ; others twice, or semi-diurnal ; some have a period of a lunar month dependent on the moon's position in her orbit as regards the sun ; others have six monthly periods dependent on the sun's declination. The moon's declination, a variable quantity with a long period of years, controls others. The position of the moon's node, and her varying distance from the earth, are responsible for considerable waves, the perigee tide being always larger than one in apogee.

The system of harmonic analysis has been adopted for the clearing of these different waves, and is, when tides are very variable, the only method by which the calculation and forecasting of tides is possible.

It is not proposed to enter into this, which forms no part of the necessary work of a marine surveyor in the field, and is a subject in itself ; but it may be mentioned that while in a few regions the time-honoured practice of calculating the time of the tide from the moon's meridian passage, and its height from the mean of tides observed in connection with that meridian passage, may serve for practical purposes, in most parts of the world the movement is so complicated that for a satisfactory forecast the employment of harmonic analysis is necessary.

General
Remarks
on Tides.

From what has been said it is evident that observation of the tide for a short period will, when tides are complicated, afford no means of predicting them. The movement may be wholly different when the sun is north of the Equator and when it is south, and many other factors make it necessary to observe for at least a year to gather an idea of the behaviour of the tides at all seasons.

The variety in complicated tides is infinite. In some cases the water will rise to nearly the same level every tide, while the low water shows great differences ; in others it is *vice versa*. In some cases the diurnal inequality, the difference in height of each successive tide, will be equally distributed between both high and low water levels ; in others it affects only one, or may vary with the season.

One feature of a tide when there is much inequality is generally regular—*i.e.*, the succession of the inequality in height. At some places the higher high water is followed by a fall to

the lower low water; the tide then rises to the lower high water, then falls to the higher low water, and finally completes its round by a rise to the higher high water again. At others this succession is reversed, but for most places the succession, whichever way it takes place, is the same throughout the year. Fig. 63 will show the two movements.

When there is great but regular inequality, the higher tide will always be in the daytime when the sun is on one side of the Equator, and in the night-time when it is on the other.

When the diurnal tides are great, the inequality in height will sometimes be such as to cause a mere stand in the tide during either the rise or fall of one tide, giving the effect of only one high and low water in the twenty-four hours.

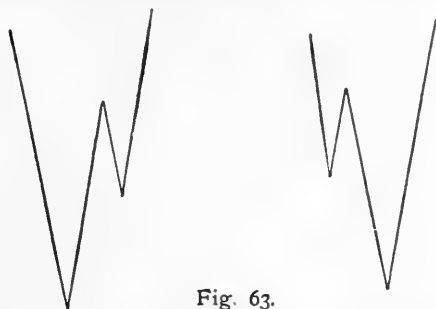


Fig. 63.

As a general rule, it may be stated that in temperate latitudes the highest tides take place at the equinoxes, whereas in the tropics these occur at the solstices.

Curiously enough, and it has affected many of our notions about tides, the tides about the British Isles are the most simple that are anywhere found—that is to say, so far as the individual movement of the tide at any one place is concerned. But they are largely affected by a further complication, known as “Interference.” By this is meant the appearance of another tidal wave, or perhaps more than one, which affects the height and time of the resultant tide at any place.

This is caused by either a tidal wave coming round from an opposite direction, or by one reflected from another coast, and it will be at once seen that if the crests of such tidal waves coincide at any point with the crest of the primary wave, the resultant tide will be higher, and if the crest of one reaches

a point at the same time as the hollow of another, the range of the tide may be, if the waves are of equal height, nothing.

To this is to be attributed the variety of the height of the tide at different parts of the coasts that appear fairly open.

Thus, in the English Channel the height of the tide varies at different places on the same day from 5 feet to 25 feet ; and the same phenomenon occurs in many parts of the world.

The tidal streams are not, however, directly affected in the same way, and there are many places where the rise of the tide is insignificant, but the tidal streams are very strong.

This is because the horizontal flow of the water is not determinable by the rise at the spot, but by the rise at other places, possibly at some distance, and by the fact that water once set in motion is not easily arrested.

The variations caused by interference are wholly distinct from the differences in the height of tide and velocity of streams caused by the conformation of the land.

The vertical movement of the water in the deep open ocean is not great, probably not more than 2 feet, and the horizontal motion is practically nil, being a merely insignificant oscillation. It is only when the water shoals that the friction of the bottom and the constriction caused by the water, which is in motion throughout its depth, being forced into a shorter column, cause the wave to become unnaturally heightened ; and horizontal movements, which we know as tidal streams, are set up, by reason of water flowing from the higher to the lower level.

Thus, on banks in mid-ocean regular tidal streams are found, and could the height of the water be measured, it would be found to vary more than in the deep water around.

The opposition of a coast and the shape of deep bays, gulfs, and channels accentuate these effects, and the height of the tide and velocity of the tidal streams vary in different places exceedingly. The oceanic tidal wave, thousands of miles in length, may have the distance between its crests shortened to hundreds, and the width of the portion that approaches the shore continuously narrowed as it passes up funnel-shaped passages.

Enough has been said to show that the tidal movements are exceedingly complicated, and though long-continued observa-

tion at one spot may enable predictions for that spot to be made, the variations are sometimes so great that at a short distance the phenomena are entirely different.

All prediction may be upset by what is known as the meteorological tide—that is, the variation in the height caused by winds, and by the difference in pressure of the air on the surface of the water.

The inconsistencies in the tide thus caused affect the height of the water more than the time of high or low water, and as they affect the mean level of the water, an unusually high tide induced by them does not mean an unusually low tide, but the reverse.

Wind will naturally have more effect when there is a funnel-shaped estuary than when it blows on to a straight, open coast, the heaped-up water at the wide mouth being forced higher and higher as it advances, for under such circumstances water will run up-hill.

From the foregoing it is evident that the mean level of the water will considerably vary. Mean Level.

When steady winds blow at certain times of the year, the variation in mean level will be seasonal ; in other places it may be constantly varying with the direction of the wind.

This variation in the mean level is important as regards navigation in some places. For instance, when a shallow flat exists which must be crossed to gain access to a harbour, and which at ordinary high water affords just sufficient depth, a change in the mean level may cause the high-water level to be 1, 2, or even 3 feet less than usual.

It is, therefore, most necessary for surveyors to acquaint themselves with the effect of wind at such places, and to record it.

Round the coasts of England and Scotland mean sea-level is usually a few inches above Ordnance datum, the Ordnance datum being 4.67 feet above the level of the old dock-sill at Liverpool. In some places, such as Sheerness and Grimsby, the difference is as much as 21 inches.

In cases where the rise of tide is great, in a funnel-shaped estuary much encumbered by sandbanks, and where there is a continuous outflow caused by a large river, the phenomena of the “ bore ” appears. Bore.

This consists in the face of the rising tidal wave becoming so steep that it rushes up the estuary in the shape of a sudden wave, sometimes almost a wall of water, breaking as it advances, and the tide thus rises many feet in a few seconds, followed by a still rapid, but more gradual, further rise, so that the whole flood may only last one or two hours, the ebb prevailing for nine or ten hours.

The main factor in the production of a bore is the retardation of the lower part of the inflowing water by the friction of the bottom in shallow water, and by the action of the down-flowing river, so that a high tide rises quicker than it can flow forward, until its height and momentum enable it to overcome these obstacles by a final grand rush.

Bores are rare, but whenever encountered the surveyor should investigate the conditions, as but little is known of the details of most of them.

CHAPTER X

TOPOGRAPHY

THE sketching in of the topography, or detail of the land, is a point on which there is more variation, as to the manner in which it is done, than in any other of the steps of a survey.

It is the least necessary part of a chart, which is destined mainly to guide over the water and not on the land ; but as we are guided over the water *by* the land, a perfect chart should have the features of the country correctly delineated, so as to assist the mariner in recognising the land by the mutual positions of peaks and other conspicuous objects. Furthermore, with our universal presence and interest all over the globe, it is impossible to say that an expedition may not want to start from some point on our chart, when information for a short distance inland will, in such a case, be most useful.

As a general rule, the land should be put in as far back from the shore as it is visible from the sea ; but this is only a very general guide, and must depend upon the distance of the back ranges, and the size of our sheet of paper. When the most distant mountains are very far back, we cannot spare time to do more than fix their summits by angles, get their heights and the extent of the range, and the country between must be a perfect blank.

Often, in savage lands, the country will be too dense with jungle to be able to do much to the topography by walking over it, which is, of course, the only way to get it correctly mapped, and we must then be content to sketch what we can see from the sea, and from the coast. By making stations in the ship, drawing a sketch at each, and getting angles to all prominent parts, such as spurs of hills, valleys, ravines, smaller peaks, etc., which will be entered on the sketch, a

Width of
Topo-
graphy
from
Coast.

Rough
Topo-
graphy.

very fair approximation of the position and shape of the more conspicuous elevations in the land, visible from seaward, will be made. The officer coast-lining will have got the entrances of all little streams fixed, and from the ship off shore we can recognise which ravines, or at any rate which of the larger ones, join on to these entrances. Topography put in in this way will present a somewhat detached appearance, and we can only fill up the hiatus by writing on the chart the general appearance of the land intervening between the hills, as far as we can see it from aloft, as, "rolling grassy plain," "densely wooded and undulating," etc. Sometimes, on a coast of this description, we can get back from time to time to an elevation we see from the ship to be partially clear, and a sketch from a position of that kind will materially improve our knowledge of the topography.

By referring to the sketch at p. 94 it will be seen how, with similar views from different points, ravines and valleys may be cut in, and roughly drawn on the chart.

Regular
Topo-
graphy.

When, however, we can spare the time to perfect our chart, and the nature of the country permits it, we should walk all over it, and sketch the topography on the ground. To do this, we must have as many conspicuous objects as possible fixed beforehand, and pricked on to a board, as for sounding or coast-line. Topography can be plotted afterwards, the same as can be done with coast-line or any other work, but it will be much more satisfactorily done if plotted at the time.

We then walk over our country, fixing ourselves with angles on commanding spots, plotting the stations by the station pointer or tracing-paper, and drawing lines from them to all things we want to plot—spurs of hills, houses, valleys, etc.—and sketching the details immediately around us. To fix details for this purpose we shall often have to content ourselves with two angles only, and as long as we do not use such points to carry on our stations with, this will be sufficient. A good deal of judgment is necessary in selecting spots to make stations, which cannot come without experience.

In placing the details on the paper on the rough board, sketch in the line of a valley first by the stream at the bottom, and then the adjacent hills or spurs.

A 10-foot or longer pole may be used with advantage in

sketching topography ; but if a range-finder is available, its use will facilitate the work enormously.

Difference of level can be at once obtained from a theodolite angle of elevation or depression by the formula :

$$\text{Diff. of level in feet} = \frac{\text{angle in secs.} \times \text{dist. in miles}}{34}.$$

Hills are best shown by contours. We do not, of course, ^{Contour} pretend that our contours are a fixed distance apart, but we ^{ing.} must endeavour to draw them approximately so, calling each contour line, 25, 50, or 100 feet apart, as the scale may require, and estimating the height of each spur with the assistance of a pocket barometer, if we have one, which will give us roughly the height of each station above the sea, if we read it when we land, and whenever we have occasion to do so. Each contour must be continued on from one hill to the other, or until it meets itself again round the hill ; and as their number and closeness together will roughly indicate the height of the hill, we must be careful not to get more on one side of a hill than another, or the value of this method will be lost, and the contours will simply show the shape of each spur, without reference to its relation in height, steepness, etc., to the next one, which is what we want to show as well. These contours will perhaps not appear in the finished chart, in which the mountains may be delineated in a different manner, but they will form an excellent guide for the amount of shade to be put on to the different hills and slopes, and it is the readiest and quickest method of showing this at the time.

The simplest form of clinometer is a bullet suspended from the zero-mark on a protractor, which is held with its edge uppermost in the direction of the slope, the angle of which it is desired to measure. The thread holding the bullet indicates the number of degrees from the horizontal at which the edge of the protractor is inclined. For different angles of slope a scale is constructed showing the distance apart at which the contours should be drawn corresponding to the scale of the chart. This is principally of service when working on large scales and dealing with long even slopes. Use of
Clinometer.

A useful and rapid method of sketching topography on a large scale is to select some commanding knoll for a station, and to

Topo-
graphy on
Large
Scales.

send two men with 10-foot pole end-boards connected by a 30-foot wire-cord to points which it is desired to fix within a radius of half a mile or so. The distance of each point is determined by measuring with sextant or theodolite the angle subtended by the 30-foot cord stretched taut between the two men, one of whom stands fast, and the other walks slowly backwards and forwards on the arc of a circle until the maximum angle is obtained. The height is found from the angle of elevation and distance, and can be taken out of the height tables supplied for work in the field.

When the angle of elevation or depression is large, it must be remembered that the distance found by the 30-foot cord method is the hypotenuse, and must be reduced to the horizontal distance, which is readily effected by means of the scale for that purpose on Craven's protractor.

Red and
Blue
Pencils.

Red and blue pencils are useful for topography. With the blue we show streams, and the red is used for marking roads. With only a black-lead pencil, the markings of these details are apt to get confused with the contour lines to express the hills.

Pocket
Sextant
and
Compass.

Much topography can be done with the pocket sextant and compass only, the latter being only used, however, when three objects to fix by cannot be got. The magnetic meridian, or several magnetic meridians, must be ruled on to the rough board, to permit the use of bearings. When the only objects available are much above us or below us, correct angles cannot be got with the sextant, and though we allow ourselves a certain amount of latitude in our angles for the purpose of topography, it will often be necessary to take a small theodolite for the purpose. A pocket sextant can be taken as well, and the theodolite, which requires more time to set up and arrange, only be used when the sextant angles will be too erroneous.

If we have a theodolite, we must take advantage of good opportunities to get a series of elevations and depressions for heights.

Diffi-
culties
with
Sextant
Angles.

In taking angles with a sextant to objects on different levels, try to find some natural mark which is exactly above or below, as the case may be, the object the farthest from your level, and nearly on a level with the other object, and

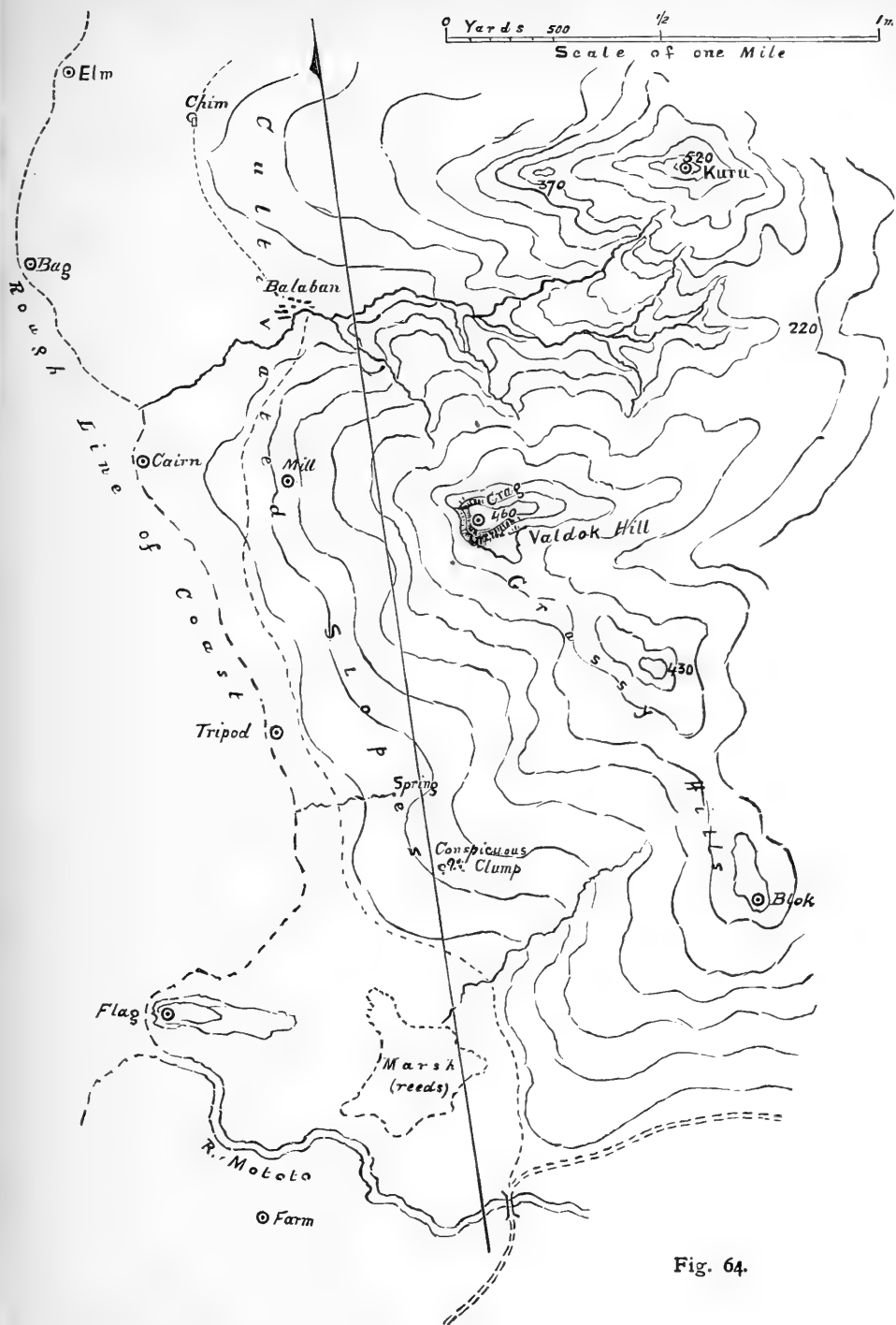


Fig. 64.

take this instead of the object itself. But it must be noted that unless this second object is nearly at the level of the observer, the angle will still be incorrect.

Fig. 64 shows a rough field-board, before any shading is placed on the sides of the valleys, which will be done with the brush before the work is considered completed.

Effect of
Light and
Shade.

In sketching topography from a distance, passing clouds or rain-squalls frequently throw up into strong relief details that would otherwise escape notice. The sun shining on a hill-side will often reveal details which may be absolutely lost a few hours later when the sun has moved round.

Baro-
meter.

The fluctuations of the barometer should be noted hourly on board the ship, and allowed for afterwards in calculating heights from readings of the aneroid.

The aneroid should be read at sea-level and at each station, the time being noted in each case.

It is more advantageous to use the aneroid to obtain differences of heights at comparatively short intervals than to rely on its indications for absolute height.

CHAPTER XI

HEIGHTS

By Theodolite—By Sextant—Obtaining Distance from Elevation of a known Height—Levelling.

FOR obtaining heights we must mainly depend on angles of elevation with sextant from afloat, and of elevation and depression with theodolite from shore stations. The pocket aneroid, though useful, as described under "Topography," to get subsidiary heights and assist in delineation of hills, is not to be depended upon.

At all main stations, and, in fact, any station well fixed and conveniently placed, angles of elevation and depression to the objects whose heights we want, should be taken throughout the course of the work. These are entered into the Height Book, and worked out when we can get the distances and occasion offers, the results being tabulated and meaned.

Elevations and depressions can be taken from any station whose height we shall eventually know ; but it is evident that any slight error in the true height of the observing station will be carried on into all heights deduced from it, and therefore it is well to get as many observations as we can from stations at the water-level, or so placed that the height above the water-level can be measured with a line.

In observing elevations and depressions with a theodolite, the instrument must be in fair adjustment, and carefully levelled, and it is further necessary to take into account the errors of level and collimation.

There are two ways of doing this. One is to take a series of observations with the telescope in its ordinary position, and then another with the telescope reversed, end for end, in the

Means used.

Stations for obtaining Heights.

Use of Theodolite.

Y's, when the mean of these two observations for each object will be the correct amount of elevation or depression. This is the best way, and eliminates all error. It may, however, be sometimes convenient to proceed as follows :

Ascertain the collimation error by directing the telescope on to an object in elevation, reading the vernier, then turning the telescope round until the level is uppermost, and again adjusting for the object and reading the vernier again. Half the difference between the readings is the collimation error, which, when the reading taken with the level uppermost is greatest, will be *added* to observations of elevations made with the telescope in its normal position, and *subtracted* from depressions. This collimation error is permanent for all positions of the horizontal arc.

For level error, at each observation of each separate object, the telescope must be brought horizontal by the level attached to it, and the vernier of the vertical arc read. Whatever it reads will be the level error.

The sign of the correction to be applied for this error is, for elevation, +, when the 0° of the arc is above the zero of the vernier when the tube is level, and - when below. For depressions the signs will be reversed. Care must be taken that no mistakes are made as to these signs. For a tyro it is slightly confusing.

Both level and collimation error must be applied to each observation.

Sextant
Eleva-
tions.

When the ship can be well fixed, sextant angles of elevation from her with a sea horizon will be very good, as good, in fact, as elevations with a small theodolite, as they are free from all possible errors of levelling, etc., and a sextant measures angles to ten seconds, whereas a small theodolite is only cut to minutes. Even when the ship is within the limits of the sea horizon, the results will be good, providing the distance of the shore line is well known, and is not under half a mile. By observing from the lowest step of the accommodation ladder, we can use a shore horizon at even less distances.

Sextant elevations, then, are very useful, but we do not generally get so many opportunities of obtaining series of heights by it ; and when at any distance from the land, only the skyline of hills will be clearly seen, so that it is principally

to the theodolite that we must look to give us a *sufficiency* of elevations.

Before dealing with the method of calculation of heights, we must refer to the effects of refraction. Refraction.

The apparent position of one object from another, as seen through our atmosphere, appears higher, whether we look up or down. The amount varies with the difference of densities of the various strata of the air, which are constantly changing.

All we can do is to take the mean refraction, and it has been found by experiment that by taking one-twelfth of the distance, regarded as minutes and seconds of arc, and applying this to the observed angle of elevation, it will give us a fair mean result for the true angle of elevation when this is small, as in all practical cases it is. It follows from this unknown amount of error in the coefficient of refraction that, when possible, objects should not be observed for elevation or depression at more than a few miles' distance. We cannot always command the maintenance of this limit, any more than we can many other theoretical points in practical hydrographical work; but when circumstances are favourable, they must be regarded.

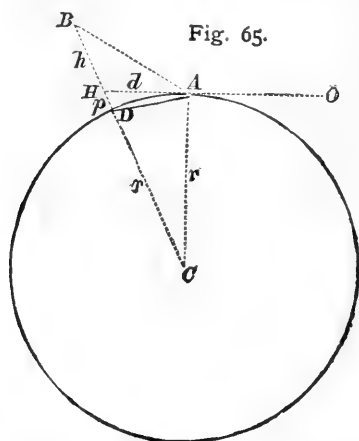
Looking upwards, or from a denser into a rarer medium, the effect of refraction is to increase the apparent elevation. This correction is therefore to be *subtracted* from elevations. As the effect, when looking downwards, is also to raise the object, or, in other words, to decrease the angle of depression, the correction for refraction must be *added* to angles of depression.

The angle of elevation measured by a theodolite, or the sextant angle when corrected for height of eye above the sea, is the angle between the tangent to the earth's surface at the observer's position and the line drawn from him to the object. Result of
Spherical
Form of
the Earth.
If the surface of the earth was a plane, all that would be necessary to obtain the height would be to work out in a right-angled triangle, $\text{Perp.} = \text{Base} \times \text{Tan angle of elevation}$, after the latter had been first corrected for the effects of refraction; but as the earth is a sphere, the tangent to it, produced, will cut the line representing the height we want, not at the point where it leaves the earth, but somewhere above that, depending upon the distance. The perpendicular, therefore, as

worked out, will only give us a portion of the height required, the other portion being that below the tangent.

Explana-
tion of
"Dip."

Thus, in Fig. 65, A is the position of the observer, A H the tangent to the earth's surface at his position, B a mountain peak whose height, B D, we want to obtain. The angle of elevation measured by a theodolite is B A H, and it is evident that the height we shall obtain by working out the triangle will be B H, leaving H D to be found independently. It will be seen that we are going to treat the angle B H A as if it was a right angle, when it is evidently more than 90° by the angle



D C A at the centre of the earth; but our figure is much exaggerated to show things clearly, and in practice the distances we use to get elevations are so insignificant, comparatively, to the diameter of the earth, and consequently the angle D C A so small, that we can neglect this quantity without introducing any error in the result. With a distance of 60 miles, when the angle is a degree, the discrepancy introduced into a height of 6,000 feet is only 2 feet.

We require, then, to get H D to add on to B H in order to get the full height of B D. This quantity, H D, is called "dip," an awkward nomenclature, as it is the same used at sea to express the angular quantity we apply to elevations taken with a sextant from a height to reduce them to the tangent to the earth, whereas here it is used to express a linear quantity.

The problem can be solved in two ways—either by finding $H D$ independently, or by adding the angle $H A D$ to $H A B$ to get the angle $B A D$, when a right-angled triangle gives us $B D$.

The latter method is the shorter, and is now employed, and the former is therefore not described ; but a table giving $H D$, the dip, or the height of the part of the object observed obscured by the horizon, is given in Appendix, Table O, as it may be sometimes useful to know how much of a mountain is below the horizon.

In the method now used, the angle $H A D$ is found as follows :

Angle $H A B$ is the elevation, corrected for refraction, and the angle $H A D$ (between the chord and tangent) is equal to half the angle at the centre—*i.e.*, half the distance in arc.

Suppose $A B$ to be 60 miles,

$$\text{Then } H A D = + 30'$$

$$\begin{array}{rcl} \text{Correction for refraction } (\frac{1}{12} \text{ of the distance} & = & - 5' \\ - \text{ to elevation, } + \text{ to depression} & & \text{-----} \end{array}$$

$$\text{Whole correction} = + 25'$$

Consequently, as $60' : 25'$

or $1' : 25''$ is a constant proportion for all angles.

Therefore, the total correction, for dip and refraction, in seconds of arc, to the observed angle of elevation or depression is :—

$$\frac{\text{Distance in sea miles} \times 100}{4}.$$

This correction is to be added to angles of elevation, and subtracted from angles of depression.

A ruled form is supplied by the Hydrographic Office, which much facilitates the calculation of heights. This form, bound into a book, constitutes the Height Book. A specimen is given on p. 262, which nearly speaks for itself.

The angle observed to the object is entered under the head of either elevations or depressions, as the case may be—as *observed*, in the case of theodolite ; *minus the correction for height of eye*, if with the sextant ; and the distance in miles and decimals is entered under its head.

Height
Form
bound
into Book.

Entering
and Cal-
culating
Eleva-
tions.

Chart of

COMPUTATION.

| | | | |
|--|--|--|---|
| <p>1</p> <p>Const. log. 3° 7 8 3 5 9 Tangt. 8° 3 5 1 5 Log. dist. 1° 0 0 0 0</p> <p>1441 15 1456</p> | <p>2</p> <p>Const. log. 3° 7 8 3 5 9 Tangt. 8° 3 5 2 4 0 Log. dist. 0° 8 5 9 7 4</p> <p>990</p> <p>2° 9 9 5 7 3</p> | <p>3</p> <p>Const. log. 3° 7 8 3 5 9 Tangt. 8° 2 3 2 6 5 Log. dist. 0° 7 2 9 9 7</p> <p>557·4 5 582 1463 901</p> <p>2° 7 4 6 2 1</p> | <p>4</p> <p>Const. log. 3° 7 8 3 5 9 Tangt. 8° 2 3 3 7 1 Log. dist. 0° 7 9 7 9 6</p> <p>653 257 910 5 905</p> <p>2° 8 1 5 2 6</p> |
| <p>5</p> <p>Const. log. 3° 7 8 3 5 9 Tangt. 8° 4 6 1 5 7 Log. dist. 0° 6 2 0 1 4</p> <p>733 908 175</p> <p>2° 8 6 5 3 0</p> | <p>6</p> <p>Const. log. 3° 7 8 3 5 9 Tangt. 8° 4 6 1 5 7 Log. dist. 0° 6 2 0 1 4</p> <p>733 908 175</p> <p>2° 8 6 5 3 0</p> | <p>7</p> <p>Const. log. 3° 7 8 3 5 9 Tangt. 8° 4 6 1 5 7 Log. dist. 0° 6 2 0 1 4</p> <p>733 908 175</p> <p>2° 8 6 5 3 0</p> | <p>8</p> <p>Const. log. 3° 7 8 3 5 9 Tangt. 8° 4 6 1 5 7 Log. dist. 0° 6 2 0 1 4</p> <p>733 908 175</p> <p>2° 8 6 5 3 0</p> |
| <p>9</p> <p>Const. log. 3° 7 8 3 5 9 Tangt. 8° 4 6 1 5 7 Log. dist. 0° 6 2 0 1 4</p> <p>733 908 175</p> <p>2° 8 6 5 3 0</p> | <p>10</p> <p>Const. log. 3° 7 8 3 5 9 Tangt. 8° 4 6 1 5 7 Log. dist. 0° 6 2 0 1 4</p> <p>733 908 175</p> <p>2° 8 6 5 3 0</p> | <p>11</p> <p>Const. log. 3° 7 8 3 5 9 Tangt. 8° 4 6 1 5 7 Log. dist. 0° 6 2 0 1 4</p> <p>733 908 175</p> <p>2° 8 6 5 3 0</p> | <p>12</p> <p>Const. log. 3° 7 8 3 5 9 Tangt. 8° 4 6 1 5 7 Log. dist. 0° 6 2 0 1 4</p> <p>733 908 175</p> <p>2° 8 6 5 3 0</p> |
| <p>13</p> <p>Const. log. 3° 7 8 3 5 9 Tangt. 8° 4 6 1 5 7 Log. dist. 0° 6 2 0 1 4</p> <p>733 908 175</p> <p>2° 8 6 5 3 0</p> | <p>14</p> <p>Const. log. 3° 7 8 3 5 9 Tangt. 8° 4 6 1 5 7 Log. dist. 0° 6 2 0 1 4</p> <p>733 908 175</p> <p>2° 8 6 5 3 0</p> | <p>15</p> <p>Const. log. 3° 7 8 3 5 9 Tangt. 8° 4 6 1 5 7 Log. dist. 0° 6 2 0 1 4</p> <p>733 908 175</p> <p>2° 8 6 5 3 0</p> | <p>16</p> <p>Const. log. 3° 7 8 3 5 9 Tangt. 8° 4 6 1 5 7 Log. dist. 0° 6 2 0 1 4</p> <p>733 908 175</p> <p>2° 8 6 5 3 0</p> |

To get this distance, if we happen to have it worked out in the triangulation, we shall of course use the calculated distance; but if not—which will be the case generally—we must measure it on our sheet, and enter the corresponding distance according to the scale.

In the column headed Corrⁿ, enter the correction for dip and refraction, obtained as above by multiplying the distance by 100 and dividing by 4.

We then work out the difference of height with these data on the opposite side of the page, the constant log being the log of feet in a mile, by which the distance must be multiplied to bring result out in feet. The log given is that for 6,075 feet, the number of feet in a mile in Lat. 44°. Theoretically, we should have a different log for different latitudes, but, as the utmost extent of error by neglect of this is 22 feet in a height of 6,000 feet, we need not regard it. This difference of height is entered in its proper column.

Tables, computed by Commander Purey Cust, for taking out the difference of height for any angle and distance, are now supplied. If these are at hand, this computation is dispensed with.

The column for height of theodolite is a little confusing, as sometimes it will be merely the height of the theodolite-telescope above the ground, and sometimes the height of it above the sea-level, which we shall enter, according as we want the height of observer's position or of object observed, as will be presently explained.

We have now all the data necessary to obtain heights.

When we have accumulated enough observations, we set about getting out results.

Height
Problems.

There are four problems for obtaining heights, and the data we have for each observation will be combined according to what we want to arrive at.

These four problems are as follows :

1. To find height of object observed, when height of observer is known, and the angle is one of *elevation*.
2. Ditto, when angle is one of *depression*.
3. To find height of observer, when height of object observed is known, and the angle is one of *elevation*.
4. Ditto, when angle is one of *depression*.

To understand the mode of combining the data, let us consider the Figs. 66 and 67.

In Fig. 66, which is the case where the angle observed is of *elevation*, and comprises Problems 1 and 3, we may have either X or Y known, and wish to obtain the other.

Suppose X to be known (Problem 1), to find Y, we have

$$Y = X + (t + h) \dots\dots\dots 1.$$

Height
Formulæ.

If Y is known (Problem 3), to find X,

$$X = Y - (t + h) \dots\dots\dots 3.$$

Fig. 66.

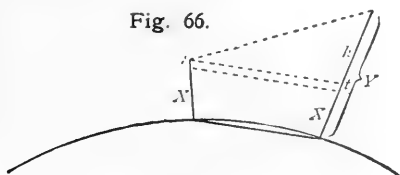
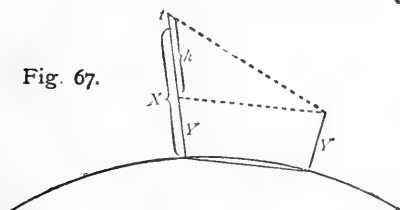


Fig. 67.



X = Height of observer's position.
Y = Height of observed position.

h = Difference of height.
t = Height of theodolite above ground.

In Fig. 67, the case where the observed angle is one of *depression*, and comprises Problems 2 and 4,

Suppose X known (Problem 2), to find Y,

$$Y = (X + t) - h \dots\dots\dots 2.$$

Suppose Y known (Problem 4), to find X,

$$X = Y + h - t \dots\dots\dots 4.$$

These four formulæ, which it is also convenient to have written for reference in the Height Book, will enable us to solve any of the problems.

When we are getting the height of the *Object observed*, we shall enter in the column of "Height of theodolite," $X + t$, or the height of theodolite above the sea; but when the observation is used to obtain the height of *Observer*, only t , the height of the theodolite above the ground, will be inserted.

Column
Height of
Theodo-
lite.

**Eleva-
tions from
Sea-level
first
meaned.**

We must commence by collecting results of elevations from stations at the sea-level, or from stations whose height above the sea-level has been measured, which will give us the heights of objects observed; and also with depressions from stations to the sea-level, or to stations whose height above sea-level has been measured, which will give us the height of the observing-stations.

**Absolute
Heights.**

Heights so obtained are termed "absolute," as being calculated directly from the sea-level.

All such heights must be obtained first; then, meaning the heights of one station which has the most observations, or of which the results agree best, we can work out all other observations from that station to other objects. We then mean another, and so on, using our observations either to obtain height of observer or of observed object, as is most convenient, as we proceed.

**Depen-
dent
Heights.**

These heights will be "dependent," as resting on the ascertained height of other stations.

No height can be considered as *exact* that is not the result of both elevations and depressions, as no matter how nicely a set of depressions, say, comes out, they will all include the refraction error, for the refraction correction is only approximate. This is with reference to detailed surveys only.

**Sextant
Eleva-
tions.**

Sextant angles of elevation must be corrected for the height of eye before being entered in the Height Book as angle observed. They are then treated in precisely the same manner as the theodolite elevations.

Aneroids.

The pocket aneroids should be tested up to a known height, to get the value of each tenth, which will be from 92 to 100 feet for a tenth, each instrument varying slightly. As before mentioned, they are useless in getting accurate heights, but will give very good approximations up to about 4,000 feet, if in good order and constantly worked; but their delicate chain-work is so liable to rust slightly at sea, that the links will frequently stick if the instrument is not carried up heights continually to work it. Placing under an air-pump will serve the same purpose. See "Barometer," p. 35.

**Only for
Approximate
Heights.**

It is useless to enter into intricate calculations of data obtained by so small a scaled instrument as a pocket aneroid; the impossibility of reading it exactly precludes any but

approximate results, and a simple multiplication of the decimal of inches by the value of a tenth, as obtained above, is quite sufficient for the purposes for which we use the instrument, the differences being taken from the barometer as observed on board.

To obtain distance from an angle of elevation of a known height is like using a lever with the ends reversed, and is seldom had recourse to in surveying, as not being correct enough.

Obtaining
Distance
from Ele-
vation of
known
Height.

As it may be, however, sometimes useful, we give a formula.

$$\left. \begin{array}{l} \text{Distance in} \\ \text{nautical miles} \end{array} \right\} = \frac{34h}{E}$$

When

h is height of mountain in feet

E is the angle of elevation in seconds,
reduced to water-level, and corrected by
the addition of the dip, as explained in
the rules for obtaining heights.

The same formula in rougher terms is—

$$\left. \begin{array}{l} \text{Distance in} \\ \text{nautical miles} \end{array} \right\} = \frac{100 h}{3 E}.$$

If the estimated distance should differ much from that given by the calculation, it should be recalculated with the correct allowance for dip and refraction.

An example is appended.

| | | | |
|--|-----------------|------------|-----------------|
| Given height of mountain observed = 2384 feet. | | | |
| Elevation | .. | .. | 0° 36' 26" |
| Height of eye | .. | .. | 16 ft. |
| Estimated distance | .. | .. | 30 miles. |
| Obs. elevation | 0° 36' 26" | 2384 | .. 3·377306 |
| Height of eye | 3 56 | 34 | .. 1·531479 |
| | <u>32 30</u> | | <u>4·908785</u> |
| | or 1950" | 2700 | .. 3·431364 |
| Corr. for Dip for 30' | | | <u>1·477421</u> |
| or $\frac{100 \times 30}{4}$ | .. 750 | Distance = | 30 miles. |
| | <u>E = 2700</u> | | |

The rougher formula will give the distance as 29·4 miles.

LEVELLING.



Simple
Levelling.

Levelling with a staff is not very much required in marine surveying. Ascertaining the height above the sea of the fixed mark used for reference for the tidal datum is the purpose for which it is most used, but it is also required to find the height of the base of a lighthouse, etc.

This is called simple levelling, and gives us the height between the two required points only, without any regard to distance.

A levelling staff and level is usually supplied to surveying ships; but a theodolite and marked boat-hook or pole will answer the purpose, if we have not got the regular apparatus.

FOR HEIGHT OF — LIGHT-HOUSE.

| Back  | | | | Fore  | | | |
|--|----|----|-------|--|----|----|------|
| Reading of Staff. | | | | Reading of Staff. | | | |
| Water-level | .. | .. | 12·64 | (1) | .. | .. | 0·63 |
| (1) | .. | .. | 13·42 | (2) | .. | .. | 1·22 |
| (2) | .. | .. | 13·81 | (3) | .. | .. | 1·52 |
| (3) | .. | .. | 12·50 | (4) | .. | .. | 0·32 |
| (4) | .. | .. | 13·06 | (5) | .. | .. | 0·87 |
| (5) | .. | .. | 12·18 | Base of L. H. | .. | .. | 3·45 |
| | | | 77·61 | | | | 8·01 |
| | | | 8·01 | | | | |
| | | | 69·60 | | | | |
| Height of base of Light-house above High-water level .. 69·6 ft. | | | | | | | |

Holding the staff at high-water mark, we place the instrument (level or theodolite) so far up the slope that we shall, when it is carefully levelled by the level attached to the telescope, read off near the top of the staff. The reading of this, called the *back station*, being taken, the staff is taken above us, and planted so that we can read just above zero of the staff, which is now at the *fore station*. The theodolite is now moved up the hill until we shall again, when levelled, read near the top of the staff; this will be another observation of *back station*, and so on until the levelled telescope reads the staff on the spot whose height we want.

There is no necessity to keep in one line directly for the spot whose height we wish to measure ; we shall do so if we can, as it is the shortest way, but in practice we are generally forced to zigzag.

The difference of the sums of the readings of back and fore stations will be the height required. Where former are the greatest, we are going up hill, and it is called *rise* ; *vice versa* is called a *fall*.

The following is another example of entries in the Field Book in connecting two bench marks, A and B.

| Back. | Fore. | + | - |
|------------------------|--------------|---------|----------------|
| 3.882 | 4.472 | | |
| <u>3.880</u> | <u>4.468</u> | | |
| 3.881 | 4.470 | | 0.589 |
| 5.035 | 5.025 | | |
| <u>5.032</u> | <u>5.018</u> | | |
| 5.033 | 5.022 | 0.011 | |
| 5.108 | 5.078 | | |
| <u>5.080</u> | <u>5.100</u> | | |
| 5.094 | 5.089 | 0.005 | |
| 5.130 | 5.000 | | |
| <u>5.128</u> | <u>5.008</u> | 0.125 | |
| 5.129 | 5.004 | | |
| 4.960 | 5.016 | | |
| <u>4.960</u> | <u>5.018</u> | | |
| 4.960 | 5.017 | | 0.057 |
| | | + 0.141 | 0.646 - |
| | | | 0.141 + |
| Difference of Level .. | .. | .. | <u>0.505 -</u> |

The levelling staves should not be too far from the theodolite ; about 50 yards is quite far enough with a 5-inch theodolite and large telescope, if accurate readings are required.

The theodolite should be, as far as possible, at equal distances from both staves, in order to avoid having to alter the focus of the telescope.

As far as possible, place the theodolite in the line joining the two staves, with two opposite levelling screws in the line of the staves.

Correc-
tions
when
necessary.

For our purposes, when the distance of the staff from the theodolite is not great, or when the distances of fore and back station from the theodolite are nearly the same, it will be sufficient to observe readings with the telescope in one position only; but when the rise of the hill is slight and distances increase, especially when the difference of distance between fore and back station is great, and we require accuracy, the telescope should be reversed in the Y's, and being again brought level by the bubble, readings should be taken a second time. The mean will be the true reading.

If the axis of the telescope and the attached level are perfectly parallel, and therefore in adjustment, it will be shown by the readings agreeing when reversed at the first station, and we shall know that we need not take this trouble; but it is necessary to ascertain this, as theodolites continually undergoing carriage by boat are liable to many accidents.

This method enables us, if necessary, to calculate the height of any of the stations where the pole is erected, but gives us no information as to the height of the spots where the theodolite stands. This can be obtained, if wished, by measuring the height of the axis of the telescope above the ground, when,

Height of theodolite position = height of back station
+ present reading of said back station — height of eye
(back station being below us).

Distances measured will enable us to make a section of the ground traversed, but, as already remarked, this is not often required from the marine surveyor, and will not be enlarged upon here.

CHAPTER XII

OBSERVATIONS FOR LATITUDE

By Circum-meridian Altitudes of Stars—By Circum-meridian
Altitudes of Sun.

ASTRONOMICAL observations are largely used in all descriptions of marine surveying. In all but small plans the eventual scale of the chart is decided by the latitude and longitude, as obtained by observations of sun or stars, and we have seen that true bearings often enter largely into the construction of charts. In running surveys, or in searching for or sounding over shoals in mid-ocean, everything depends on the positions astronomically found, and every method of correctly finding the latitude and longitude is in requisition. In considering this subject, we will take first shore observations with artificial horizons, where we require results as accurate as we can obtain with the sextant, to which instrument remarks will be confined, excepting so far as the theodolite is used for true bearings, and afterwards sea observations.

General
Remarks.

In all observations of the heavenly bodies, instrumental errors, atmospheric effects, and personal differences largely influence the results. No matter how correctly we may take the actual observations, unless we can eliminate these variable quantities, the positions obtained will be in error.

Elimina-
tion of
Errors.

On every occasion, therefore, where accuracy is aimed at, the mode in which this elimination can be best carried out must be considered. The general principle used in doing this is to get two sets of observations for one result in such a manner that the errors of all kinds will act in opposite directions in each set, and therefore disappear when the mean is taken. The precise way in which this is done will be described under each different observation.

LATITUDE BY CIRCUM-MERIDIAN ALTITUDES OF STARS.

Latitude
by Obser-
vations
"Abso-
lute."

Determinations of latitude are more simple in one respect than those for longitude, as they are "absolute"—that is to say, they depend solely upon themselves; whereas longitude has to be obtained by the difference of two sets of observations at two different places, and is further complicated by the eccentricities of the chronometers upon which, when there is no telegraph, we have to rely.

But more
Difficult.

But, on the other hand, the observations required for correct latitude are more difficult to take, as, to arrive at anything like an exact result, we must use stars, and each step of the observation of these in an artificial horizon is rendered less easy by the fact of their being made at night. It is much easier to become a good day observer than a good night one.

Elimina-
tion of
Errors in
Observa-
tions for
Latitude.

The errors to be eliminated as far as possible in observing for latitude are, firstly, errors of observation; secondly, instrumental errors, as centring error, index error of sextant, error caused by refraction in the rays passing through the glasses of the roof of the horizon, etc.; thirdly, atmospheric refraction, which varies much, and for which no known rule of correction thoroughly suffices; fourthly, personal errors, caused by each individual's mode of observing the contacts.

Another source of error in observations both for time and latitude, when using sextant and artificial horizon, is due to there being no means of ensuring that each observation is made precisely in the same spot in the artificial horizon. Great care is therefore necessary in this respect.

Roof error is minimised at altitudes between 40° and 50° .

Errors of observation are eliminated by taking as many observations of altitude as we can, and we must therefore observe off the meridian, or what are known as circum-meridian altitudes, which consist in observing from a short time before the meridian passage to a short time after it, and adding a certain correction to each altitude to make it equal to the meridian altitude, and thus get a mean meridian altitude, which, if we can calculate the correction exactly, will be of much more value than actual observation on the meridian only.

There remain the other errors, some of which may be directly allowed for, but only approximately; others cannot be corrected at all, and the latitude resulting from observations of a single body—as, *e.g.*, the sun—will be therefore always more or less in error.

The only way satisfactorily to clear these errors is to observe Pairs of Stars. stars, in pairs, of as equal altitude as can be found, one north, and one south, of the zenith. These errors will then act in opposite directions, as everything tending to increase or diminish the altitude on one side of the zenith will act similarly on the other; but, in working out the latitude, the resulting error will increase the latitude in one case and decrease it in the other, so that the mean of the latitudes obtained by each star of such a pair will approximate very closely to the correct one.

To eliminate the artificial horizon roof error when observing pairs of stars, the roof must always be in the same position with respect to the observer, and therefore must be reversed when changing from face north to face south, and *vice versa*. If observing a single object, as the sun, the roof must be reversed when half-way through the observation.

The use of a sextant stand, when once the observer has got Sextant Stand. thoroughly accustomed to it, is an immense assistance to good observations, as the images of the stars, instead of quivering and shaking with every slight motion of the hand of the observer, remain perfectly still, and can be made to pass over one another with great accuracy.

Certain preparations are necessary for *good* star observations, for all scurry that can be avoided should be. Preparations necessary.

In the first place, stars must be selected and arranged for observations according to their pairs.

If stars given in the “Nautical Almanac” only are used, the chances are very much against a sufficient number of pairs being obtainable, as only a small proportion of observable stars are there included, though the number has been lately much increased.

A surveying vessel will have the Greenwich and Cape Star Catalogues. Observatory Catalogues of Stars, and out of these enough pairs can nearly always be picked to enable us to get a satisfactory latitude in one night, including stars down to the fourth

magnitude, which can be easily observed on an average night by a practised observer with good instruments.

A special list of stars is supplied to all surveying ships, which includes all stars in the Greenwich and Cape Catalogues, and also a diagram with an ingenious method of pairing stars.*

The German "Jahrbuch," the French "Connaissance des Temps," and the American "Ephemerides," contain many stars, with their apparent place for the day, which do not appear in the "Nautical Almanac." If any of these publications are at hand, it is convenient to make use of the information therein given, instead of calculating the apparent place from the data in the Greenwich and Cape Catalogues.

Choosing
Stars.

By observing stars of small magnitude, a sufficient number of pairs can generally be obtained by midnight or a little later.

The time of meridian passage and approximate altitude of all stars down to the fourth magnitude passing the meridian before that time must be calculated. Only those stars whose altitudes range from 20° to 60° need be considered.

A general list of stars being prepared and arranged consecutively in the order of their passing the meridian, pairs of stars at corresponding altitudes north and south are selected and inserted in the Angle Book, together with the time of the meridian passage of each star, the time that will be shown by the pocket chronometer that is used for taking time, double altitude, magnitude, whether it is north or south of the zenith, and each pair must be numbered.

The nearer together in point of time the two stars of a pair can be placed, the greater will be the chances of the elimination of the refraction errors, as in a few hours temperature often varies much, dews form, and many differences may arise in the atmospherical conditions.

Stars over 60° of altitude are not usually good to observe, as, though a sextant will measure over 120° , the image of the star will not be sharp when reflected from the index glass at such a large angle unless the glass be unusually good.

Altitudes of stars selected for pairs should not differ more

* These are respectively compiled and devised by Lieut. H. B. T. Somerville and Commander Pury Cust, and much facilitate the selection of stars.

than 2° or 3° if possible. Generally pairs within this limit can be found.

If one star of a pair be lost, it is useless taking the other, unless a substitute for the one lost can be had. It is well, therefore, to be provided with spare stars for pairs, as this may often happen from clouds intervening, etc.

Care must be taken in choosing pairs to leave sufficient time between each meridian passage for the due observation of each star before and after culmination.

This will vary with the latitude and declination, as a star should not be observed so far from the meridian as to bring the mean of the altitudes observed less than a minute or so under the meridian altitude. Fifteen minutes elapsing between each passage will give plenty of time under most circumstances.

Time must also be allowed for changing the position from north to south, and *vice versa*, but all this will vary with the quickness and experience of the observer. Beginners must be satisfied with a few stars, and must allow more time.

In preparing the ground, we must look out for a spot whence we can see clear in the line of the meridian north and south, and one far enough from the beach to be beyond the distance where surf will shake our quicksilver. The latter point is sometimes—as, for instance, where jungle comes down to the very beach—difficult to find, but it is well worth looking for, and going inland a bit to get it, as otherwise good observations may be rendered impossible from the vibration set up. The more solid the ground the better, as it is astonishing what slight causes will suffice to set the surface of the mercury in motion. The use of the amalgamated trough mentioned on p. 15 will, however, enable observations to be obtained when impossible with the older form of horizon.

Wind is a frequent source of quaking mercury, and care should be taken to have the horizon trough firmly placed, and the roof so fitted that the wind cannot get under its lower edge.*

A screen of canvas to windward is sometimes a good thing, but on some ground this causes such vibration of the earth as to be worse than the free blast of the wind.

* *Vide* Artificial Horizon, p. 14.

If more than one officer is to observe, a screen of some kind should be put up north and south between the observers to keep the lights out of one another's eyes.

In the tropics and some other localities mosquitoes must not be forgotten. In places where these plagues abound, it is preferable to court the wind instead of shutting it out, in order to free ourselves if possible of them. Sand-flies are perhaps worse, as nothing will get rid of them, and many an otherwise favourable opportunity of getting stars has been spoilt by these wretched little insects.

The spot for the artificial horizon being settled on, and the direction of the meridian taken with a compass, it is a good plan to dig holes, if the nature of the soil permits, in which to place the lantern used for reading off when not required, so as to avoid unnecessary glare.* The best place for these will be on the left side of, and a little behind, the observer's seat, and two will be wanted for each observer, one for the north stars' position, the other when facing south. If the ground will not admit of digging holes, buckets will answer the same purpose well enough, but not so well.

If special lanterns can be got, these precautions will not be necessary; but we are assuming observations with ordinary ships' lanterns.

All these kinds of preparations should be made before sunset, if possible; confusion will be sure to occur if things are delayed till after dark.

For observations with a sextant stand, a small stool is wanted, as described under "Sextant Stand," and another for the observer's seat. It much facilitates good observations for the observer to be comfortable, especially when he is about to observe for several hours consecutively.

Star Map. A good star map is very useful to assist in recognising the objects chosen.

Error of Chronometer required. It will have been necessary to obtain the error of the chronometer on the day of our star observations (by single altitude is sufficient), unless we have recently obtained error at the same place, and have confidence that the chronometers are going sufficiently well to give us the true time of place to, say, two seconds.

* Some officers now fit their sextants with small electric lights as a luxury.

Single altitudes of a star near the prime vertical, either before or after observations for latitude, will give the error of the chronometer with sufficient accuracy for the purpose of calculating the reduction to the meridian.

Having thus made all preparations, and compared the pocket watches with the standard chronometer, and another as a check, we land to observe. We may here remark that we must again compare on returning on board.

Placing the artificial horizon north and south, we put on the roof, with the mark on it in the settled direction, according as our first star is north or south of the zenith. Observing Stars.

We then place the sextant on its stand, having first screwed in the inverting tube with the weakest-powered eyepiece. This should be adjusted to focus, as near as possible, before screwing into the collar.

Place the sextant and stand on the stool, so that one of the three legs which support the stand is at right angles to the meridian, and on the right of the observer.

Set the vernier to a few minutes less than the estimated double altitude of the star.

Move the stool with the instrument on it, so that, looking over the tube, we can see the reflection of our star in the artificial horizon.

Point the telescope at this, and set taut the screw that fixes the handle of the sextant on to the bearing of the stand, then, working the sextant right and left in the stand pivots, the other image of the star will soon dash across the field.

Unless the star is moving rapidly in altitude, there will be no further need to move the sextant on its bearing, and care should be taken that the screw is tight enough to prevent its moving while turning the sextant up to read, or the telescope will not point to the star when directed to the horizon again, which it should do at once, and so save time in redirecting. Beginners often neglect this, failing to see the necessity for it; and, losing time in looking again for the star after each reading, miss one of the great advantages of a stand—viz., that, once fixed, the stars will always be in the field without any bother.

With faint stars, when there are several of nearly the same brightness close together, it is sometimes rather confusing,

and a beginner will find it very difficult to be sure of his star ; but comparison with the star atlas, and consideration of how the star wanted lies with regard to the others, will, after a little experience, clear up the difficulty.

If the star be a faint one, it is difficult to bring it down to meet its image in the horizon by taking the sextant off its stand ; and if it is a bright one, there is no need to do so, as it cannot be mistaken if the estimated altitude is anywhere near the truth. There is then, in regularly pre-arranged observations, no necessity for doing this, and we shall trust entirely to the vernier being set to the calculated altitude for finding the star.

Advant-
age of
Levels on
Sextant
and
Sextant
Stand.

The levels on the index-bar of the sextant and on the arm of the sextant stand having been accurately adjusted by means of the sun as already described, and knowing the exact spot on the roof of the artificial horizon to which the telescope should point on looking along its upper surface when it is truly directed to the centre of the artificial horizon, the calculated double altitude of the star is set on the sextant. The bubble of the sextant level is brought into the centre of its run by turning the sextant on its bearing in the stand in a vertical plane. The slow motion given by the tangent screw on the sextant stand, if so fitted, enables this to be done with great nicety. The bubble on the arm of the stand is also brought to the centre of its run by making the arm horizontal and working the foot-screw.

The sextant and stand are then moved bodily in the direction of the artificial horizon until the telescope points correctly to the centre, the bubbles of both levels being kept accurately in the centre of their runs.

On looking through the telescope there can be no possibility of mistaking the star to be observed.

Having brought the two images into proximity by hand, place the right hand on the screw at the end of the stand-leg that has been arranged at right angles, and the left hand on the tangent screw of the sextant, when, by working these two screws, the images of the stars can be made to pass over one another exactly, and the word "stop" given.

At this signal the attendant bluejacket will hold the lantern up for reading off. The light should be thrown on to the arc

from a direction as nearly at right angles to the plane of the sextant as possible, to avoid parallax.

A small electric light fitted to the sextant, as described on p. 11, obviates the necessity for using a lantern, and is a great convenience.

If the horizon stand is raised off the ground by about a foot, and placed on a firm foundation, thus bringing the artificial horizon closer to the telescope, faint stars are more easily observed, and the movement of the sextant necessary to keep the star in the field, owing to its motion in the heavens, will be lessened. A lantern placed on the ground behind, or a little on one side of, the observer, and faintly showing on the artificial horizon, will sufficiently illuminate the wires of the telescope on a dark night.

In taking the next observation, turn the tangent screw on or back alternately before commencing to bring the images together again, so as to be entirely free from bias as to whether the star is rising or falling, and also to compensate for small centring errors which are liable to be introduced by the action of the tangent screw pressing the centre, upon which the index-bar turns, in opposite directions, according to the way in which the screw is moved.

Resetting
for each
Observa-
tion.

The amount of time to observe before and after culmination varies with the position of the star and the latitude of the place, as before mentioned ; but, as a rule, commencing six minutes before the calculated time of meridian passage, and continuing for a like time afterwards, will be ample, as we do not wish the correction eventually to be applied to the mean of the observed altitudes to bring it up to the meridian altitude to be more than one minute if we can manage it.

Time from
Meridian.

Observing as close to the meridian as is recommended above, the decimal parts of seconds need not be recorded in taking the time.

Decimals
unneces-
sary.

The observed altitude of a heavenly body can be corrected to the meridian when the hour angle and the latitude are known ; but if the hour angle is large, the calculation of the quantity to be added to the altitude is a complicated process, and it is only when the hour angle is small, and very little in error, that the formula assumes a simple and practical form. The error introduced by working with an assumed latitude also

Reduction
to
Meridian.

increases rapidly with the hour angle, so that we are confined in using this method to about twenty minutes of the time of meridian passage in ordinary latitudes; but in observations such as we are discussing now, which have for their object as correct a determination of the latitude as we can obtain, we must not observe more than about ten minutes from the meridian, and perhaps less.

The best method to use in reducing observations to the meridian is that known as Raper's, in which the principle is to add on to the observed latitude the amount necessary to make it equal to the meridian altitude, and then to calculate the latitude as in a meridian observation.

This amount is known as the "Reduction to the Meridian," and is so called because it is subtractive from the observed zenith distance.

The formula used by Raper is*—

$$\text{Reduction in } \left. \begin{array}{l} \text{secs. of arc} \end{array} \right\} = \cos \text{ dec.} \times \cos \text{ lat.} \times \sec \text{ alt.} \times \frac{\text{Vers hour angle}}{\sin 1''}$$

Raper gives a table of $\frac{\text{Vers H A}}{\sin 1''}$ for every minute and second of hour angle up to thirty minutes from the meridian, which is very convenient, and is given in Appendix M. It saves a considerable amount of figures in the calculation, and thereby diminishes the chances of clerical errors; but the formula, as given above, can be worked out if Raper's table is not at hand.

Sidereal
Hour
Angle.

The hour angle as marked by a watch beating mean time, or nearly mean time, will not be strictly correct either for a star or the sun, as, for the former, it should be taken by a watch beating sidereal time, which gets over its twenty-four hours while a mean solar watch has only advanced $23^h 56^m 04^s$ nearly; and apparent solar time varies from day to day as the speed of the earth in her orbit varies; but, within the limits we observe, the difference in ordinary latitudes is scarcely perceptible, and if we observe the stars of a pair about the same distance from the meridian, any little discrepancy will disappear in the mean.

If it be desired to correct for the difference when observing

* For proof, see Appendix D.

stars, a constant log of 0.002000 added to the other parts of the equation will give a close approximation to the exact reduction.

In working out the circum-meridian observations of a star, it is not necessary to calculate the reduction for each individual observation. As $\frac{\text{Vers H A}}{\text{Sin } 1''}$ is the only variable quantity in the equation, and as it varies with the hour angle, by taking this out for each observation, and meaning these quantities,

Calculating the Reduction.

we obtain a mean value of $\frac{\text{Vers H A}}{\text{Sin } 1''}$, which we insert in the

equation, and add the mean reduction so found to the mean of the altitudes. If working with Raper's tables, we take out the whole quantity, $\frac{\text{Vers H A}}{\text{Sin } 1''}$; if without them, we look out the Vers H A only, and introduce the log Sin 1" into the calculation with the other logarithms.

Knowing then the error of the watch used on mean time of place, and the approximate latitude and longitude, the rule for working out circum-meridian observations of stars will stand thus :

1. Calculate Greenwich date.
2. Correct right ascension of mean sun (sidereal time of "Nautical Almanac") for this Greenwich date, and subtract it from the right ascension of the star, which will give the mean time of star's meridian passage.
3. Apply to this the error of the watch, which will give the time shown by the watch at the star's meridian passage.
4. Mean the observed altitudes, correct this mean for index error, and divide it by two. To this apply refraction (corrected for thermometer and barometer), which will give true altitude.
5. Write down the times of each observation in a column, and taking the difference between each, and the time shown by the watch at meridian passage, we get the hour angle at each observation, which place in another parallel column.
6. Take out for each of these hour angles the quantity from Raper's Reduction Table, or, if we have not got that, the natural versine.

Rule for Calculating the Reduction.

7. Add these quantities together, and divide the sum by the number of observations, to get a mean.

8. Add together log cosine declination, log cosine estimated latitude, log secant true altitude, and the logarithm of the result of No. 7. If we are using versines, a constant log 9.316400 is also to be added. (This is log cosec $1'' + 4 + 0.002000$.)

9. Look out the sum of these logs as a natural number, which will be the number of seconds of reduction required.

10. Add this to the mean observed true altitude, which will give the calculated meridian altitude, from which the latitude is obtained in the usual manner.

An example is given on pp. 283 and 284.

Pole Star. When the pole star is observed, it must be worked out by the rule given in the "Nautical Almanac," care being taken to take out all the quantities from the tables exactly by interpolation.

Moon not Convenient. The moon is but of little use for observations of any kind. Its rapid motion necessitates very careful corrections, which take more time than they are worth, and besides, we have nothing to put against it to eliminate errors.

Results of Pairs. The separate stars being worked out, we mean the result of each pair, and the mean of these again will give us the mean latitude.

Although from circumstances many more observations may be got of one star of a pair than of the other, no value can be assigned to one over the other, and the direct mean must be taken; but in meaning up the results of pairs, less value would be given to a pair in which the observations of one star are few in number than to a pair where a proper number of observations of each star has been obtained. The necessity for assigning this value is increased where the observations are not only few but indifferent; but it would be a question whether it would not be better to omit such a pair from the final result altogether, and certainly it would be best to do so, if there are several other good pairs.

An example of the method of tabulating the different observations and pairs is given on p. 285.

The sets of stars given as an example were taken under favourable circumstances of sky and weather, and are not

On July 11th, 1879. At Buyuk Chekmejeh Δ the following observations were taken. Index error of sextant $-35''$
 Latitude (approx.) $40^{\circ} 57' 45''$ N. Longitude $28^{\circ} 30'$ E.
 Mean Time of Transit of star calculated, Xh. 12m.

| Time by watch (Breguet 2086). | | | Altitude α Ophiuchi. | | |
|-------------------------------|----|----|-----------------------------|----|------|
| h. | m. | s. | ° | ' | " |
| 10 | 09 | 36 | 123 | 22 | 00 |
| | 10 | 11 | | 22 | 40 |
| | 10 | 42 | | 22 | 50 |
| | 11 | 14 | | 23 | 15 |
| | 11 | 43 | | 23 | 25 |
| | 12 | 09 | | 23 | 35 |
| | 12 | 50 | | 23 | 50 |
| | 13 | 14 | | 23 | 50 |
| | 13 | 38 | | 23 | 55 |
| | 14 | 06 | | 24 | 00 |
| | 14 | 33 | | 24 | 00 |
| | 14 | 55 | | 23 | 50 |
| | 15 | 16 | | 23 | 50 |
| | 15 | 37 | | 23 | 45 |
| | 16 | 04 | | 23 | 30 |
| | 16 | 24 | | 23 | 15 |
| | 16 | 50 | | 23 | 00 |
| | 17 | 20 | | 22 | 40 |
| | 17 | 46 | | 22 | 30 |
| | 18 | 15 | | 21 | 55 |
| Mean | | | 123 | 23 | 16.7 |

The calculation will appear as follows:—

| Watch Times. | | | Hour Angle. | | $\frac{\text{Vers H. A.}}{\text{Sin. 1''}}$ |
|--------------|----|----|-------------|----|---|
| h. | m. | s. | m. | s. | |
| 10 | 09 | 36 | 4 | 27 | 38.9 |
| | 10 | 11 | 3 | 52 | 29.4 |
| | 10 | 42 | 3 | 21 | 22.0 |
| | 11 | 14 | 2 | 49 | 15.6 |
| | 11 | 43 | 2 | 20 | 10.7 |
| | 12 | 09 | 1 | 54 | 7.1 |
| | 12 | 50 | 1 | 13 | 2.9 |
| | 13 | 14 | 0 | 49 | 1.3 |
| | 13 | 33 | 0 | 25 | 0.3 |
| | 14 | 06 | 0 | 03 | 0.0 |
| | 14 | 33 | 0 | 30 | 0.5 |
| | 14 | 55 | 0 | 42 | 1.0 |

LATITUDE BY CIRCUM-MERIDIAN STARS AT BUYUK CHEKMEJEH, JULY 11TH, 1879. ARRANGED ACCORDING TO
ALTITUDES AND PAIRS.

| LATITUDE BY CIRCUM-MERIDIAN STARS AT BUYUK CHEKMEEH, JULY 11TH, 1879. ARRANGED ACCORDING TO ALTITUDES AND PAIRS. | | | | | | | | | | |
|--|------------|---------------|--------------|------|------------------|---------------|--------------|---------------------------------|--|----------|
| SOUTH STARS. | | | | | NORTH STARS. | | | | | REMARKS. |
| Alt. | # | No. of Obsns. | Latitude. | Alt. | # | No. of Obsns. | Latitude. | Latitude by means of each Pair. | | |
| ° 38½ | ζ Ophiuchi | 20 | ° 40 57 52.6 | ° 40 | Polaris | 20 | ° 40 57 42.1 | Taken consecutively. | | |
| 45½ | δ " | 20 | 55.8 | 44½ | δ Ursæ Min. | 20 | 49.1 | | | |
| 51½ | δ Aquilæ | 20 | 47.1 | 49 | ε " | 20 | 48.7 | | | |
| 58½ | κ Ophiuchi | 16 | 44.2 | 58 | χ Draconis | 23 | 51.4 | | | |
| 61½ | α " | 20 | 46.0 | 61 | ε " | 20 | 53.2 | Taken consecutively. | | |
| 62½ | ζ Aquilæ | 20 | 39.2 | 63½ | δ " | 17 | 54.4 | | | |
| | | | | | Mean Latitude .. | | | | | |
| | | | | | 40 57 48.6 N. | | | | | |

be remarked that the latitude obtained from south stars diminishes as the altitude increases, and *vice versa* with the north stars. This is wholly attributable to the centring error, and from these observations a very fair idea of that error for each altitude may be obtained, as explained on p. 9. It is therefore well, though the result will not be affected, to apply the centring errors if known.

In the example given only the direct mean was taken, though the second pair was open to suspicion, the seconds of both north and south stars being somewhat out of the order observable in the remainder.

Valuing
Results.

If, however, the observations had been less uniform, and there were not sufficient pairs manifestly better than others, by which we could elect to stand, value would be assigned by some such system as the following.

Assume a number as perfection, say 10, and give to each pair its value in that scale. Then the sum of the products of the value and the seconds of latitude by each pair, divided by the sum of the values, will give the mean seconds of latitude. The values should not be given by the results, or we shall arrive at pretty much the same conclusion as we should by assuming the latitude directly, but by the circumstances of observation of each star, and the number of observations, remembering that reasons to doubt one star will as equally affect the pair as if both stars were bad.

Values must also be given when meaning the results by different observers; but here again, if one observer is admittedly the best, and his observations are also good, a more probable result will be obtained by ignoring the latitudes of the others altogether. They will not be totally lost, as experience in good observing is only to be obtained by plenty of practice, and a young observer is more likely to take pains when there is a chance, if his observations turn out well, of having them included in the operations of the survey, than when merely observing for what one may term barren practice.

"Probable
Error."

To determine the "probable error" and the value of the result of one set of observations as compared with others, for the purpose of meaning the whole, we may proceed as follows:

Let S = sum of the differences between each observation and the mean of all the observations, irrespective of sign.

N = number of observations.

Then—

Probable error of one observation $= \pm \frac{S}{N}$

Probable error of arithmetical mean of all observations $= \pm \frac{S}{N^2}$

Value for meaning with others, or "Weight of the Set" $= \frac{N^2}{S}$

This question of giving values is a very difficult one, and many experienced observers prefer to take a direct mean, both for the result by each individual and when finally meaning the different observers' results, omitting those altogether which seem least worthy of trust, and giving an equal value to

Difficulty of Valuing.

| Observer. | Latitude. | Value of each result. | Product of Value and Secs. of Lat. | |
|-----------|--------------|-----------------------|------------------------------------|--|
| A | 5° 14' 16" 3 | 3 | 48·9 | Mean Latitude by 4 observers :— 5° 14' 09"·4 S. |
| B | 08·8 | 5 | 44·0 | |
| C | 03·0 | 5 | 15·0 | |
| D | 12·0 | 6 | 72·0 | |
| | | 19 | 179·9 | = 09"·4 |
| | | | 19 | |

everything else. It is very much a case of judgment under the circumstances ; on some occasions one system will be best, and at others the other, and we must so leave it, simply remarking that whenever observations vary enough to make the consideration of this question necessary, the final result must always be regarded with suspicion.

An example is given on p. 288 of latitude arrived at by giving values.

LATITUDE BY CIRCUM-MERIDIAN STARS AT MESALE 1st AUG. 31st, 1878 (RESULT BY VALUES).

| SOUTH STARS. | | | | NORTH STARS. | | | | Latitude by mean of each Pair. | Value. | Product of Value and Secs. of Lat. | REMARKS. |
|--------------|------------------|------------------------|-----------|--------------|--------------------|------------------------|-----------|--------------------------------|--------|------------------------------------|---|
| Alt. | * | N ^o of Obs. | Latitude. | Alt. | * | N ^o of Obs. | Latitude. | | | | |
| 40 | α Cygni | 4 | 5 13 38.6 | 38 | α Pavonis | 22 | 5 14 33.1 | 0 5 14 05.8 | 8 | 46.4 | Obs. of α Cygni, good. |
| 44 | Vega | 25 | 37.6 | 51 | α Indi | 12 | 49.0 | 13.3 | 4 | 53.2 | Two stars four hours apart. |
| 51 | ϵ Cygni | 3 | 37.5 | 51 | β Sagittarii | 19 | 45.7 | 11.5 | 5 | 57.5 | α Indi misty, Alts 7° different. |
| 60 | κ Pegasi | 14 | 33.5 | 61 | ϵ " | 3 | 42.5 | 08.0 | 7 | 56.0 | ϵ Cygni only fair. |
| Sums | | | | | | | | 24 | 213.1 | | ϵ Sagittarii, pretty good. |
| | | | | | | | | | | $\frac{213.1}{24} = 8''.8$ | |

Mean Latitude 5° 14' 08".8 S.

(Observer B)

.. ..

In observing stars under the pole, we must not forget that the reduction will be subtractive from the observed altitude.

Stars
under
Pole
Planets.

The larger planets are not good for observation, as they are so much bigger and brighter than the point which a star shows. On occasions, however, they must be used. The R.A. and Dec. must be calculated exactly.

In using stars from the Greenwich or Cape Catalogues, it is necessary to calculate the apparent place of the star for the day. The method of doing this is given in the "Nautical Almanac," in the explanation under the head of "Stars," where examples of a north and of a south star are shown worked out. Care must be taken to give the proper signs + or - to each logarithm.

Calculat-
ing
Apparent
Places of
Stars
from
Cata-
logues.

Next to the observation of stars in pairs, the circum-meridian observation of the sun in the artificial horizon is the most correct and simple method we have of obtaining latitude; but it is evident that we cannot use it when the altitude exceeds 65° , as a sextant will not measure the double angle. We must, in the case of the sun, be doubly careful in correcting the refraction if we wish to get as near the truth as possible. There is nothing to be gained by observing both limbs of the sun, as the motion in altitude will be so small that it will not matter whether the images are opening or closing.

Latitude
by Cir-
cum-
Meridian
Altitudes
of Sun.¹

The roof of the horizon should be reversed at about noon, and the sights worked out as two sets, roof one way and roof the other.

However careful we may be, we shall not expect our latitude by the sun only to be exact, and in many cases where we are going to be satisfied with this observation it will not matter if the latitude be a quarter of a mile or so in error, and the reversal of the roof may often be dispensed with.

If, however, we know our centring error, and can depend upon the sextant, we can by its application get a vastly improved result, and none of these precautions should be omitted.

An observation of the sun cannot be meaned with an observation of a star the other side of the zenith, as all refraction errors, as well as errors introduced into the instrument by the heat of the sun, will be entirely different.

Sun and
Stars can
not be
Paired.

Circum-meridian observations of the sun are worked out in precisely the same manner as those of a star, the only dif-

Calcula-
tion of
Sun Ob-
serva-
tions.

ference being in the ordinary corrections to declination, etc. We want the error of the watch on apparent time to calculate what it shows at apparent noon, the time the sun will be on the meridian.

An example is given on next page.

On November 15, 1876, circum-meridian altitudes of the ☉ were observed with artificial horizon at Maghabiyeh I^d. Approximate latitude, $18^{\circ} 15'$ north; longitude, $40^{\circ} 44'$ east. Barometer, 30.00 inches; thermometer, 80° . Mean of observed altitudes Sun's Upper Limb, $106^{\circ} 47' 09.1''$.

Local
Attrac-
tion.

There is a source of error in obtaining an accurate altitude which must not be forgotten, especially when the scale of a chart depends on a difference of latitude, and that is the local attraction due to the irregular disposition of masses of land in the vicinity of an observation spot. A mountain mass on one side, or deep sea, will cause the local direction of gravity to slightly diverge from the vertical. The surface of the mercury will not in such a case be truly horizontal, and the altitude will be in error. This may often largely account for differences between triangulation and observation, and when the former is good, it may be in certain cases desirable to rest on it rather than the scale by observations. Formulæ have been drawn up for correction, but they rest largely on assumptions of mass which cannot be verified.

This attraction will also affect determination of time, and therefore longitude.

In mountainous countries, as near the Alps and in the Caucasus, deflections have been observed to the amount of as much as twenty-nine seconds. On the other hand, deflections have been observed in flat countries. In the vicinity of Moscow within a distance of sixteen miles the plumb-line varies as much as sixteen seconds in such a manner as to indicate a vast deficiency of matter in the underlying strata. But these are exceptional cases. On the north coast of Banffshire the deflection amounts to ten seconds. There is the sea to the north and an undulating country to the south, which, however, to a spectator at the station does not suggest any great disturbance of gravity.

There are many instances of disturbances of three to four seconds.

| Time by Watch. | | | Hour Angle. | | Vers. H. A. Sin. 1" |
|----------------|----|----|-------------|----|------------------------|
| h. | m. | s. | m. | s. | |
| 5 | 36 | 39 | 6 | 09 | 74.3 |
| | 37 | 06 | 5 | 42 | 63.8 |
| | 37 | 46 | 5 | 02 | 49.7 |
| | 38 | 07 | 4 | 41 | 43.1 |
| | 38 | 28 | 4 | 20 | 36.9 |
| | 38 | 53 | 3 | 55 | 30.1 |
| | 39 | 21 | 3 | 27 | 23.4 |
| | 41 | 32 | 1 | 16 | 03.1 |
| | 41 | 54 | 0 | 54 | 01.6 |
| | 42 | 16 | 0 | 32 | 0.6 |
| | 42 | 37 | 0 | 11 | 0.1 |
| | 42 | 57 | 0 | 09 | 0.0 |
| | 43 | 21 | 0 | 33 | 0.6 |
| | 43 | 43 | 0 | 55 | 01.6 |
| | 44 | 44 | 1 | 56 | 07.3 |
| | 45 | 24 | 2 | 36 | 13.3 |
| | 45 | 40 | 2 | 52 | 16.1 |
| | 46 | 04 | 3 | 16 | 20.9 |
| | 46 | 29 | 3 | 41 | 26.6 |
| | 47 | 12 | 4 | 24 | 33.0 |
| | 47 | 32 | 4 | 44 | 44.0 |
| | 48 | 18 | 5 | 30 | 59.4 |
| | 48 | 51 | 6 | 03 | 71.9 |
| | | | | | 23) 626.4 |
| | | | | | Mean 27.23 |

| | | | | h. | m. | s. | | | | | h. | m. | s. |
|-------------------|----|----|----|-----|-----|----------|--------------------|-------------------|----|----|----|--------|---------|
| App. noon.. | .. | .. | .. | 12 | 0 | 0 | App. noon | .. | .. | .. | 0 | 0 | 0 |
| Watch slow | .. | .. | .. | 6 | 17 | 12 | Long. | .. | .. | .. | 2 | 43 | 0 |
| Watch at Transit | .. | .. | .. | 5 | 42 | 48 | Gr. Date | .. | .. | .. | 2 | 43 | 0 |
| | | | | 0 | 1 | 0 | | | | | 0 | 1 | 0 |
| Obs. alt. \odot | .. | .. | .. | 106 | 47 | 09.1 | Dec. ap. noon. Gr. | .. | .. | .. | 18 | 39 | 47.8 |
| I. E. | .. | .. | .. | | | -40.0 | | | | | -1 | 42.4 | |
| | | | | 2) | 106 | 46 | 29.1 | Dec. at Transit.. | .. | .. | 18 | 38 | 05.4 S. |
| | | | | 53 | 23 | 14.5 | | | | | | | |
| S. D. | .. | .. | .. | | | -16 | 13 | Var. | .. | .. | .. | 37.80 | |
| | | | | 53 | 07 | 01.5 | | | | | | 2.71 | |
| Refraction.. | .. | .. | .. | | | -35.0 | | | | | | 378 | |
| True alt. | .. | .. | .. | 53 | 06 | 26.5 | | | | | | 2646 | |
| | | | | | | | | | | | | 756 | |
| Cos dec. | .. | .. | .. | | | 9.976613 | Corr. | .. | .. | .. | .. | 102.43 | |
| Cos lat. | .. | .. | .. | | | 9.977586 | | | | | 0 | 1 | 0 |
| Sec alt. | .. | .. | .. | | | 0.221713 | Tr. alt. | .. | .. | .. | 53 | 06 | 26.5 |
| 27.23 | .. | .. | .. | | | 1.435048 | Red. | .. | .. | .. | | | +40.8 |
| | | | | | | 1.610960 | Mer. alt. | .. | .. | .. | 53 | 07 | 07.3 |
| Reduction .. | .. | .. | .. | | | 40".8 | Z. D. | .. | .. | .. | 36 | 52 | 52.7 |
| | | | | | | | Dec. | .. | .. | .. | 18 | 38 | 05.4 |
| | | | | | | | Latitude.. | .. | .. | .. | 18 | 14 | 47.3 N. |

CHAPTER XIII

OBSERVATIONS FOR ERROR OF CHRONOMETER

General Remarks on Obtaining Longitude—Error by Equal Altitudes
Error by Two Stars at Equal Altitudes.

Shad-
well's
"Notes on
Chrono-
meters."

THE whole question of obtaining longitude by means of chronometers is so ably and exhaustively treated by Captain Shadwell in his "Notes on the Management of Chronometers," both as regards the treatment of the watches, the method of observation, and the various systems of obtaining meridian distances, that we refer the reader to that work for full information on the subject. Here we do not pretend to give more than the broader principles of the general question, but a work of this kind, intended for the perusal of young surveyors, would be incomplete without some reference to it.

Absolute
Longi-
tudes.

The methods of obtaining longitude, called "absolute methods," which give the longitude of the place as measured from the first meridian, directly and independently, such as observations of occultations of the stars by the moon, moon culminating stars, eclipses of Jupiter satellites, etc., are now rarely employed in nautical surveying, and may be said to be decidedly inferior in value to the results of good chronometric runs.

Similarly, altazimuths, portable transits, and other like astronomical instruments, are now seldom or never supplied to a surveying vessel. The sextant in a practised hand will give results equal to those obtainable by fixed instruments of small size, and has the great advantage of being more portable, and always ready.

Remarks
confined
to Differ-
ential
Longitude
and Sex-
tant.

To the sextant, telegraph, and chronometers, therefore, our remarks will be confined.

By the use of the two latter we obtain only the "difference

of longitude," or "meridian distance," between two places, neither of which may have its meridian distance from the primary meridian of Greenwich determined; but by the accumulation of such observations, the absolute longitudes of certain places are from time to time decided.

These, then, become secondary meridians, on which the longitude of places in their vicinity depend. Secondary Meridians.

When, therefore, a secondary meridian is changed in its value as regards its distance from the first meridian, all places whose longitude has been measured from it are changed also.

This is the work of the Hydrographic Office, which receives and collates all information. The nautical surveyor simply finds the difference of longitude, and transmits that information only.

A list of secondary meridians is given in the Instructions for Hydrographic Surveyors.

For our purposes we may look upon difference of longitude as divided into two main cases. The first, where the scale of a chart we are making depends on the astronomical observations for latitude and longitude at either extremity of our piece of coast. The second, when we wish to determine the relative positions of places more or less far apart, which are mainly required for the purposes of navigation. Cases defined.

In the first, we can nearly always use the system, hereafter described, of "travelling rates," which much adds to the accuracy of the result.

In the second, time, distance, and general circumstances often prevent our obtaining these, and compel us to use what we can get.

In obtaining the meridian distance between two places, either by means of a telegraph, or by carrying chronometers between them, the principle is the same, and is this—viz., that the difference of mean time of place at any moment at the two places is their difference of longitude in time. Principle of Differential Longitude.

If, therefore, we can find out that at the time that at a position A it is 9 o'clock, it is 8 o'clock at B, we know that the difference of longitude is equal to one hour of time, and that A is east of B. The telegraph enables us to do this in its simplest form, as, ascertaining the exact time at each end

by astronomical observations, we can find out by an exchange of signals what is the difference of time.

Chrono-
metric
Difference
of Longi-
tude.

In chronometric difference of longitude we have literally to carry the time from one place to the other. We ascertain the time at one place on a certain day, or, what is the same thing, we find out the Error of our chronometer on local time.

Supposing for the moment that our chronometer is keeping exact mean time, by carrying it to the other place and finding out its Error on local time there, we can deduce the difference of longitude by the difference of the two Errors. Thus, if our chronometer is four hours slow on mean time at A, and we find when we get to B that it is three hours slow, we know the difference of longitude is one hour, and that A is east of B. Unfortunately, chronometers do not keep mean time, and the problem is complicated by having to ascertain what time they *do* keep, or, in other words, what they gain or lose in each day, which is called the rate. If we can find this, we shall be able to get the difference of longitude just as accurately as if the chronometer was keeping mean time, as we can correct for this rate; but here, again, chronometers are not, and probably never will be, perfect instruments, and are liable to change of rate, and it by no means stands to reason that because a chronometer gains five seconds a day one week, it will do so the next, especially when the ship has been at anchor during one period, and under way for the other, and the temperature has not been invariable.

Chronometric runs are therefore liable to the errors arising from change of rate. To overcome this as far as possible, a number of chronometers are carried instead of one, and, if possible, what is called the travelling rate is obtained.

Travelling
Rate.

If the rate of chronometers is obtained at a station A, and we then go to another station, B, and obtain rate again there, and apply the mean of these rates as the assumed rate of the chronometers while being carried from A to B, we have no guarantee whatever that this assumption is correct, as the time employed in carrying the chronometers does not enter into the calculation at all, and they may have been going quite differently when the ship was at sea, with the vibration of the engines, motion of the ship, etc., to influence them, to what they were when the ship was at anchor, besides the

important factor of change of temperature. If, however, we can return at once to A, and obtain the Error again, we can positively say that the chronometers have gained or lost so much between the first and second observations at A. Assuming this loss or gain to have been uniformly carried on throughout the interval, we shall have a travelling rate which will give a far nearer result than by using rates obtained at either end of our required base. By this means we only obtain one meridian distance for our double run forwards and backwards, but it will be of more value than two separate meridian distances obtained by fixed rates.

Even if we have to stop at B a few days, by observing on arrival, and immediately before departure, we can eliminate the gain or loss of the chronometers during the stay there, by subtracting it from the total gain or loss during the time of our absence from A, and dividing the remainder by the number of days actually travelling. We shall thus still get a fair travelling rate, if the chronometers are at all trustworthy as timekeepers.

Modifica-
tion of
Travel-
ling Rate.

This, then, is the system of travelling rates, which can be generally, and always should be, if possible, used in determining difference of longitude for the scale of a chart.

Whatever be the system of rates employed, good observations must of course be regarded as the foundation of all of them. We cannot control the irregularities of our chronometers, but we can, to a certain extent, make sure of getting fairly correct time by using the proper means.

To ascertain the Error of the chronometer as exactly as we can with sextant and artificial horizon, we must endeavour to get rid of the atmospheric and other errors, as we do in observations of stars for latitude, which in this case is attained by observing at equal altitudes east and west of the meridian. It will be evident that, whatever be the instrumental and other errors (excepting those of observation), supposing them to remain unaltered, the middle time between the observations will be the same, as whatever tends to make the observed altitude more or less in the forenoon will act in the same manner in the afternoon, and as we do not want to know at all what that altitude is, but merely to ensure that it is equal, a.m. and p.m., the amount of the errors is immaterial.

Elimina-
tion of
Errors by
Equal
Altitudes.

The method of equal altitudes, therefore, must be used whenever we wish to get Error exactly. The Error of watch obtained by single altitudes, called "absolute observations," will depend for its accuracy upon the corrections for each source of error, which, as we have before stated, can only be considered as approximate.

Superior
and In-
ferior
Transit.

Equal altitudes of the sun can be taken either in the forenoon and afternoon of the same day, so as to find the Error at noon, called Error at superior transit; or in the afternoon of one day and the forenoon of the next, by which means we obtain the Error at midnight, or at inferior transit. Theoretically, these are equally correct, but in practice it is better to get Error at noon, if we can, as the elapsed time being less, gives less latitude to the chronometers or hack-watches for eccentricity. The alternative is, however, very valuable, and saves many a day, as when, for instance, we arrive at the place we wish to observe at an hour or two too late for forenoon sights. We can then begin our set in the afternoon, and get away, if we wish to do so, the next morning after forenoon sights, and thus save several hours, a considerable consideration in running meridian distances.

Principle
of Equal
Altitudes.

The principle of finding the Error of a timekeeper by observation of equal altitudes is that, the earth revolving at a uniform rate, equal altitudes of a body, on either side of the meridian, will be found at equal intervals from the time of transit of that body over the meridian, and that therefore the mean of the times of such equal altitudes will give the time at transit.

In the case of a body whose declination is practically invariable, as a star, this is strictly true, and the calculation of the Error of the watch is confined to taking the difference of the time shown by the watch, and the true calculated time of transit.

Equation
of Equal
Altitudes.

In the case of the sun, however, the declination is constantly changing; the altitudes are thereby affected, and an altitude equal to that observed before transit will be reached after transit, sooner or later, according to the direction of change in the declination.

It is therefore necessary to make a calculation of the correction resulting from this change of declination, to be applied

to the middle time, to reduce it to apparent noon, which correction is termed the "equation of equal altitudes."

The observation of stars at equal altitudes will therefore be, Stars and Sun compared. theoretically, the best to use, as being the simplest, and they will indeed give as good results as those of the sun; but practically the latter has generally been observed in marine surveying for the purpose of obtaining time. In many cases the inconvenience of landing, and carrying watches backwards and forwards for comparison, etc., by night, besides the increased difficulties of observing and reading instruments by lamp-light, lead to the choice of day observations; but in places where clouds persistently veil the sun in forenoon or afternoon, the nights are often clear, and equal altitudes of stars become most valuable.

There are two other methods of obtaining Error by stars, Other Methods by Stars. that are both good: the first, observing stars of corresponding altitude on either side of the meridian, working them out as absolute altitudes and meaning the results; the other is a method to which attention has been drawn by Captain A. M. Field, R.N., of observing two different stars of nearly the same declination at equal altitudes. This is published as a pamphlet by the Hydrographic Office, but is briefly described on p. 316. The great advantage of either of these is that the complete observation is made in a very short time, thereby obviating the disadvantages of star observations mentioned above.

In taking the observations of equal altitudes in the artificial horizon, we are limited, as always, to altitudes between 20° and 60° , as the horizon will not permit us to observe a lower altitude than 20° , and the sextant will not measure much more than 120° . These restrictions will, however, only be inconveniences, as regards the sun, in extreme latitudes, as we must choose as our time of observation, so as to minimise the effects of errors of observation, the period at which its motion in altitude is the greatest—i.e., when it is near the prime vertical—at which time, in all but high altitudes, the altitude will come between our limits. When the place of observation is near the Equator, and the latitude and declination are nearly the same, we could observe up to a very short time of noon, the sun's motion in altitude being nearly uniform Limitations of Observations.

throughout the day ; but we are in this case limited by the range of the sextant.

It is difficult to lay down any rule as to what is the smallest rate of motion in altitude we should observe at, as the greatest motion in altitude during the day varies so much with the latitude and declination. We can only say that we should, when we have any choice, not observe beyond an hour when the time of changing 10' of double altitude exceeds 30 seconds.

Sets of
Observa-
tions.

Opinions have much differed on the number of consecutive observations that it is best to take to comprise in a set. The only theoretical limit is that the equation of equal altitudes should be practically the same throughout the set, as the variation in the time required by the sun to traverse the number of minutes of altitude between observations at the beginning and end of the set will not matter, as we do not care whether the mean of the times agrees exactly with the mean of the altitudes.

It seems well, therefore, to observe tolerably long sets, as errors of observation are thereby eliminated. The same result in the end will be attained by a large number of shorter sets ; but the value of each set is much enhanced if composed of a considerable number of observations, and it saves time and trouble in the calculations.

Too long sets are to be deprecated as wearying to the eye and hand, and the observations will therefore suffer from that cause, especially in hot countries, where the necessity for observing in the full glare of the sun makes it a trying operation.

We prefer to take eleven observations in a set. This allows the observer to commence his second set, of lower limbs (a.m. observations), at exactly 1° more altitude than his first one, of upper limbs. It does not much matter, as each one has his own plans for these details, and soon falls into a regular method, which is the great thing to prevent mistakes.

Observa-
tions
must be
Similar.

The only point, in fact, of importance is *always to observe in the same way*. Not only does it save time and errors, but it is necessary in combining observations, whether for rate or meridian distance, that they be as *similar in all respects* as we can get them. The whole system is a system of differences, and it is manifest that the result is the better the more like the observations are. It follows from this that the observers

employed in any string of meridian distances should be the same, the instruments and watches the same, the temperature and time of observing the same, as far as possible. Also, supposing temperature to be the same, that a rate will be probably more correct if obtained by combining single altitudes of different days, both either a.m. or p.m., than by taking equal altitudes one day and single altitudes the other.

If clouds prevent observation at *precisely* the same altitude, after transit, the mean of Error obtained by absolute sights, a.m. and p.m., at *nearly* the same altitudes, will be almost as good as equal altitude sights.

When the observers are good, the greatest error is frequently introduced in comparing the hack-watches to be used for taking time with the chronometers, and great pains should therefore be taken with this operation. Com-
paring
Watches.

The watches used for taking time must be compared before, and after, both forenoon and afternoon sights, with the standard and another chronometer; and at noon all the chronometers should be compared with the standard, and the hack-watches with the same two chronometers.

In the case of stars, when return cannot be made to compare, the proper comparison for middle time must be deduced by interpolation from the comparisons before leaving and on returning.

Before saying more about comparing, we must remark that the seconds hands of pocket-chronometers are rarely placed symmetrically in the centre of the dial on which the seconds are marked. Defects in
Pocket-
Chrono-
meters.

These watches beat five times in 2 seconds, commencing with the even minute. The beat of the watch should therefore coincide exactly with every even second; the first beat from the minute being 0.4 second, the second 0.8, the third 1.2, the fourth 1.6, the fifth at 2.0, and so on throughout the whole 60 seconds.

But it will be found, in nearly every watch, that the hand does not fall over the even second on some parts of the dial, although it may on others, and each watch must be examined by counting the beats from the even minute, to ascertain how the hand falls in different parts of the dial, or the time-taker will be at a loss to know what is the exact decimal which his

watch is beating. For instance, supposing that at the 40-seconds' mark on the dial the hand falls a little short of the mark one beat and a little in advance at the next, unless he knows which of those beats is meant for the 40 seconds, he may be giving the time four-tenths of a second wrong. This, of course, refers both to comparing and taking time for the observations.

Method of
Compar-
ing.

In comparing, which is perhaps best done by two persons, the "Stop" is given on an exact second of the box-chronometer (which beats half-seconds), and the time by the pocket-watch noted. A check is then taken by comparing the reverse way, calling the "Stop" at an exact second by the pocket-watch, and noting the seconds and parts of seconds of the box-chronometer. As parts of seconds have to be estimated on both watches, these two comparisons will frequently differ two-tenths of a second. This is as near as we shall probably be able to arrive at the truth; but if the difference exceeds this, more checks should be taken, until we are satisfied which was wrong.

No operation requires more care or more practice than comparing, and while the simple method given above will give good results when observers are experienced, varying the beat on which the word "Stop" is given has the effect of varying the fraction of a second to be estimated by the other observer, and tends to avoid bias on his part.

Thus, an even second, an odd second, and a half-second beat should be used successively in giving the word "Stop" in the case of a chronometer, and any or all of the five beats in the case of a pocket-chronometer.

At least six comparisons on different beats should be interchanged to secure a good result.

In the same manner checks should be taken when comparing one box-chronometer with another.

If two pocket-watches are available, it will not be amiss to use both, even when there is only one observer, as it helps to eliminate errors of comparison. In this case half the sets will be taken with one watch, and the other half with the other.

Com-
parison of
Solar and
Sidereal
Chrono-
meters.

There will be a coincidence of beats at intervals of 3 minutes 4 seconds when using chronometers beating half-seconds, and at intervals of 6 minutes 8 seconds in the case of clocks beating seconds. The comparison is made when the chrono-

meters are beating together, and it may be noted that an error of 10 seconds, in estimating the instant at which it takes place, will produce an error of only 0.025 second. It is, therefore, an extremely accurate method to compare solar chronometers through the intermediary of a sidereal chronometer, when one is available.

A chronograph has recently been fitted to a small chronometer, with specially adapted "quick-train" movement. This beats too rapidly to allow time to be taken by counting the beats, but it has the advantage of enabling the chronometer to be carried with less liability to disturbance of rate than either a pocket-chronometer or a box-chronometer beating half-seconds.

Chronograph for use in the Field.

The current from a small electric battery being broken at intervals of two seconds by an arrangement fitted to the chronometer escapement, a lever is caused thereby to strike an anvil over which a tape is unwound at a uniform rate by means of clockwork mechanism. The observer causes a similar lever to strike the anvil on completing the circuit of another battery at the instant of observation. The fine points with which the levers are armed are adjusted to strike at points on the tape exactly opposite and close to each other. The position of the perforations in the tape caused by the lever actuated by the observer, relatively to the perforations at two seconds' intervals made by the beating of the chronometer, indicates the exact time at which the observation was made. The time results are obtained from the tape by applying to it a piece of glass, ruled with eleven straight lines converging to a point. The ends of these lines on the base of the triangle so formed are equidistant on the edge of the glass, so that when the first and last lines are so placed as to coincide with the perforations at the beginning and end of the two seconds' interval, it is divided into ten equal parts. The base of the triangle is always kept parallel with the line of the perforations on the tape.

Whilst the observations are in process the tape is watched by an assistant, and the perforations marked at suitable intervals to correspond with the time shown by the chronometer.

The rate at which the tape is unwound can be varied within certain limits, thus giving a larger or smaller time scale.

Much greater precision is obtainable by the use of the chronograph than by the eye-and-ear method, or by an assistant taking the time.

It also affords the means of comparing mean-time chronometers with each other within a very small margin of error, and gives results only slightly inferior to those obtained by comparing mean-time and sidereal chronometers by coincidence of beats.

Having compared the watches, we land to observe.

Care in
carrying
Hack-
Watches.

The watches to be used on the ground should always be carried in their boxes, and great care must be taken not to jerk them, and above all to avoid any circular motion.

Method of
Observa-
tion.

The method of observation for time differs from that already described of stars for latitude, inasmuch as we observe at stated altitudes, generally at every 10', setting the sextant for the purpose, and noting when the contact takes place. In observing with the stand, therefore, we only need to work the screw of the stand leg to get the suns vertically under one another.

Preparation of the ground is not necessary, as in the case of observing stars, excepting so far as selecting the spot, to ensure being able to see in both the a.m. and p.m. directions.

Observing
both
Limbs.

It is well to observe both upper and lower limbs, as, though it will make no difference to the result, it is good to have constant practice at both opening and closing suns, and not have all one way in the forenoon and the other in the afternoon. If we begin by a set of upper limbs, and immediately after take a set of lower, as an invariable practice, there will be no confusion, and we shall soon naturally fall into the system.

It may be here noted that with the inverting tube, the movable sun (the sun reflected from index and horizon-glass) is *above* the other, when we are observing upper limb, and *below* when lower limb. Also that upper limbs in the forenoon are closing suns, and in the afternoon opening suns. It is necessarily *vice versa* for lower limb.

Dark Eye-
pieces to
be used.

Always use the dark eye-pieces, of which there should be several of different degrees of shade, as, if the brilliancy of the sun varies by passing clouds, no inherent error is introduced by changing these, which is the case with the hinged

shades on the sextant. Moreover, the suns having been once equalized as to brilliancy with one eye-piece, by moving the up-and-down piece screw they will remain equal, no matter what shade of eye-piece we use ; but with the hinged shades, the position of up-and-down piece, which equalizes the suns, as seen through one set of them, will be different to that required for others, besides the possibility of error thus introduced.

The suns should be as dark as possible. If too light shades are used, the irradiation spoils the sharpness of the limb.

Use the eye-piece with the greatest magnifying power, as it much facilitates correct contacts.

Great care must be taken in setting the vernier, and we must see that the tangent screw at the commencement of each set is run back to its full extent, so as to avoid risk of being "two blocks" in the middle of the set, and so probably lose an observation.

After bringing the zero of the vernier into what we believe to be coincidence with the minute of arc required, glance right and left to see that the marks on vernier and arc are displaced in a symmetrical manner on either side. The eye will easier catch any inaccuracy in the setting by this means.

In setting, turn the tangent screw for the final adjustment the same way both forenoon and afternoon. Thus, if with altitudes increasing the tangent screw is turned to the right to attain coincidence, with altitudes diminishing the vernier must be set back below the required altitude, so as again to turn the screw to the right for final adjustment. This tends to eliminate error from slackness of screw.

Some observers, after giving a preliminary "Ready" at the commencement of each set, give no warning after, and simply "Stop" at each observation. With very careful time-takers this is sufficient, but experience of human nature leads us to say that it is better to call "Ready" about three seconds or so before each "Stop," and thereby avoid all chance of the time-taker having his eye and ear off the watch.

If a chornometer fitted for use with a chronograph is available, the accuracy of the observations will be much improved.

Suns
should not
be too
Bright.

Should be
as large as
possible.

Setting
the
Vernier.

Warning
Calls.

Use of the
Chrono-
graph.

The observer himself registers the time of each observation on the chronograph, using his left hand to make electric contact and his right hand to work the foot-screw of sextant stand.

An assistant, roughly noting the time of each observation to the nearest second or two, marks the tape at suitable intervals, as before mentioned.

The exact time is read off from the tape at leisure, and inserted in its proper column in the Sight Book.

Allowance
for Clouds
after
Transit.

We should take more observations in our first half of equal altitudes than will be absolutely needed, so as to allow of some losses in the observations after transit, from obscurations of the sun.

Index
Error.

At the conclusion of observations it is always well to take the index error. It tells us whether our sextant is keeping a steady error, and also, by calculation of semi-diameter therefrom, whether the instrument is in adjustment for side error, and also, if we lose the other half of equal altitudes, and decide to work single altitudes instead, we shall have the index error observed at the time.

Calculat-
ing Time
for Obser-
vations
after
Transit.

After sights before transit, we must calculate the time the observations after transit will commence. By far the simplest plan, when engaged in observations, is to have an ordinary watch set to apparent time, which time the ship herself will in many cases be keeping, when, by noting the time by this watch at the last observation, the time of commencement of the first observation after transit will be found by taking the time noted from twelve hours.

If we do not do this, and the ship be keeping mean time, we must find the mean time of the last observation by applying the approximate Error of the watch. Subtract this from twelve hours, and apply twice the equation of time, subtracting if apparent noon is before mean noon, and adding if *vice versa*. This will give mean time of the first observation after transit, which can be re-transferred to the watch by the application of the Error.

We want the time by the ship's clock, to ensure leaving her at the right time, and the time by our watch, to avoid the chance of being too late, on one hand, and scurry after reaching the observation spot, on the other.

Algebraically we can express this :

$$T = 12 - (t + e) + 2q,$$

where T is mean time of first observation required.

t is time by watch of last observation.

e is error of watch slow of mean time.

q is equation of time.

The book in which the observations are registered should be ruled as in annexed specimen. (See p. 306.)

Form in
Rough
Sight
Book.

The first column is for the intervals between each observation of the first set of sights ; the second, the time taken at those sights ; the third, the double altitude ; the fourth, the sum of the seconds of the two times ; the fifth, the time at second set of sights ; and the sixth for the intervals between the latter.

On next page is an example of a set of sights as written in the Angle Book.

When the time-taker is practised, it is well for him to note down the interval between the sights as he notes down the time, as it enables the observer at once to know, when the set is over, whether he has been getting good observations or not, as the intervals should theoretically be precisely the same.

Noting
Intervals
between
Sights.

We do not usually attempt to estimate time with a pocket-watch to less than two-tenths of a second, so we shall not find the intervals agreeing exactly, even supposing no errors of observation to exist. Taking everything together, if the difference in these intervals does not exceed one second and a half, we may consider that we have obtained very good observations. Another reason for noting these intervals is that we shall see if the sun's motion is becoming too slow.

In working out sights for equal altitudes, it is merely the mean of the middle times of each set that we wish to get, so that we need not mean up each column of times, but merely the sum of the seconds of each corresponding times, which we have in the fourth column of our Sight Book. Then, taking the times of the middle observation, meaning those, and substituting for the seconds of this mean the mean of the seconds just found from the fourth column, we shall have the true mean middle time of our observations. Thus, in our example

Method of
Meaning.

the mean of the fourth column is 14.5 seconds. We substitute this for the 14.0 seconds obtained by adding the two times at the middle observation, and, dividing by two, we get the mean middle time by the set as 11 hours 51 minutes 37.22 seconds.

In cases where the equation of equal altitudes is varying rapidly, we shall not find the middle times of two successive sets agreeing exactly, as they should differ by the amount of the variation of the equation of equal altitudes in the time.

$\frac{1}{2}$ April 3rd, 1880, at Nagara Light-house Δ .

Time by Breguet (2086).

| Interval in seconds. | Time by watch. | \odot | Sum of secs. | Time by watch. | Interval in seconds. |
|---|----------------|--------------|--------------|----------------|----------------------|
| | h. m. s. | $^{\circ}$ ' | s. | h. m. s. | |
| | 8 03 10.8 | 57 20 | 14.8 | 3 40 04.0 | 27.6 |
| 28.0 | 38.8 | 30 | 15.2 | 36.4 | 28.2 |
| 27.6 | 04 06.4 | 40 | 14.6 | 39 08.2 | 27.2 |
| 27.2 | 33.6 | 50 | 14.6 | 41.0 | 27.8 |
| 28.0 | 05 01.6 | 58 00 | 14.8 | 38 13.2 | 28.0 |
| 27.2 | 28.8 | 10 | 14.0 | 45.2 | 27.6 |
| 27.4 | 56.2 | 20 | 13.8 | 37 17.6 | 28.0 |
| 28.2 | 06 24.4 | 30 | 14.0 | 49.6 | 27.4 |
| 27.2 | 51.6 | 40 | 13.8 | 36 22.2 | 27.2 |
| 28.0 | 07 19.6 | 50 | 14.6 | 55.0 | 27.8 |
| 28.0 | 47.6 | 59.00 | 14.8 | 35 27.2 | |
| <div> <div>11) 5.0</div> <div>14.45</div> </div> | | | | | |
| <div> <div>h. m. s.</div> <div>Times at middle sight { 8 05 28.8</div> <div>15 37 45.2</div> </div> | | | | | |
| <div> <div>2) 23 43 14.0</div> </div> | | | | | |
| Mean mid. time by watch 11 51 37.22 | | | | | |

N.B.—The seconds of the result are obtained by halving the mean of Column 4.

Theoretically, this is an objection to long sets of observation, but practically the errors of observation will exceed any little discrepancy introduced by assuming the equation to be uniformly variable during a set of eleven to fifteen observations.

If a contact is missed in either half-set, it is no use to interpolate a time. The sight must be missed out of the double set altogether. It will not affect anything but the number of observations in the set, except when it happens to be the first

Sights
missed.

or last of a set, when the set will become one with an even number of observations, and to get elapsed time we must, instead of the central observation, take the mean of the two times corresponding to the two central observations.

Four sets of eleven observations each, half upper limb and half lower, ought to give accurate time; but it must be understood that equal altitudes do not eliminate personal errors, only instrumental and atmospherical ones. It is evident that, if an observer is, for instance, habitually too slow in recording his contacts, the resulting middle time will be so much after the true time of transit. Taking the case of an observer recording the time of contacts of opening limbs correctly, and of closing ones habitually too late, the middle time will still be in error. It is only when the observer records opening limbs in error one way, and closing in error in the contrary way, that these personal errors can be eliminated. This is, of course, not likely to happen with many people, and we must consider the time resulting from any observer's sights as being always in error by the amount of his personal equation.

In running a meridian distance, however, this personal error, supposing it to be tolerably constant, will disappear, as his time being equally in error, and in the same direction (either too fast or too slow) at both places, the difference of time, on which alone longitude depends, will not be affected. From this it results that in running a meridian distance, the same observers must always be employed.

The formula for finding the equation of equal altitudes is as follows :

$$\text{Equation} = A + B.$$

$$A \text{ (in seconds of time)} = \frac{1}{15} \times \frac{c}{2} \tan \text{ lat} \times \text{Cosec} \frac{\text{elapsed time}}{2}.$$

$$B \text{ (do.)} = \frac{1}{15} \times \frac{c}{2} \tan \text{ dec} \times \text{Cot} \frac{\text{elapsed time}}{2}.$$

Formula
for Equa-
tion of
Equal
Altitudes.

where $\frac{c}{2}$ is half the change of declination in the elapsed time,

or, as we use it in the computation, the change of declination in half the elapsed time.

The rules for noting the algebraic signs of A and B will be given hereafter.

Error
obtained
of Hack-
Watch

In making the observations it is most convenient to ascertain the Error of the hack-watch, and thence, by using the comparisons, to arrive at the Error of the standard. In the case where the watch has a large rate, as shown by the comparisons before and after sights, the elapsed time must be corrected for the amount gained or lost by the watch in the interval on mean time, which can be roughly calculated from the known rate of the standard.

Having meaned the sights, and obtained the mean middle time for each set, and knowing the estimated latitude and longitude, the rule for working a set of equal altitudes at superior transit will stand thus :

Practical
Rule for
Calcula-
tion of
Equation.

1. Ascertain elapsed time by subtracting the central time of observation before transit from the central time after transit, increased, if necessary, by twelve hours. Halve this, and if there are decimals, they can be rejected, the nearest even second being used.

2. To 0 hours apply longitude to find Greenwich date of apparent noon at place.

3. Correct declination at apparent noon in "Nautical Almanac" for the Greenwich date of apparent noon.

4. Correct equation of time at apparent noon for Greenwich date of apparent noon.

5. Multiply the variation in declination for one hour by the half-elapsed time, to get $\frac{c}{2}$.

N.B.—The variation we want is that at Greenwich time of local noon ; we must therefore correct the variation given in the "Nautical Almanac" for the longitude.

It is worth noting that in turning the minutes and seconds of $\frac{1}{2}$ E.T. into the decimal part of an hour, an exact and more easily workable decimal may be obtained by altering $\frac{1}{2}$ E.T. by a few seconds. The equation of equal altitudes is practically unaffected by so doing, if the log. cosecant and log. cotangent of the amended $\frac{1}{2}$ E.T. are used in the calculation, because the omission or addition of a single observation would of itself alter $\frac{1}{2}$ E.T. by a greater amount than that suggested.

$\frac{1}{2}$ E T. will thus be expressed to two places of decimals instead of three, giving fewer figures by which to multiply the

hourly variation ; and in taking out the log. cosecant and log. cotangent $\frac{1}{2}$ E.T. we shall be dealing with whole seconds instead of fractions.

6. For A, add together the logarithms of $\frac{c}{2}$, the tangent of the latitude, and cosecant of half-elapsed time ; and for B the logarithms of $\frac{c}{2}$, the tangent of the declination, and the cotangent of half-elapsed time. Either subtract the log. of 15 from each of these sums, to reduce the results to time at once, or take out the natural numbers of the sums as they stand, and when A and B have been added or subtracted, divide the result by 15, to reduce it to time.

N.B.—Tables are given in various works on nautical astronomy to facilitate the calculation of A and B ; but as these are only made out for every so many minutes of elapsed time, interpolation is necessary when working with any pretence to accuracy, and very little is gained by their use in their present form.

7. To the mean middle time of the set, apply the equation of equal altitudes with its proper sign (rule given below), which will give the time shown by the watch at apparent noon. N.B.—When working several sets, calculate them simultaneously as far as this, and mean the results, thus getting the mean time shown by the watch at apparent noon.

8. Find the mean time of apparent noon, by applying the equation of time with its proper sign to 0 or 24 hours, and take the difference between this and mean time shown by the watch, for the Error of the latter, subtracting one from the other, according as it is intended to show the watch as fast or slow on mean time.

A universal system must be adopted of showing all chronometers and hack-watches as fast or slow of the standard and on mean time, not some one way and some another, which leads to confusion. It does not much matter which is taken. The writer has always shown them as slow on mean time. Thus all chronometers are shown slow of the standard, and the standard and all others slow on mean time of place, or of Greenwich, as the case may be.

All
Watches
to be
either
Slow or
Fast of
Mean
Time.

The rule for giving A and B their proper algebraic signs is as follows :

Signs of
Equation.

$$\begin{array}{l} \text{At} \\ \text{Superior} \\ \text{Transit.} \end{array} \left\{ \begin{array}{l} \text{A is } \left\{ \begin{array}{l} + \text{ if declination is decreasing and of same} \\ \text{name.} \\ + \text{ if declination is increasing and of different} \\ \text{name.} \\ - \text{ if otherwise.} \end{array} \right. \\ \\ \text{B is } \left\{ \begin{array}{l} + \text{ if declination is} \\ \text{increasing} \\ - \text{ if declination is} \\ \text{decreasing} \end{array} \right\} \left. \begin{array}{l} \text{When elapsed time is} \\ \text{less than 12 hours.} \end{array} \right\} \\ \text{Reversed when elapsed time is greater than} \\ \text{12 hours.} \end{array} \right.$$

The above rule for the algebraic signs of A and B may be stated more briefly thus :

A is + from summer to winter solstice, and *vice versa*.
B is + the equinoxes and the solstices, and *vice versa*.

$$\begin{array}{l} \text{At} \\ \text{Inferior} \\ \text{Transit.} \end{array} \left\{ \begin{array}{l} \text{A is reversed from what it would be at superior transit.} \\ \text{B is the same as at superior transit.} \end{array} \right.$$

Change of
Declina-
tion in
Inferior
Transit.

In working with inferior transit, whereby we find the Error at midnight, there is no difference in the rule, except that in calculating the change during the half-elapsed time, we use the variation of declination found by interpolation for the Greenwich time of local midnight.

Calculat-
ing a
Mean
Com-
parison
for Hack-
Watch.

The next step is to calculate, from the comparisons taken with the standard before and after sights, a mean comparison to apply to the Error of watch found above, to arrive at the Error of the standard.

To do this, we take any sight, and by interpolation calculate the comparison at the a.m., and also the p.m. time corresponding. The mean of these two will give the comparison at noon. This should, if the watch has been going well, correspond very closely with the comparison actually taken at noon, and it will be satisfactory if it does so. If it does not, we cannot help it ; but we shall know that the Error of the standard will be slightly incorrect from a jump in the watch, and shall be prepared to give the result a smaller value in consequence, in event of discrepancies with others.

Noon
Com-
parison
not to be
used.

The mean comparison, as found above, must always be used, not the comparison *taken* at noon, which is done solely to ascertain how the watch has been going.

An example of the calculation follows :

AT MESALE I^d Δ AUG. 31st, 1878.

SIGHTS OBTAINED FOR ERROR BY EQUAL ALTITUDES. LAT. 5° 14' S.
LONG. 39° 40' E.

| | | | |
|-----------------------------------|------------|-------------------|-----------|
| P.M. Time by watch .. | h. m. s. | Ship app. T. .. | h. m. s. |
| A.M. " " .. | 10 16 22.4 | Long. .. | 0 00 00 |
| | 3 55 50.6 | | 2 38 44 E |
| El. time .. | 6 20 31.8 | G. date Aug. 30th | 21 21 16 |
| $\frac{1}{2}$ El. time for use .. | 3 10 16 | | |

| | | | | | |
|------------|---------|---------|----------|-----------|----------|
| Dec. .. | 8 36 30 | Var. .. | 54.18 | Eq. T. .. | m. s. |
| Correction | 2 23 | | 2.64 | | 2.03 |
| Dec .. | 8 38 53 | | 21672 | Eq. T .. | 0 14.12 |
| | | | 32508 | | |
| | | | 10836 | | |
| Var. .. | 54.18 | | " | Var .. | s. 0.768 |
| E.T. | 3.17 | | 143-0352 | | 2.64 |
| 2 | | | | | 3072 |
| | 37926 | | | | 4608 |
| | 5418 | | | | 1536 |
| | 16248 | | | | |

$$\frac{c}{2} = 171.6906$$

$$2.02752$$

| | | | |
|---------------------------|----------|-------------------------|----------|
| Tan lat .. | 8.961866 | Tan dec .. | 9.182106 |
| Cosec $\frac{E.T.}{2}$.. | 1.131907 | Cot $\frac{E.T.}{2}$.. | 9.961038 |
| $\frac{c}{2}$.. | 2.234770 | $\frac{c}{2}$.. | 2.234770 |
| | 1.328543 | | 1.377914 |
| 15 .. | 1.176091 | 15 .. | 1.176091 |
| | 0.152452 | | 0.201823 |
| A = - | 1.421 | B = - | 1.592 |
| | | A = - | 1.421 |

Equation of equal alt. - 3.013 secs.

| | | |
|----------------------------|------------|------------|
| Mean mid. time .. | h. m. s. | 7 06 06.51 |
| Eq. of Eq. alts. .. | | -03.01 |
| Time by watch at app. noon | 7 06 03.50 | |

| | | | | |
|-----------------------------------|---------|------------|--------------------|------------|
| Time by watch by 11 observations. | \odot | h. m. s. | } Mean of two sets | h. m. s. |
| 11 " | \odot | 7 06 03.41 | | 7 06 03.45 |
| 11 " | \odot | 03.49 | } " " | 03.44 |
| 11 " | \odot | 03.30 | | |
| 11 " | \odot | 33.59 | | |

| | |
|-------------------------|-------------|
| Mean Time by watch .. | 7 06 03.45 |
| Mean time of App. Noon | 12 00 14.12 |
| Watch (Breguet) slow .. | 4 54 10.67 |

To calculate the comparison between standard (A) and watch at noon, we have the following comparisons observed :

| Before A.M. Sights. | | | | Check. | After A.M. Sights. | | | | Check. |
|---------------------|----|----|------|-----------|---------------------|----|----|------|--------|
| | h. | m. | s. | secs. | | h. | m. | s. | secs. |
| A. .. | 4 | 16 | 55.0 | 02.6 | .. | 5 | 37 | 10 | 19.2 |
| Breguet .. | 3 | 30 | 02.4 | 10.0 | .. | 4 | 50 | 17 | 26.0 |
| | | | | 0 46 52.6 | 52.6 | | | | |
| | | | | Mean 52.6 | | | | | |
| | | | | | | | | | |
| Noon. | | | | Check. | Before P.M. Sights. | | | | Checks |
| | h. | m. | s. | secs. | | h. | m. | s. | secs. |
| A .. | 7 | 50 | 50 | 06.2 | .. | 9 | 47 | 25.0 | 38.8 |
| Breguet .. | 7 | 03 | 55.8 | 12.0 | .. | 8 | 50 | 30.6 | 44.0 |
| | | | | 0 46 54.2 | 54.2 | .. | 0 | 46 | 54.4 |
| | | | | Mean 54.2 | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | After P.M. Sights. | | | | Check. |
| | h. | m. | s. | | | h. | m. | s. | secs. |
| A .. | .. | .. | .. | | | 11 | 26 | 10.0 | 21.0 |
| Breguet .. | .. | .. | .. | | | 10 | 39 | 14.8 | 25.8 |
| | | | | | | 0 | 46 | 55.2 | 55.2 |
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Observations are sometimes necessary for the determination of "personal equation" before or after the exchange of telegraphic signals with a fixed observatory for telegraphic meridian distance. Sextant Observations for Time at an Observatory.

In such a case it is advisable to allow the standard sidereal clock to carry on the time between the forenoon and afternoon observations, and to eliminate the chronometer used for the sextant observations, except during the intervals a.m. and p.m. over which they extended. This is specially important when using the inferior transit.

The chronometer will be compared with sidereal standard clock immediately before and after exchanging signals, if a telegraphic meridian distance is being carried out.

The following example illustrates the method of treating the comparison between the sidereal standard clock (Hardy) and a mean solar chronometer (Mercer). Example.

On June 22, 1889, the following comparisons were made between Hardy and Mercer by coincidence of beats :

| | | A.M. | | | | | | P.M. | | | | | |
|--------|----|----------------------|----|------|---------------------|----|------|----------------------|----|------|---------------------|----|------|
| | | Before Observations. | | | After Observations. | | | Before Observations. | | | After Observations. | | |
| | | h. | m. | sec. | h. | m. | sec. | h. | m. | sec. | h. | m. | sec. |
| Hardy | .. | 2 | 32 | 22.5 | 4 | 09 | 38.0 | 8 | 06 | 22.5 | 9 | 50 | 09.5 |
| Mercer | .. | 8 | 28 | 23.0 | 10 | 05 | 22.5 | 2 | 01 | 28.0 | 3 | 44 | 58.0 |

| | | | | | | |
|------------------|---------|-----------------------------|----|-------------|----|---------|
| Mercer showed at | time of | middle sight of a.m. series | .. | h. | m. | sec. |
| " | " | corresponding | " | 9 | 48 | 20 |
| | | " | " | p.m. series | .. | 2 14 00 |

Required, the comparison at apparent noon.

| | | | | | | |
|----------------------------------|---------------|----------------------|----|-------------|----|---------|
| Mercer at | middle | sight of a.m. series | .. | h. | m. | sec. |
| " | corresponding | " | " | 9 | 48 | 20 |
| | | " | " | p.m. series | .. | 2 14 00 |
| Mercer at apparent noon (nearly) | .. | .. | .. | 0 | 01 | 10 |

| Before A.M. Observations. | | | | After A.M. Observations. | | | |
|--|----|----|------|--------------------------|----|----|------|
| | h. | m. | sec. | | h. | m. | sec. |
| Hardy .. | 2 | 32 | 22.5 | Hardy .. | 4 | 09 | 38.0 |
| Mercer .. | 8 | 28 | 23.0 | Mercer .. | 10 | 05 | 22.5 |
| Mercer slow on Hardy | 6 | 03 | 59.5 | Mercer slow on Hardy | 6 | 04 | 15.5 |
| Retardation for sidereal interval since first comparison | | | | .. | | | |
| | | | | 15.936 | | | |
| Mercer slow on Hardy, allowing for acceleration of sidereal time | | | | 6 03 59.564 | | | |

This method eliminates that portion of the rate of the sidereal standard on mean solar time which is due to the acceleration of sidereal time on mean solar time. It thus enables the rate of the mean solar chronometer to be compared with that of the solar sidereal clock reduced to mean solar time, besides giving smaller quantities to deal with.

The comparison at apparent noon may be worked up equally well by treating the four comparisons in the ordinary manner, as stated on p. 312.

TABLE I.

TABLE FOR VALUING SETS OF EQUAL ALTITUDES.

| | | | | Variations in Sums of Seconds of Middle Time. | | | | | | |
|---|-----------|--------------|----|---|-----|-----|-----|-----|-----|-----|
| | | | | s. | s. | s. | s. | s. | s. | s. |
| | | | | 1.5 | 2.0 | 2.4 | 3.0 | 3.5 | 4.0 | 5.0 |
| 4 | sets of 9 | observations | .. | 100 | 95 | 90 | 75 | 55 | 50 | 30 |
| 2 | " | 9 | " | .. | 90 | 80 | 70 | 50 | 35 | 30 |
| 4 | " | 7 | " | .. | 90 | 85 | 80 | 60 | 45 | 40 |
| 2 | " | 7 | " | .. | 60 | 55 | 50 | 40 | 25 | 20 |
| 4 | " | 5 | " | .. | 80 | 70 | 60 | 50 | 35 | 30 |
| 2 | " | 5 | " | .. | 50 | 45 | 40 | 30 | 20 | 15 |

TABLE II.

| | | Range in Time by Different Sets. | | | | | | | | | | | |
|---------------------------|-----|----------------------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|----|
| | | s. | s. | s. | s. | s. | s. | s. | s. | s. | s. | s. | s. |
| | | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 1.0 | 1.2 | 1.8 | 2.5 | |
| 4 sets of 9 observations. | .. | 100 | 90 | 80 | 70 | 60 | 50 | 40 | 30 | 25 | 15 | 10 | |
| 2 " | 9 " | 80 | 70 | 60 | 50 | 40 | 30 | 20 | 12 | 8 | 5 | 3 | |
| 4 " | 7 " | 100 | 90 | 80 | 65 | 50 | 40 | 30 | 25 | 20 | 15 | 8 | |
| 2 " | 7 " | 70 | 60 | 55 | 45 | 30 | 25 | 20 | 10 | 5 | 4 | 2 | |
| 4 " | 5 " | 90 | 80 | 70 | 60 | 40 | 30 | 25 | 18 | 15 | 10 | 7 | |
| 2 " | 5 " | 50 | 40 | 30 | 20 | 15 | 12 | 10 | 5 | 3 | 2 | 1 | |

The weight to be given to the results of each observer's observations in determining the final mean result presents some difficulty; but it must obviously have some relation to the number of observations in each set, and the variation in the sums of the seconds of middle time of the observations com-

Comparing the Values of Equal Altitudes by different Observers.

posing the set, combined with the range in the results of the various sets.

The foregoing tables give empirical values, which experience shows may be used to obtain weights that are at least reasonable.

The value of each set is found by taking from Tables I. and II. the proper quantities, and multiplying one by the other. The weight to be assigned to the total number of observations by each observer is the sum of the values of each set.

ERROR BY EQUAL ALTITUDES OF TWO STARS ON OPPOSITE SIDES OF THE MERIDIAN, BY CAPTAIN A. M. FIELD, R.N.

The principle of this method depends upon the sidereal time of passing the meridian of a place, by an imaginary star having the mean right ascension of the two stars selected, being compared with the time shown by a sidereal chronometer at that instant; the difference is its error on sidereal time. A mean solar chronometer can be used equally well. The sidereal time required is the mean of the right ascensions of the two selected stars. The chronometer time at that instant (mean or sidereal) is the mean of the times at which the eastern and western stars had equal altitudes, with the "equation of equal altitudes" applied with its proper sign.

When preparing a list of stars for observing, it will be necessary to first find the R. A. of the meridian for the times between which it is required to carry on the observations. As stars will generally be observed within 4^h of the meridian, the limits of R. A. of the stars falling within the required period may be obtained by subtracting 4^h from the first R. A. of the meridian, and adding 4^h to the last R. A. of the meridian as found above.

To arrange the stars in pairs, it is necessary to select two bright stars of nearly the same declination, not differing much from the latitude, but differing in R. A. by from 4^h to 8^h . The time at which they will be simultaneously of equal altitude will be about the time when the mean of their R. A.'s is on the meridian.

The time at which it will be necessary to begin observing will be governed by this, and the observations of one star

should be completed shortly before that time, in order to allow an interval to prepare for observing the other.

It may be found that when two stars are of equal altitudes, they are too near the meridian for obtaining the best results, or that the altitudes are too great for the sextant ; in which case it would be necessary to commence observations of the eastern star as much as an hour earlier, and the western star will then have to be observed the same time later than when they would be simultaneously of equal altitude.

As a general rule, if the difference in right ascension is less than 6^h , the eastern star should be observed first ; if it exceeds 6^h , then the western star ; this is in order that the stars may be observed as favourably as possible with respect to the "prime vertical," but it will vary according to the latitude and declination. It will be noticed that if the observations are commenced with the eastern star, then they are, as a whole, taken farther from the meridian than in the other case.

If the difference in R. A. exceeds 8^h , then there will probably be an interval between finishing the observations of one star and beginning those of the other, and part of the advantages of the method are lost ; the same remark applies if the difference in R. A. is less than 4^h .

Having decided on which star to begin with, observe it continuously in the ordinary way, until the sidereal time is nearly equal to the mean R. A. of the two stars (the error of chronometer on sidereal time should be roughly known), when prepare to observe the other star, commencing at the same altitude as the last observation of the first star, and complete the series, which may be divided into sets in the usual way.

Owing to the more rapid changes in "equation of equal altitudes," when there is a large difference in the declinations of the stars, it will be remarked that the "middle times" vary more rapidly than in the case of the sun; and the rapidity of this change increases as the observations get farther away from the sidereal time at which the "imaginary star" passes the meridian.

If a mean solar chronometer be used, the chronometer interval (corrected for rate) must be turned into a sidereal interval, and the resulting "error of chronometer" will be the error on sidereal time at that particular instant, from which the error on mean time can be readily deduced.

Equation of Equal Altitudes.

The rigorous expression, according to Chauvenet, is :

$$\sin a = \cot \frac{1}{2} \text{ E.T.} \tan d \cdot \tan \delta \cdot \cos. a - \operatorname{cosec}. \frac{1}{2} \text{ E.T.} \tan l \cdot \tan \delta.$$

where a = equation of equal altitudes = $\frac{h - h^1}{2}$

$$\frac{1}{2} \text{ E.T.} = \frac{1}{2} \text{ elapsed time} = \frac{h + h^1}{2}$$

d = declination at upper meridian passage.

$d - \delta$ = „ „ observation E. of meridian.

$d + \delta$ = „ „ „ W. „

$\delta = \frac{1}{2}$ difference of the declination at the two times of observation.

h and h^1 = Hour angles from noon at East and West observations respectively.

In the case of equal altitudes of two stars, one east and the other west of meridian, the above formula is strictly accurate, whatever may be the difference in the declinations ; the half-elapsed time being found as follows :

$$\begin{aligned} \frac{1}{2} \text{ E.T.} &= \frac{h + h^1}{2} \\ &= \frac{(R - S) + (S^1 - R^1)}{2} \\ &= \frac{(R - R^1) + (S^1 - S)}{2} \end{aligned}$$

Where, R and R^1 = Right ascensions of star E. and star W.

S and S^1 = Sidereal time of observation of star E. and star W. respectively.

$S^1 - S$, the difference of the sidereal times, may be taken as equal to the difference of times (as shown by a sidereal chronometer) of the observation of the two stars ; the chronometer being, of course, corrected for rate.

If $R^1 > R$, add 24^h to R .

If the western star be observed first, in which case $S > S^1$, then S and S^1 are treated algebraically.

d = the mean of the declinations of the two stars.

$\delta = \frac{1}{2}$ difference „ „ „

In working the rigorous expression, an approximate value for a is first obtained, disregarding the term $\cos. a$, and then rework, using the value of a thus found.

Notes on the Computation.

The constant logarithm 4.138339 is an abridgment of Chauvenet's formula, by adopting it to circular measure, and is correct so long as the equation of equal altitude is small; certainly up to four minutes no error is introduced by its use.

The cosine of a with an equation of equal altitude under four minutes changes so slowly that it is practically the same for each set unless separated by a very long interval.

The R. A. M. S. must be found for the Greenwich date corresponding to the chronometer middle time. In cases of a small equation of equal altitude and changing slowly, this is the same for each set; but where it is large, and therefore changing rapidly, as in the second example, the R. A. M. S. must be corrected for every set. But this merely involves applying the acceleration due to the difference of the chronometer middle time of each successive set to the R. A. M. S. for the first set.

A more or less accurate knowledge of the G. M. T. is therefore necessary, but this is inherent to the use of stars for time under all circumstances, and a second approximation must be made if the error on local time has been assumed more than three or four seconds in error.

It is worthy of remark that $\tan d \times \tan \delta \times 4.138339$ and $\tan l \times \tan \delta \times 4.138339$ form absolutely constant logarithms for the same stars, at the same place, for any night in the year, and it is only necessary to add the logarithm of $\frac{1}{2} E. T.$

PAGE 319, lines 32, 33, 34.—*Cancel and substitute.*

The equation of equal altitude is nearly always so large as to exclude all possibility of doubt as to which way to apply it, but the investigation gives the rule. If the declinations of the two stars are nearly equal, it is necessary to pay close attention to the signs of the expressions of the formula to avoid any chance of making a mistake.

Equation of Equal Altitudes.

The rigorous expression, according to Chauvenet, is :

$$\sin a = \cot \frac{1}{2} \text{ E.T. } \tan d. \tan \delta. \cos. a - \operatorname{cosec.} \frac{1}{2} \text{ E.T. } \tan. l. \tan \delta.$$

where a = equation of equal altitudes = $\frac{h - h^1}{2}$

$$\frac{1}{2} \text{ E.T. } = \frac{1}{2} \text{ elapsed time } = \frac{h + h^1}{2}$$

d = declination at upper meridian passage.

$d - \delta$ = „ „ observation E. of meridian.

$d + \delta$ = „ „ „ W. „

δ = $\frac{1}{2}$ difference of the declination at the two times of observation.

h and h^1 = Hour angles from noon at East and West observations respectively.

In the case of equal altitudes of two stars, one east and the other west of meridian, the above formula is strictly accurate, whatever may be the difference in the declinations ; the half-elapsed time being found as follows :

$$\begin{aligned} \frac{1}{2} \text{ E.T. } &= \frac{h + h^1}{2} \\ &= \frac{(R - S) + (S^1 - R^1)}{2} \\ &= \frac{(R - R^1) + (S^1 - S)}{2} \end{aligned}$$

Where, R and R^1 = Right ascensions of star E. and star W.

S and S^1 = Sidereal time of observation of star E. and star W. respectively.

$S^1 - S$, the difference of the sidereal times, may be taken as

In working the rigorous expression, an approximate value for a is first obtained, disregarding the term $\text{Cos. } a$, and then rework, using the value of a thus found.

Notes on the Computation.

The constant logarithm 4.138339 is an abridgment of Chauvenet's formula, by adopting it to circular measure, and is correct so long as the equation of equal altitude is small; certainly up to four minutes no error is introduced by its use.

The cosine of a with an equation of equal altitude under four minutes changes so slowly that it is practically the same for each set unless separated by a very long interval.

The R. A. M. S. must be found for the Greenwich date corresponding to the chronometer middle time. In cases of a small equation of equal altitude and changing slowly, this is the same for each set; but where it is large, and therefore changing rapidly, as in the second example, the R. A. M. S. must be corrected for every set. But this merely involves applying the acceleration due to the difference of the chronometer middle time of each successive set to the R. A. M. S. for the first set.

A more or less accurate knowledge of the G. M. T. is therefore necessary, but this is inherent to the use of stars for time under all circumstances, and a second approximation must be made if the error on local time has been assumed more than three or four seconds in error.

It is worthy of remark that $\tan d \times \tan \delta \times 4.138339$ and $\tan l \times \tan \delta \times 4.138339$ form absolutely constant logarithms for the same stars, at the same place, for any night in the year, and it is only necessary to add the logarithm of $\frac{1}{2}$ E. T. to each to obtain the equation of equal altitude corresponding to that $\frac{1}{2}$ E. T.

The equation of equal altitude is always so large as to exclude all possibility of doubt as to which way to apply it, but the investigation gives the rule. Two examples are given; the first illustrates the cases of two stars differing by $1^{\circ} 40'$ in declination, and the second where they differ by $8^{\circ} 25'$.

In the first instance the equation of equal altitudes hardly

changes between the first observation and the last, and therefore the sums of the middle times do not vary. This is due to the difference in declination being small, and to the fact that the middle time of observation is only 10 minutes 16 seconds from the "time of crossing," or the time at which the stars had equal altitudes, the one rising and the other setting, and is represented by the " $\frac{1}{2}$ interval."

In the second instance the equation of equal altitude changes very rapidly; the difference in declination is large, and the " $\frac{1}{2}$ interval" is also somewhat large—viz., 44 minutes 20 seconds; but the " $\frac{1}{2}$ interval" may, nevertheless, be extended to upwards of an hour, if necessary, without affecting the accuracy of the result.

The acceleration in the change of equation of equal altitudes may be considered as practically uniform for the short intervals of four or five minutes necessary to obtain sets of observations, and therefore, although it will be noticed that the sums of the middle times change rapidly, yet it will nevertheless be perfectly accurate to take the means and calculate the equation of equal altitude corresponding to the " $\frac{1}{2}$ interval" for the middle observation, no matter how rapidly the equations may change.

It should be observed that in the case of two stars differing much in declination, a small error in the latitude may produce an appreciable error in the equation of equal altitudes.

For the same elapsed time and differences of declination, this error varies as sec. lat., the formula being—

$$\text{Error in equation of equal altitudes} = \frac{\text{error in latitude}}{15} \times \text{sec. } l. \times \text{cosec } \frac{1}{2} \text{ E. T.} \times \tan \delta.$$

In latitude 45° the error from this cause is twice what it would be at the Equator, and in latitude 60° it is four times as great, since $\text{sec. } 45^\circ = 2$, and $\text{sec. } 60^\circ = 4$.

Also, if the latitude and elapsed time remain the same, the error varies as $\tan \delta$, or practically directly as δ , for small values of δ up to 10° .

The following table shows the error in equation of equal altitudes resulting from an error of $5''$ in latitude, using stars with varying differences of declination in different

latitudes, assuming $\frac{1}{2}$ E. T. to be 2 hours 30 minutes throughout.

| Latitude. | Difference in Declination. | | |
|-----------|----------------------------|--------------|--------------|
| | 5° | 10° | 15° |
| 0° | sec. ·024 | sec. ·048 | sec. ·072 |
| 30° | ·032 | ·064 | ·096 |
| 45° | ·048 | ·096 | ·144 |
| 60° | ·096 | ·192 | 288 |

The most favourable conditions for the use of this method are thus shown to be in low latitudes, and with stars not differing much in declination.

EXAMPLE 1.

On the evening of 28th November, 1892, at Hong Kong Dockyard, latitude $22^{\circ} 16' 55''$ N., longitude $114^{\circ} 9' 49''$ E., the following observations of Markab (West of meridian) and Aldebaran (East of meridian) were obtained:

| Interval. | Markab (W.) Time by watch. | Double Altitude. | Sums. | Aldebaran (E.) Time by watch. | Interval. |
|---------------|-------------------------------|------------------|-----------------|----------------------------------|------------------------|
| " | | | " | | |
| — | | | | | |
| 43.2 | h. m. s. 1 15 14.8 | 106 20 | 55.2 | h. m. s. 1 38 40.4 | 42.8 |
| 43.2 | 1 15 58.0 | 106 00 | 55.6 | 1 37 57.6 | 43.2 |
| 43.2 | 1 16 41.2 | 105 40 | 55.6 | 1 37 14.4 | 43.6 |
| 43.6 | 1 17 24.8 | 105 20 | 55.6 | 1 36 30.8 | 43.2 |
| 42.8 | 1 18 07.6 | 105 00 | 55.2 | 1 35 47.6 | — |
| | | | 5) 2.2 | | |
| | | | <u>2) 55.44</u> | | |
| | | | <u>57.72</u> | | |
| Markab .. | h. m. s. .. 1 16 41 | | | | |
| Aldebaran .. | .. 1 37 14 | | | | |
| | 2) 2 53 55 | | | | |
| Middle time | .. 1 26 57 | | | | |
| ½ Interval .. | .. 0 10 16 | | | | |
| | | | | Mean middle time by watch = | h. m. s. 1 26 57.72 |
| | | | | ½ Interval .. | .. = 0 10 16 |

COMPUTATION.

| | | | | | | | | | |
|--|-------|------------------|--|--|----------|--------------|----------------------------------|-------|--------------|
| $\frac{1}{2}$ Interval | | h. m. s. | h. m. s. | R. A. Markab | | h. m. s. | Declination | | ° ' " |
| | | 0 10 16 | 22 53 24.93 | R. A. Aldebaran | | 4 29 47.08 | Declination | | 14 37 47 N. |
| Mean Middle Time | | 1 26 57.72 | 3 29 12.01 | Mean R. A. | | 1 44 36.005 | | | 30 55 32 |
| Equation of equal altitude | | + 1 01.99 | | $\frac{1}{2}$ Difference | | 2 45 11 | d | = | 15 27 46 |
| Time by watch of crossing | | 1 27 59.71 | 0 10 18 | $\frac{1}{2}$ Interval (Sidereal Time) | | 0 10 18 | δ | = | 0 49 59 |
| | | | | $\frac{1}{2}$ Elapsed Time | | 2 34 53 | | | |
| GREENWICH DATE. | | | | | | | | | |
| Tan d | | 9.441883 | Tan l | | | 9.672550 | Time by watch of crossing | | h. m. s. |
| Tan δ | | 8.462536 | Tan δ | | | 8.162536 | Approximate error | | 1 27 59.7 |
| Constant logarithm | | 4.138339 | Constant logarithm | | | 4.138339 | | | 7 44 57 |
| Constant logarithm for all sets with these stars | | 1.742758 | Constant logarithm for all sets with these stars | | | 1.913425 | S. M. T. | | 9 12 57 |
| Cot $\frac{1}{2}$ E. T. | | 10.095662 | Cosec $\frac{1}{2}$ E. T. | | | 10.203754 | Longitude in time | | 7 36 39 E |
| Cos equation equal altitude | | 9.999996 | | | | 2.117179 | Greenwich date, 28th Nov. .. | | 1 36 18 |
| | | 1.838716 | | | | | | | |
| Equation of equal altitude | | 61.99 or 1 01.99 | Sidereal time | | h. m. s. | 16 31 23.380 | | | |
| | | | Acceleration, 1 h. | | | 9.856 | | | |
| | | | 30 m. | | | 5.914 | | | |
| | | | 15 sec. | | | 0.050 | | | |
| | | | | | | | R. A. M. S. | | 16 31 39.200 |
| | | | | | | | Mean R. A. | | 1 44 36.005 |
| | | | | | | | S. M. T. of crossing | | 9 12 56.805 |
| | | | | | | | Time by watch of crossing | | 1 27 59.710 |
| | | | | | | | Watch slow on M. T. at 9.13 p.m. | 7 44 | 57.095 |

COMPUTATION.

| | | | | | | | | |
|--|-------|-------------------|--|-------|--------------|-----------------------------------|--------------|---------------|
| $\frac{1}{2}$ Interval | | h. m. s. | R. A. Markab | | h. m. s. | Declination | | ° ' " |
| Mean Middle Time | | 0 44 21 | R. A. γ Orionis | | 22 59 25.100 | Declination | | 14 37 47.2 N. |
| Equation of equal altitude | | 3 27 43.63 | | | 5 19 23.548 | | | 6 12 55.5 N. |
| | | + 3 34.23 | | | | | | |
| Time by watch of crossing | | 3 31 17.86 | Mean R. A. .. | | 4 18 48.648 | | | 20 50 42.7 |
| | | | $\frac{1}{2}$ Difference .. | | 2 09 24.324 | d | = | 10 25 21 |
| | | | $\frac{1}{2}$ Interval (Sidereal Time) .. | | 3 09 59.2 | δ | = | 4 12 26 |
| | | | $\frac{1}{2}$ Elapsed Time | | 2 25 30.7 | GREENWICH DATE. | | |
| Tan d | | 9.564681 | Tan l .. | | 8.351616 | Time by watch of crossing | h. m. secs. | |
| Tan δ | | 8.866649 | Tan δ .. | | 8.866649 | Approximate error | 3 31 17.9 | |
| Constant logarithm | | 4.138339 | Constant logarithm | | 4.138339 | | 7 05 14 | |
| Constant logarithm for all sets with these stars | | 2.269669 | Constant logarithm for all sets with these stars | | 1.356604 | S. M. T. .. | 10 36 32 | |
| Cot $\frac{1}{2}$ E. T. .. | | 10.132734 | Cosec $\frac{1}{2}$ E. T. .. | | 10.226870 | Longitude in time | 6 55 25 E. | |
| Cos equation of equal altitude | | 9.999947 | | | 1.583474 | Greenwich date, 13th Nov. | 3 41 07 | |
| | | | | | | Sidereal time | h. m. secs. | |
| | | | | | | Acceleration, 3 h. | 15 32 15.020 | |
| | | | | | | 41 m. | 29.569 | |
| | | | | | | 7 sec. | 6.735 | |
| | | | | | | | 0.019 | |
| | | | | | | R. A. M. S. .. | 15 32 51.343 | |
| | | | | | | Mean R. A. .. | 2 09 24.324 | |
| | | | | | | S. M. T. of crossing .. | 10 33 32.981 | |
| | | | | | | Time by watch of crossing | 3 31 17.860 | |
| | | | | | | Watch slow on M. T. at 10.30 p.m. | 7 05 15.121 | |
| | | | | | | Mean R. A. .. | 2 09 24.324 | |
| | | | | | | S. M. T. of crossing .. | 10 36 32.981 | |
| | | | | | | Time by watch of crossing | 3 31 17.860 | |
| | | | | | | Watch slow on R. T. at 10.30 p.m. | 7 05 15.121 | |
| Equation of equal altitude | | 214.23 or 3 34.23 | | | | | | |

CHAPTER XIV

MERIDIAN DISTANCES

Telegraphic—Chronometric.

UNDER this head we shall consider all the methods available for our purposes of obtaining difference of longitude.

TELEGRAPHIC MERIDIAN DISTANCE.

Where a telegraph can be used, it is of course the best, and at the same time the simplest, means of obtaining difference of longitude.

This method consists in sending a current through the wire at a known local time from one place, the local time of arrival at the other place being noted. The difference of these is the difference of longitude.

Retarda-
tion.

In theory, the passage of the current through the wire is instantaneous ; but in practice it takes an appreciable time, when the distance is considerable, and the electrical condition of the wire is not first-rate ; and to eliminate this, and also to decrease errors of sending and receiving, we must send several sets of signals in both directions equally, the mean of which will give the true time.

A little consideration will show that if a signal is sent from A to B, a place to the westward, and it takes two seconds to traverse the wire, the time at B will have had those two seconds in which to catch up the A time, which is so much ahead ; or, in other words, the difference of the two times as shown will be two seconds too little. Whereas, if the signal is sent from B to A, the watch at A, already ahead, will advance another two seconds before the signal arrives, and

the difference of the two times will be two seconds too much. The mean of the two will therefore be correct, and half the difference of the two will give the "retardation of the wire"—a matter, however, purely of curiosity as far as our results are concerned. The mean also eliminates "personal equation" in sending and receiving signals. In land lines the retardation is about one-tenth of a second per thousand miles; with submarine cables it is larger, and varies.

To eliminate all personal errors, the only satisfactory method is for the observers to change ends. When this is not possible, personal errors, when obtainable, should be applied. These can only be determined by a considerable series of observations for time, and the error of each observer or the person chosen as standard being recorded and meaned. The steadiness, and therefore the value, of the personal error can thus be judged.

Personal equation in sending and receiving signals can best be determined at an observatory by means of a chronograph; it is usually very much smaller than that found in the sextant observations of different observers.

In this case a number of chronometers is not necessary. Only One Watch Necessary. All that is wanted is one good time-keeper. If, however, two watches are at hand, it is not amiss to ascertain the Errors of each separately, and use them both in transmitting the signals.

A box-chronometer is the best for sending and receiving signals by, and, if practicable, it may be a good plan to land one, and let it stand in the telegraph office for a few days beforehand to settle down, comparing the watch actually used at sights with it before and after observations.

Sights must be obtained on the day of sending the signals, and the latter should be transmitted at or about noon or midnight. Where the places are far apart in longitude, it can only be near noon at one place, and Error must be obtained at the other, either on the day before or after, as well, so as to be able to correct the Error to the time of interchange of signals. Using land-lines, day time is most favourable for exchange of signals.

If the observation spot can be at, or close to, the telegraph office, it is convenient, as the watch will not have to be carried

Time to
exchange
Signals.

Observa-
tion Spot.

about ; but in many instances the local arrangements will not admit of this.

Galvano-
meter.

Telegraphic instruments differ very much ; but it does not much matter which are used, as long as they are similar at both ends. The deflection of an ordinary galvanometer needle of Wheatstone's instrument, or of the Morse recorder, or of the more delicate mirror of long submarine cables, will all serve our purpose. Preference is given to one or the other by different observers. The writer prefers an instrument giving a sound to the silent movement or the suspended mirror.

Each signal will consist of one deflection, and the key should be kept pressed down for about a second.

Pre-
arranged
Method of
sending
Signals.

In sending the signals, it must be clearly arranged beforehand what is going to be done.

A good plan is as follows :

In commencing, give a warning, say of three rapid signals, at ten seconds before an even minute by the sender's watch. The first signal will then go at the even minute, and at every ten seconds another, *missing the fifty seconds, to mark the even minute*, for three minutes, ending with another even minute.

After an interval of three or four minutes, a similar set will be sent in the reverse direction.

If at the receiving ends the signals agree, this will be quite sufficient, unless we intend to use another watch.

An example of a telegraphic meridian distance is appended.

It will be seen that the resistance of the wire and instrumental retardation was less on one day than on the other, amounting at one time to nearly a tenth of a second, and at the other to only twenty-five thousandths.

Use of
Sidereal
Chrono-
meter.

If possible, a sidereal chronometer should be used at one end and a solar chronometer at the other. A different fraction of a second having to be estimated at each signal prevents any bias on the part of the observer receiving the signals, which is the weak point when using two solar chronometers. The coincidence of beats occurring every 3 minutes 4 seconds is worth more as an accurate comparison than any number of estimations of fractions of a second, and should be used in preference, provided the signals have been sent automatically by a clock.

MERIDIAN DISTANCE BY TELEGRAPH SIGNALS EXCHANGED BETWEEN
CONSTANTINOPLE AND DARDANELLES.

April 3rd and 18th, 1880.

Observation spot at Constantinople was at Leander's Tower, observers having to go 1½ miles to Telegraph Office by caïque.

Observation spot at Dardanelles at Nagara Light-house, observers having to go three miles to Telegraph Station by steam pinnace.

| Series. | Watch Times. | | Local Times. | | Meridian Distance. | Remarks. |
|--------------------------------|----------------------|------------------------|------------------------|------------------------|-----------------------|---|
| | Sending. | Receiving. | Sending. | Receiving. | | |
| Dardanelles to Constantinople. | h. m. s. 11 31 00 | Missed. | h. m. s. 11 42 48.2 | Missed. | | |
| | | h. m. s. 11 44 15.2 | 58.2 | h. m. s. 11 53 22.1 | h. m. s. 0 10 23.9 | Sent by Breguet 2084. Received by Dent 6119. |
| | 10 | 25.3 | 43 08.2 | 32.2 | 24.0 | |
| | 30 | 35.2 | 18.2 | 42.1 | 23.9 | |
| | 40 | 45.3 | 28.2 | 52.2 | 24.0 | |
| | 32 00 | 45 05.3 | 43 48.2 | 54 12.2 | 24.0 | |
| | 10 | 15.2 | 58.2 | 22.1 | 23.9 | |
| | 20 | 25.2 | 44 08.2 | 32.1 | 23.9 | |
| | 30 | 35.2 | 18.2 | 42.1 | 23.9 | |
| | 40 | 45.3 | 28.2 | 52.2 | 24.0 | |
| | 33 00 | 46 05.2 | 48.2 | 55 12.1 | 23.9 | |
| | 10 | 15.2 | 58.2 | 22.1 | 23.9 | |
| | 20 | 25.2 | 45 08.2 | 32.1 | 23.9 | |
| | 30 | 35.2 | 18.2 | 42.1 | 23.9 | |
| | 40 | 45.2 | 28.2 | 52.1 | 23.9 | |
| | 34 00 | 47 05.2 | 48.2 | 56 12.1 | 23.9 | |
| | | | | Mean .. | 0 10 23.93 | |
| Constantinople to Dardanelles. | 11 49 00 | 11 35 55.2 | 11 58 06.9 | 11 47 43.4 | 0 10 23.5 | Sent by Dent 6119. Received by Breguet 2084. |
| | 10 | 05.0 | 16.9 | 53.2 | .7 | |
| | 20 | 15.0 | 26.9 | 48 03.2 | .7 | |
| | 30 | 25.0 | 36.9 | 13.2 | .7 | |
| | 40 | 35.0 | 46.9 | 23.2 | .7 | |
| | 50 00 | 54.9 | 59 06.9 | 43.1 | .8 | |
| | 10 | 37 04.8 | 16.9 | 53.0 | .9 | |
| | 20 | 14.8 | 26.9 | 49 03 0 | .9 | |
| | 30 | 24.9 | 36.9 | 13.1 | .8 | |
| | 40 | 35.0 | 46.9 | 23.2 | .7 | |
| | 51 00 | 54.8 | 12 00 06.9 | 43.0 | .9 | |
| | 10 | 04.9 | 16.9 | 53.1 | .8 | |
| | 20 | 14.9 | 26.9 | 50 03.1 | .8 | |
| | 30 | Missed. | — | — | — | |
| | 40 | 34.9 | 46.9 | 23.1 | .8 | |
| | 52.00 | 55.0 | 01 06.9 | 43.1 | .7 | |
| | | | C. to D. | Mean .. | 0 10 23.76 | |
| | | | D. to C. | " .. | 23.93 | |
| | April 3rd | Mean Mer. | Dist. .. | 0 10 23.84 | | |

In the event of chronographs being available for recording the signals received at each end, a sidereal chronometer is not necessary.

The following example of a telegraphic meridian distance Example. between Penang and Singapore illustrates the case of using a sidereal chronometer in connection with a mean solar chronometer. At Singapore the time was carried on by means of a sidereal standard clock, a sidereal chronometer (Mercer) being used for exchanging signals. At Penang a mean solar chronometer (I) was used for both purposes.

| AT PENANG. | | | AT SINGAPORE. | | |
|----------------------------------|----|-----------|---------------------------------------|----|-----------|
| Signals sent by I. | | | Signals received by Mercer. | | |
| | h. | m. sec. | | h. | m. sec. |
| Mean of 16 signals sent by I. .. | 5 | 08 26·850 | Mean of 16 signals received by Mercer | 7 | 10 26·400 |
| I slow on M.T. .. | 6 | 41 48·184 | Mercer slow on sidereal standard .. | 23 | 34 26·778 |
| Penang M.T. sending .. | 11 | 50 15·034 | Sidereal standard | 6 | 44 53·178 |
| Singapore M.T. receiving .. | 0 | 04 21·227 | Sidereal standard slow .. | | 51·641 |
| Meridian distance | 0 | 14 06·193 | Sidereal time .. | 6 | 45 44·819 |
| | | | Sidereal time at midnight .. | 6 | 41 22·877 |
| | | | Sidereal interval .. | 0 | 04 21·942 |
| | | | Reduction to solar time .. | | 0·715 |
| | | | Singapore M.T. receiving signal .. | 0 | 04 21·227 |

The arrangements recently carried out for exchanging telegraphic signals over very long distances may be mentioned. The cable between Ascension and Porthcornow being joined up at the intermediate landing-point, syphon recorders with two syphons were used at both ends. At Ascension one syphon was connected through a battery with the chronometer specially fitted to record its beats on a tape, the other syphon being connected with the cable. The syphons were adjusted for parallax. Similar arrangements were made at Porthcornow, the sidereal clock at Greenwich recording its beats on the tape at the former place by means of a direct and

Tele-
graphic
Meridian
Distance
between
Green-
wich and
Ascension.

return wire. These arrangements were entirely successful over a distance of more than 4,000 miles, which is the longest distance over which such work has yet been carried out.

The indications on the tapes were not sharply defined, but it was possible to detect unmistakably the first indication of the movement of the syphon recorder, which was the particular phase selected for measurement in each case.

Personal Equation. The following formulæ show the effect of the observer's personal equation on the results :

A and B are the observers at X.

C and D are the observers at Y.

+ b = B's personal equation on A as determined by equal altitude observations.

+ c = C's personal equation on A as determined by equal altitude observations.

+ d = D's personal equation on A as determined by equal altitude observations.

A = local time of the signal at X by A's observations.

B = " " " " X " B's "

C = " " " " Y " C's "

D = " " " " Y " D's "

If X is situated to the eastward of Y,

Meridian distance = local time at X - local time at Y.

$$\begin{aligned} &= \frac{A + (B + b)}{2} - \frac{(C + c) + (D + d)}{2} \\ &= \frac{(A + B) - (C + D)}{2} + \frac{b - c - d}{2}. \end{aligned}$$

If Y is situated to the eastward of X,

Meridian distance = local time at Y - local time at X.

$$\begin{aligned} &= \frac{(C + c) + (D + d)}{2} - \frac{A + (B + b)}{2} \\ &= \frac{(C + D) - (A + B)}{2} - \frac{b - c - d}{2}. \end{aligned}$$

The amount and effect of personal equation being known beforehand, it is sometimes possible to combine the observers

in pairs in such a manner that it shall produce little or no effect on the result, which is a desirable object to attain.

In order to secure reliable results, the observations should extend over two or more days, exchanging signals at noon and midnight on each consecutive day. The error of the chronometer being obtained at regular intervals, a satisfactory check is thus afforded on the observations themselves, and also on the regularity of the rate of the chronometers throughout the whole period. The observers should then change ends and repeat the series. A full report of the operations should be rendered, giving details of the observations for time, the various comparisons between the chronometers used for sending and receiving signals, and the standard chronometer used at each end for carrying on the time. When it is impracticable for the observers to change ends, the results of any previous observations that may be available for determination of personal equation should also be included. For the form in which these details may conveniently be rendered, reference should be made to the various pamphlets that have been issued from time to time by the Hydrographical Department, giving the results of such observations.

Return
showing
Details of
Tele-
graphic
Meridian
Distance

CHRONOMETRIC MERIDIAN DISTANCES.

When we have no telegraph, we must have recourse to chronometers for conveying the time.

Having obtained sights at the two places whose meridian distance we require, we come to the consideration of the rate to be used.

If we have been able to run backwards and forwards, as recommended on p. 294, we shall use a travelling rate.

The algebraic formula for finding the meridian distance by travelling rate, when we return at once to the original station, is as follows :

Formula
for
Travel-
ling Rate

$$M = \beta - a - n \frac{a' - a}{m + n},$$

where M = meridian distance,

a = error at place A, before starting,

a' = „ „ A, on returning,

β = „ „ B,

n = No. of days between first observations at A and those at B,

m = No. of days between observations at B and those at A, on returning.

Then $\frac{a' - a}{m + n}$ = travelling rate.

This can be put another way, by example, as follows :

Example
of Travel-
ling Rate.

Let us suppose we have obtained Error at Maziwi at noon, August 27; we have been to Mesale, and there obtained Error at noon on the 31st; and then, returning to Maziwi, have obtained another Error there at midnight of September 1-2.

To find rate in this case we simply divide the difference of the Errors ascertained on the 27th and 1st by $5\frac{1}{2}$ (the interval between them). This rate, multiplied by 4 (the interval between sights at Maziwi on 27th and Mesale on 31st), will give the quantity to be applied to the error of the chronometer in question at Maziwi on the 27th, to give the error on the 31st on mean time of Maziwi. The difference of this and the error of the same chronometer on mean time of Mesale, as ascertained on that day, will be the meridian distance by that chronometer.

Form for
Meridian
Distance.

In working out a meridian distance with several chronometers, it is convenient to use a form, as shown in the example of the above-cited instance (p. 336).

Rejection
of Results.

Here so many of the chronometers agree closely that the result by D seems doubtful, and looking at the comparisons taken every day with the standard, we see it has been going very irregularly; we therefore reject it. This should not be done without some independent evidence of this kind; and in a meridian distance, where the interval of time is great, or where all the chronometers have been going but fairly, as shown by the daily comparisons, it is very unsafe to reject chronometers solely because they vary from a small majority of the others.

The performances of chronometers used during a chronometric meridian distance may be shown graphically by the aid of a diagram on squared paper, the vertical ordinates representing the observed daily differences, and the horizontal ordinates the days during which the meridian distance was carried out.

Diagram
showing
Chrono-
meter
Rates.

A similar diagram will show the variation in the rates of the different chronometers as ascertained from time to time.

Supposing that we had had to stay at Mesale for a few days before returning to Maziwi, we can still find a fair travelling rate.

Another
Case of
Travel-
ling Rates.

The formula for this is as follows :

$$M = \beta - a - n \frac{(a^1 - a) - (\beta^1 - \beta)}{m + n},$$

where, the other letters representing the same values,

β^1 is the error at place B before leaving.

Here the travelling rate is,

$$\frac{(a' - a) - (\beta^1 - \beta)}{m + n}.$$

This can be exemplified thus. We obtain sights at Maziwi on 27th and at Mesale on 31st as before. Then sights again at Mesale on the 4th noon, and again at Maziwi on the 6th noon.

From the difference of the errors on the 27th and 6th we should deduct the difference of the errors on the 31st and 4th, and divide the remainder by 6, the sum of the intervals from the 27th to the 31st, and of the 4th to the 6th, or, in other words, the number of days actually travelling, which will give us the rate. We then proceed as before.

The travelling rate obtained in this instance will not be as good as in the former case, as the chronometers will have had two disturbances instead of one, and the rate they may settle into on starting the second time, after four days' quiet at anchor, may not be the same as before; but it will still be better than obtainable by any other method, and if circumstances of weather, sea, and temperature are nearly alike on

both journeys, and the intervals are not long, we shall probably get a very good result.

Travelling rates obtained thus should always, as already remarked, be used when the scale of the chart depends on the observations.

The method is very simple, and, used for this purpose, none of the considerations of temperature, etc., hereafter mentioned need be thought of, as the time is short.

We now come to the consideration of the rates to be used on other occasions, especially when voyages are long, and circumstances change much during them. Other Rates.

This is a very wide subject, and besides the fact that it has already been fully discussed by Captain Shadwell, in his masterly treatise before referred to, neither space nor the intention of these pages permits our going very far into it, and we shall content ourselves with giving general descriptions of cases, together with formulæ for them, with just sufficient reasons to allow of their being understood.

The whole question rests on: What makes chronometers vary?

The labours of many observers show us that the answer is:

1. Imperfection in the workmanship of the watch.
2. Changes of temperature.
3. The quality of the oil in the pivots, and its age (*i.e.*, the time elapsed since the watch was last cleaned).
4. Accidental shocks or vibrations imparted to the watch.

Why
Watches
change
their
Rates.

A supplementary question may be asked: Which of these is the most important? To which the general answer is that, according to circumstances, any one may be.

1. *Imperfection of Workmanship.*—For this manifestly there is nothing to be done. A badly made chronometer will go so erratically that we shall soon lose confidence in it, and reject it from all results, returning it as soon as we can. There are, however, but few chronometers that pass through the hands of the Royal Observatory which will come under this head, and doubtless many a chronometer has been classed in this category from ignorance of the circumstances of its compensation, and its resulting variation under change of temperature. If on a voyage *during which temperature is uniform* a chronometer placed with others, under the same conditions of protection

Imperfection.

from injury, etc., goes erratically, while the others maintain their rates pretty steadily, we may fairly conclude it to be inferior.

The uniformity of rate of a chronometer while on shore, or when the ship is at rest, cannot be taken as a conclusive test.

Change of
Tempera-
ture.

2. *Change of Temperature.*—A chronometer is supposed to be compensated in such a manner that at two temperatures, a varying number of degrees apart, the rates will be equal. At all other temperatures the rates will vary, reaching a maximum at about the mean temperature between the other two.

Let us, for brevity, call this temperature of maximum rate T .

Tempera-
ture of
Maximum
Rate.

If we then examine the rates of a chronometer, we should find a steady change of rate in one direction (nearly always in the direction of acceleration of gaining) from low temperatures to high, until we reach T , when the change of rate should vary in the opposite direction.

For every chronometer we shall have a different quantity for T , and different coefficients of change. Many chronometers are supposed to be compensated for $T = 60^\circ$, the mean temperature generally experienced over the globe; but it would seem that makers cannot command the point T ; anyway, many have T over 90° , so that for such a watch, in practice, the direction of change is invariable, which will result in a great accumulation of difference of rate when passing through hot and cold climates, and where the coefficient is large, in great absolute change of rate.

Different
Con-
clusions.

Different observers on the performances of chronometers have come to different conclusions on the subject of the law of change for a degree on all parts of the scale, which can only be accounted for by supposing that they have experimented on different classes of time-keepers.

Some have stated that they vary regularly, so as to have the same rate at an equal number of degrees above or below T , and have established the proportion of variation at the square of the difference of T , and the temperature required.

Other experiments have shown that the manner in which watches vary is not quite so regular as this, and that the coefficient of change is generally less at temperatures higher than T than at those below.

The fact is that there is no invariable law on the subject, a watch being too complicated a machine to admit of any practical conclusion, unbased on actual experiment with each individual watch. No Strict Law.

Experiment, however, does give results that can be practically used, and tables of rates can be formed from observation of the watch at different fixed temperatures, which, with some watches, will undoubtedly give better results than by using invariable rates. Practical Experiments.

Tables of this kind are now furnished to ships sailing from Liverpool, whose chronometers are rated at the Bidston Observatory, the director of which, Mr. Hartnup, has studied the question for many years. Liverpool Observatory.

The rate of the watch to be used for determining the position at sea is then taken day by day from the table, according to the temperature experienced, and added to the accumulated rate since departure, obtained in a similar manner.

It seems pretty well established that the coefficient of change for a degree remains the same, or nearly the same, for each individual watch, although the absolute rates of the watch (which depend upon many things) may vary.

The chronometers issued to H.M. ships have no such information sent with them, for this reason. Chronometers in H.M. Ships.

The time-keepers are carefully chosen from many sent to the Royal Observatory by different makers for trial, and only those whose compensation is such that they show very little change of rate at a great variation of temperature, or, in other words, whose compensation is as perfect as may be, are taken, the limit allowed being one and a quarter seconds of change of daily rate for 45° of temperature.

This reduces the variation of rate arising from change of temperature probable in a voyage to very small quantities, which would be lost in the variation arising from other causes, and it is not considered necessary under these circumstances to give data for allowing it.

3. *The Oil in the Pivots.*—With good oil the inequality arising from age shows itself in the shape of a gradual and tolerably uniform acceleration of rate, generally in the direction of gaining, with a new chronometer, and when the instrument is older and in all parts somewhat worn, in the contrary direction. Quality Oil.

It should be excessively small, and our opinion is that in the practical question of meridian distances the labour of ascertaining it is not repaid by the result. It is difficult to separate the error due to this from that originating in defective mechanism, and though formulæ have been elaborated for its detection, we do not propose to give them here.

Vibra-
tions and
Shocks.

4. *Vibrations and Shocks*.—However well chronometers may be stowed, the jars from seas striking the ship, and other like accidents, must be communicated more or less to the chronometers. The vibration of the screw is in some vessels sufficient to pass through all the soft cushions in which they may lie, and must have its effect, more especially from the fact that the watches themselves are hanging in the metal gimbals, in which there must be play sufficient to allow them to swing easily, and therefore enough to set up small shocks on any violent movements of the ship.*

* In connection with the observations of the transit of Venus of 1874, Lord Lindsay conveyed nearly sixty chronometers to Mauritius. These were kindly permitted by him to be used in assisting to determine the meridian distance between Mauritius and Rodriguez, when they were shipped on board H.M.S. *Shearwater*, under the author's command. As the results by these watches, both of the distance between Mauritius and Rodriguez, and Mauritius and Aden (between which latter places they were conveyed in the mail steamer), were remarkably good, and as the results by the *Shearwater's* chronometers which were admitted into the distance Mauritius to Rodriguez were not so satisfactory, a description of the manner in which Lord Lindsay's watches were stowed may not be out of place. We may add that the *Shearwater* had to beat up for eight days against a strong trade-wind on one occasion, and was a very lively ship.

The watches were taken out of their gimbals and placed in square boxes, which held nine of them each. The partitions of these boxes were thickly stuffed with very soft material (cotton-wool) covered with satin, so that each watch lay in a bed of down which was made exactly to fit it.

Each box was fitted with a metal framework after the fashion of gimbals, the outer pivots of which fitted into carefully turned sockets, in two upright columns of wood, which were firmly screwed to the deck. Each pair of uprights carried three boxes of watches.

The effect of this was that any slight shocks to the boxes caused by seas striking the ship, or by longitudinal slipping of the pivots, were entirely deadened before reaching the watches themselves.

This mode of stowing necessitated taking the watch up bodily in the hand to wind, which at first sight seems dangerous, and undoubtedly does present more opportunity for accident than the ordinary method; but, as far as the author is aware, none took place during the five or six months the watches were thus treated, and the admirable agreement of the results seems to show that this system was unusually successful.

Whether it could be adopted on board men-of-war, especially small ones, which are usually employed in surveying duties, is another matter, as it

In our opinion the variation of rate arising from these causes is, with the generality of Admiralty watches, the larger proportion of the total change.

The only notice that can be taken of variation of rate due to this, is to consider it as detracting from the general value of the meridian distance; and the nature of the passage, whether rough or smooth, should therefore be noted in the returns.

Magnetism is another disturbing cause to which irregularities of chronometers have been referred. As no trustworthy conclusion as to this has been arrived at, we do no more than mention it.

It follows as a matter of course, from the preceding observations, that not only will the rate of the chronometer as ascertained before leaving a port be different to that found on arrival at another port, but that the sea rate for the interval will probably be different from either of them.

We have now to consider the means at our disposal for approximating to the true rate under different circumstances.

The most satisfactory circumstance under which we can determine meridian distance (after those already described) is when, having left a port A, called at B (the position we want), where we have only stayed long enough to get Error, and eventually arrived at K without further stoppage, *the longitudes of A and K are sufficiently well known to take them as secondary meridians.* Interpolating between Places whose Longitude is established.

In this case, by applying the known difference of longitude between A and K to the observations at A, we find the Error on mean time at K at the epoch of starting from A. The difference between this and the Error ascertained on arrival at K, divided by the duration of the voyage, will give us a fair sea rate, which we shall assume to be uniform and invariable during the voyage. A simple application, then, of accumulated rate up to the time of observations at B, will give us the

certainly demands more space, both for the swinging of the box and to allow of free access for handling the watches.

It was very convenient for comparing, as one watch could be held to the ear while the eye took the time by the standard.

It seems probable, also, that the temperature would be more constant, from the fact of the watch being embedded in thick soft material. The lid of each box was also stuffed softly, and, when in place, pressed on the glass of each watch, excluding all air.

meridian distance from A to B, dependent upon A and K being in certain longitudes.

We can use the same means if we call at more places than one on the way between A and K, but each stoppage will probably detract from the value of the sea rate.

We are here using the sea rate only, and therefore shall take the date of the last observations at departure and first on arrival as the epochs for calculation. If we have obtained rate on departure and arrival, we shall gain valuable information about our chronometers, as we shall be able to see how far they have obeyed any theory as to gradual or uniform change of rate, according to the ordinary assumption that the sea rate is the mean of the two harbour rates.

The value of a meridian distance by this method will, as always, be influenced by the conditions of temperature, fair passage, etc., which must therefore be taken into consideration and recorded.

It will be remarked that by this method a large amount of time is saved, and opportunities otherwise wasted are utilised to their full extent. Instead of the necessity of waiting, certainly at A and K, and perhaps at B as well, for from five to eight days, a simple call of a few hours at each is sufficient to obtain an excellent result. Moreover, instead of involving the eccentricities of chronometers during the time in harbour at each end, we only include in the calculation the actual time while travelling at sea, and thereby save the irregularities of a good many extra days.

Shad-
well's
Treat-
ment. Captain Shadwell, in treating of this case, does not use an invariable sea rate pure and simple, but supposes that the rate of departure has gradually and uniformly changed into the sea rate, which he considers as the rate on the middle day of the passage only. He therefore applies for his determination of B from A an intermediate rate between the sea rate and rate of departure; but our experience does not lead us to think that this is an advantage, although by doing the same to the sea rate and rate of arrival, he gets a second meridian distance from B to K, and takes the mean of the two as his result. Our opinion is that, temperature being left out of the question, a better result is likely by using a uniform sea rate.

The algebraic formula for meridian distance by above method of uniform sea rate is—

Formula
for Inter-
polation.

$$M_1 = \lambda_1 - \lambda + \tau \frac{\lambda_2 - \lambda - M}{t}.$$

Where M is meridian distance between the terminal points A and K of the voyage.

M_1 is meridian distance between port of departure and a port B touched at on the voyage.

λ is error at A on leaving.

λ_1 is „ „ B on touching.

λ_2 is „ „ K on arriving.

t is interval between observations at A and K .

τ „ „ „ „ A and B .

In any case of a ship's calling at a place as an intermediate port on her voyage between two other places, it may be well to send home, beside the meridian distance obtained in the ordinary manner, the information which would enable the office, *if or when it possesses the true difference of longitude between the terminal ports*, to calculate the difference of longitude of the intermediate place by the last formula.

This necessary information will be—

$\lambda, \lambda_1, \lambda_2, t$, and τ .

In transmitting this information, we could, for the facilitation of computation afterwards, give only the mean of the Errors of all the chronometers, instead of the individual error of each, or, in other words, assume an imaginary watch, the result of which will give the same meridian distance at the mean of the meridian distances by each chronometer; but the adoption of this method will of course preclude any estimation of the value of the distance by the concurrence of individual results, and should be therefore only adopted when we have reason to believe from intercomparisons during the voyage that the watches have been going well together.*

Meaning
the Errors

There is another adaptation of the methods of sea rates as obtained by Error at two places whose longitude is known, which is often useful.

* This method of interpolation is not recognised as being as valuable as I believe it to be, and the remarks on it must be taken as my private opinion only.—W. J. L. W.

Adapta-
tion of
Fore-
going.

If we obtain Error before leaving A, and after some days call at B, whose difference of longitude from A is known, and there obtain Error again, we get a very good sea rate for the subsequent part of our voyage, which we can utilise to determine the position of C, any third place at which we may hereafter soon call, with a probable better result than by means of harbour rates. If absolute altitudes are used, they must, of course, be both either a.m. or p.m.

This method is especially useful for navigational purposes. Suppose a ship to leave Portsmouth and to call at Gibraltar for a few hours only. Error can be obtained, and by means of the known difference of longitude a sea rate deduced, which will give a better landfall for Malta than the harbour rates at Portsmouth.

Mean
Harbour
Rates.

When our voyage is simply from one port to another, and we wish to find the meridian distance between them, we must depend mainly upon the harbour rates ascertained before departure and on arrival.

The ordinary and rougher method is to assume that the rate has changed uniformly from the rate of departure to that of arrival, and that therefore the mean of the two rates will represent the mean rate during the passage. We believe that (owing to the many causes of variation impossible to formulate) in most cases, and especially where temperature has been, in the chronometer room, fairly uniform, this method will give as good a result as any other; but where temperature has changed much, the result of long meridian distances with such rates will have but very little value, and that a correction for temperature will much improve the result, if we can apply it.

French naval officers have done much in working out this question, and Captain Shadwell gives their separate theories and formulæ. To our mind the method of M. Mouchez is the most practical; and not undertaking to enter into the question of acceleration, nor depending on observations on the watches while in the observatory, it is more adapted to actual work.

Mouchez's
Rule.

Mouchez proceeds on the assumption, which is near enough to truth for the method, that the rate varies uniformly with the temperature; but in working on this hypothesis, we must not forget that for each chronometer there is a point of temperature at which the rate is at a maximum, and that the sign of the variation will change as we pass it.

He ascertains by observations for rate at different temperatures, undertaken by the officers when the chronometers are embarked, the coefficient for temperature by simply dividing the difference of rate by the difference of mean temperatures during the intervals of rating.

This coefficient of change will remain constant for some period, though the actual rates themselves will alter from other causes ; nevertheless, the more these observations are multiplied the better, and the latest determinations will be used in practice.

In determining the sea rate for a meridian distance, he applies to the rate of departure the change of rate due to the difference between the mean temperature during rating and the mean temperature during the passage, which gives one value for the sea rate. Doing the same for the rates of arrival, he gets another value for sea rate. The mean of these two he takes as the final mean sea rate to be used. One weak point here is that the mean temperature, T , of the compensation will not be indicated, unless many observations at different temperatures are made. It will therefore add considerably to the value of this method if we can find T .

It will be more satisfactory if we can get this from the Lieussou's
Observatory ; but a formula for ascertaining it is given by
Captain Shadwell, from M. Lieussou, which we here append,
but we apprehend that in practice not many opportunities will
present themselves for making use of it. It depends on the
results of four observations for rates, at equal intervals of
time, and at different temperatures, a difficult condition to
satisfy except with artificial aid for the temperature. M. Lieus-
sou remarks, " that four rates and four temperatures, observed
at intervals of ten days, determine the constants for each
chronometer with a precision sufficiently remarkable." With
the other constants we do not propose to deal, but solely to give
his formula for ascertaining T , which is—

$$T = \frac{1}{2} \cdot \frac{(m_1 - 2m_2 + m_3) (t_2^2 - 2t_3^2 + t_4^2) - (m_2 - 2m_3 + m_4) (t_1^2 - 2t_2^2 + t_3^2)}{(m_1 - 2m_2 + m_3) (t_2 - 2t_3 + t_4) - (m_2 - 2m_3 + m_4) (t_1 - 2t_2 + t_3)}.$$

Here T = mean temperature of compensation required.

$m_1 m_2 m_3 m_4$ are the four observed rates corresponding to
 $t_1 t_2 t_3 t_4$, the four temperatures.

The intervals between the sets of observations for rates should be between ten and thirty days.

**Hartnup's
Formulæ.** Mr. Hartnup's formulæ are somewhat different, and do not give exactly the same results with the same data.

He observes the rate at three different temperatures not less than 15° apart, but there must be an equal number of degrees between them.

The same remark already made as to M. Lieussou's method will apply here—viz., that in service afloat it will be difficult to fulfil the conditions of observation. His formulæ are as follows :

$$C = \frac{2(d-d_1)}{p^2}$$

$$T = t_1 + \frac{d+d_1}{2 C p}$$

$$R = r_1 - (T - t_1) \frac{d+d_1}{2 p}.$$

Where C is the coefficient of change of rate,

T is temperature of maximum rate,

R is rate at that temperature,

t_1 is the middle temperature,

r_1 is observed rate at temperature t_1 ,

d is difference of rate between that at lowest temperature and t_1 ,

d_1 is difference of rate between t_1 and that at highest temperature,

p is difference between highest and lowest temperatures observed at.

Then to find the rate in any required temperature.

If N = any number of degrees from T,

Rate at $T \pm N = R + C N^2$.

**Epochs of
Calcula-
tion.**

In using the rates of departure and arrival in calculating a meridian distance, the Errors at the last observation at departure and first at arrival should not be taken for the epochs of calculation, but the mean of the two should be used for the purpose, for it is at the mean date between the two observations for each rate at which the latter is actually fixed. Thus, if we observe at a place A on the 2nd and 8th, and again

on arrival at B on the 20th and 27th, we should take the mean of the two Errors on 2nd and 8th, and call it the Error at A on the 5th, and similarly at B on the 23rd·5, and use the interval between these two epochs for the multiplication of the mean rate.

The formula given by Tiarks, and generally adopted, for calculating the meridian distance between two places by rates at departure and arrival, without any consideration of temperature, is—

$$M = \lambda^1 - \left\{ \lambda + t \left(a + \frac{b}{2} \right) \right\}$$

Where M is meridian distance required,

λ the Error at mean epoch of departure,

λ^1 „ „ „ „ arrival,

t the interval between the two epochs,

a the rate at departure,

b the difference between rate at departure and arrival.

In calculating t , the difference of time, due to difference of longitude between the two places, must not be forgotten ; but, being reduced to the decimal of a day, must be added or subtracted to the interval between the epochs, according as we have moved westward or eastward.

Thus, if our mean epoch at A is at noon on the 20th, and at another place, B, 30° to the westward, at noon on the 30th, the interval of time for accumulated rate will not be ten days, but ten plus the difference of longitude of the two places, or 10·08 days ; for the sun, having completed the ten days by returning to the meridian of A, will take yet another ·08 of a day to be on the meridian of B.

Similarly, in calculating sea rate from observations at different places where longitude is known, we must allow for this difference of time.

Thus, having taken sights at A at noon on the 2nd, and at B, 20° eastward, on the 11th at noon, the interval with which to divide the difference of Error at A (corrected for difference of longitude) and Error at B, to ascertain the daily rate, will be 8·94 days, as the sun will be on the meridian at B ·06 of a day earlier than at A.

Tiarks' Formula with Temperature Correction.

The same formula, when intending to correct for temperature, will stand thus :

$$M = \lambda^1 - \left\{ \lambda + t \left(a + \frac{b}{2} \right) + t \left(\frac{\theta + \theta^1}{2} - \theta_2 \right) y \right\}$$

Where, the other letters signifying as before,

θ is mean temperature during rating at departure,

θ^1 " " " " arrival,

θ_2 " " during the passage,

y is the coefficient for temperature found from previous observations.

Algebraic Signs.

In all cases of correction for temperature the algebraic sign of y must be remembered ; that is, it must be applied according to the observed effect in altering the rate.

The same remark applies to the algebraic signs of all quantities in the formulæ.

Thus in the formula

$$M = \lambda^1 - \left\{ \lambda + t \left(a + \frac{b}{2} \right) \right\}$$

the signs which are here given, as throughout, for chronometers slow of mean time and losing rates, will only be true under those circumstances with increasing losing rates and when moving eastward. A consideration of the facts, and obvious effects of the corrections, is perhaps the best course to take to determine these signs.

A meridian distance, founded only upon rates obtained at one end, without any further correction, cannot be considered as of any value whatever, unless the voyage be very short.

Tiarks' Formula for Interpolation with Harbour Rates.

When using the combination of harbour rates at each end of a voyage, A to K, to determine the position of some intermediate place, B, we must, to be consistent, remember that we are assuming that the rate has gradually and uniformly changed from that of departure to that of arrival, and that the rate to be used for a portion of the voyage will not therefore be the same as that for the whole of it.

Tiarks, interpreted by Captain Shadwell, gives us the following formula :

$$M_1 = \lambda_1 - \left\{ \lambda + \left(\tau a + \frac{\tau^2}{2t} b \right) \right\}$$

Where M_1 is meridian distance A to B,

λ_1 is Error at B,

λ is Error at mean epoch at A,

a is rate of departure,

b is difference of rates of departure and arrival,

t is interval between mean epochs of rating at A and K,

τ is interval between mean epoch at A and observations at B.

It is to this case that our observations on p. 343 refer, to the effect that the data for calculating the position of B, as interpolated between A and K, may be also transmitted home.

A very good way of measuring meridian distance for the scale of a chart, when the actual distance between the stations is not too far, is by rockets. Parties landed at either end of the base whose difference of longitude is to be measured ascertain the Error of their pocket-chronometers. The ship, midway between the two, fires rockets vertically, and the bursting of these, an instantaneous phenomenon, is noted by the watches at either end. Use of
Rockets.

An ordinary service signal rocket can be depended on to mount 1,200 feet, and should reach 1,600. The bursting, if it occurs, as it should, at the highest point, will therefore be visible nearly 40 miles on either side, which will permit a base of 75 miles to be measured under very favourable circumstances of dark night and clear atmosphere, when the stations are east and west of one another.

Rockets will not often, however, be seen this full distance; the balls of fire, released on bursting, are scarcely bright enough; and supposing the observers to be at the sea-level, the burst of the rockets will only just be above the horizon, in which position atmospheric disturbances are greatest, and may disperse the rays of light before they can reach the observer. Ascending a hill, therefore, will greatly assist clear vision, and the use of a pair of field-glasses will do wonders. Twenty-five miles, on either side, should be measured in this way without any great difficulty.

It is important, in transmitting to the Hydrographic Office the results of a Meridian Distance, that sufficient information Transmit-
ting
Results.

is given to enable it to be valued and compared with others between the same places. The form appended is that now employed.

| No. 24. | | | | | | | | | |
|--|--------------------|------------------|--------------------|-------|----------------------|-------|------------|---|--|
| RETURN OF MERIDIAN DISTANCE, H.M.S. , 1874. | | | | | | | | | |
| Captain. | | | | | | | | | |
| From <i>Seychelles</i> . | | | | | To <i>Zanzibar</i> . | | | | |
| Observation spot, Seychelles—Hondouls Jetty, Mahé .. | | | | | Lat. 4 37 15 S. | | | | |
| " " Zanzibar—Old British Consulate Garden .. | | | | | " 6 09 45 S. | | | | |
| Rates used—Mean rates of departure and arrival. | | | | | | | | | |
| Error at Seychelles on Jan. 13th by Eq. Alts. ☉ | | | | | | | | | |
| " " " " " 18th " " " | | | | | | | | | |
| " " Zanzibar " Feb. 1st " " " | | | | | | | | | |
| " " " " " 9th " " " | | | | | | | | | |
| Duration of passage, Jan. 18th, 6 P.M. to Jan. 30th, 4 P.M. | | | | | | | | | |
| Epochs for calculating accumulated rate, Jan. 15½, Feb. 5th = 20·545 days. | | | | | | | | | |
| Chronos. | By Observer I. | | | Date. | Mean Temp. | Date. | Mean Temp. | Remarks. | |
| | Rate of Departure. | Rate of Arrival. | Meridian Distance. | | | | | | |
| | s. | s. | h. m. s. | Jan. | ° | | ° | | |
| A | -1·280 | -1·544 | 1 05 04·54 | 13 | 80 | 29 | 80 | Sea smooth during passage. | |
| B | -1·158 | -1·211 | 05·40 | 14 | 81 | 30 | 81 | Steaming 7 days. | |
| C | -1·888 | -2·849 | 04 59·48 | 15 | 82 | 31 | 79 | Sailing 5 days. | |
| D | +2·212 | +2·026 | 05 05·03 | 16 | 80 | Feb | | Head generally West. | |
| E | -2·068 | -2·361 | 04·96 | 17 | 79 | 1 | 80 | C. H. & F. going irregularly by intercomparisons. | |
| F | -4·908 | -5·267 | 02·80 | 18 | 81 | 2 | 78 | | |
| G | +4·832 | +4·864 | 03·40 | 19 | 78 | 3 | 80 | | |
| H | -2·668 | -5·261 | None calculated. | 20 | 77 | 4 | 81 | | |
| J | | | | 21 | 78 | 5 | 80 | | |
| K | | | | 22 | 76 | 6 | 79 | | |
| | | | | 23 | 77 | 7 | 81 | | |
| | | | | 24 | 78 | 8 | 80 | | |
| | | | | 25 | 75 | 9 | 81 | | |
| | | | | 26 | 77 | | | | |
| | | | | 27 | 79 | | | | |
| | | | | 28 | 80 | | | | |

| | | | | | |
|---|----|----|----|-----------------|-----------------|
| Chronometers rejected C. F. & H. | | | | Number used, 5. | |
| Mean Meridian Distance by Observer 1 | h. | m. | s. | .. | 1 05 04·59 |
| " " " " " " | " | " | " | 2 | 05·43 |
| " " " " " " | " | " | " | 3 | |
| " " " " " " | " | " | " | 4 | |
| Final Mean Meridian distance by arithmetical mean | h. | m. | s. | | 1 05 05·0 W. |
| " " " " " " | " | " | " | " | values assigned |

CHAPTER XV

TRUE BEARING

By Theodolite—By Sextant—Variation.

In nearly all descriptions of surveys true bearings will be used.

The most correct method, from a shore station, is to use the theodolite, which will alone give a very good result for azimuth ; but it is better to get the altitudes with a sextant and artificial horizon, when two observers are available. By Theo-
dolite and
Sextant.

The theodolite in this case is only used for taking the horizontal angle between the sun and the zero.

There are three principal methods in use for obtaining the azimuth. By observations at equal altitude a.m. and p.m., by observations a.m. and p.m. at nearly the same altitudes, or by single observations. Three
Methods.

The former is theoretically the more correct, as many errors are eliminated ; the second is nearly as good ; but our experience is that with single observations taken with the sun near the prime vertical, with instruments in good order, the result is quite as near the truth as is generally requisite in marine surveys. When an extensive piece of coast is being surveyed, we shall, as before stated, depend partly upon the astronomical positions for the scale and bearing of the chart ; but, nevertheless, accuracy in obtaining the original bearing for working is necessary. Single
Altitude
generally
Sufficient.

A very important point is careful levelling, which should be done with the telescope pointing in the direction of the sun, and the accuracy of the movement of the telescope in a vertical plane should be tested, as no method will entirely eliminate error due to want of such accuracy.

In the first method, the sun will be observed at an even

stated altitude, and the sextant will be set beforehand, the observer using it giving the "Stop" to the theodolite observer.

In the others, the theodolite observer generally calls the "Stop," and the sextant observer takes whatever altitude it happens to be.

At very low altitudes, it is difficult to observe the sun in the artificial horizon, and in such cases the altitude is read off on the vertical arc of the theodolite.

Elimination of Errors of Verticality of Azimuthal Axis and of Collimation.

It is desirable to make the observations at a low altitude, in order to avoid the effect of any error in the verticality of the azimuthal axis arising from inaccurate levelling or instrumental defect, which increases with the altitude. Errors arising from this cause are eliminated by reversing the telescope in its Y's for half the sets of observations. Errors due to the adjustment of collimation not being perfect may be eliminated by taking half the sets of observations with the telescope turned half round in the Y's with the bubble upwards, and the remainder of the observations with the telescope in its ordinary position.

Changing Degree of Zero.

To arrive at a satisfactory result in either case, it is necessary to take several sets, with a different degree of the arc pointed at the zero in each, so as to eliminate the errors of the horizontal arc of the instrument.

Correcting to Sun's Centre.

As it is the bearing of the sun's centre which we obtain by working out the azimuth, the aim of the theodolite observations is to get the horizontal angle between that centre and our zero; but it is manifest that we cannot trust our eye to place the cross-wires of the telescope exactly on the centre of the sun, nor can we place the wires truly vertical and horizontal.

It we could do the latter, we could arrive at the angles to the centre by merely observing the sun in one quadrant, and applying the semi-diameter \times sec. alt.; but we must not trust this, if we want fair accuracy.

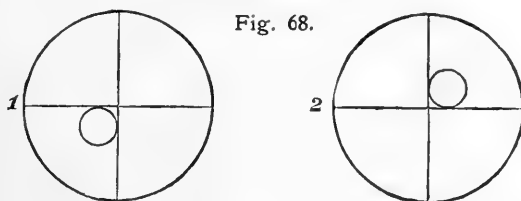
Method of Bearing by Equal Altitudes.

In equal altitude observations, the method is to fix on an altitude for both sextant and theodolite, and set the vertical arc of the latter at it. In the forenoon, bring the sun so that it is in the lower half of the field, and approaching the vertical wire. The theodolite observer then keeps the limb of the sun

in contact with the vertical wire, and below the horizontal one. If the theodolite is truly levelled, he will not need to touch his vertical tangent screw, but, if necessary, he must do so, to keep the upper limb of the sun as nearly touching the horizontal wire as he can. When the upper limbs of the sun in the artificial horizon are in contact, the observer calls "Stop," and the motion of both tangent screws of the theodolite ceases. The horizontal arc is then read.

Then, without moving the theodolite in altitude, the other limb of the sun is brought on the other side of the vertical wire, and the reading made when the artificial horizon observer gives "Stop," on the lower limbs of the sun coming in contact.

The sun will thus have passed between opposite quadrants of the cross-wires, as in the diagram Fig. 68.



Similar observations are made at the same altitude in the afternoon, the lower limb coming first. Each set will thus consist of two observations a.m. and two p.m. In this method the time must be taken exactly, which is a drawback, as it either requires three persons or that one should take time as well as his observation. There is, however, no necessity to know the *local* time very exactly; all we want is the true elapsed time.

To work out the equal altitude observation, the means of the times, and of the horizontal angles of a.m. and p.m. respectively, are taken.

If the sun had no motion in declination, the mean of a.m. and p.m. horizontal angle would be the angle on the horizontal arc corresponding to the true meridian, or, in other words, the bearing of the zero; but as this is not so, we must work a correction similar to the equation of equal altitudes when obtaining time, to be applied to this mean of the angles.

Calculating
Bearing
by Equal
Altitudes.

The formula for this is—

$$\text{Correction} = \frac{c}{2} \operatorname{Cosec} \frac{\text{time elapsed}}{2} \operatorname{Sec} \text{ lat.},$$

where $\frac{c}{2}$ is half the change of declination in elapsed time.

This correction is additive to the angle when the sun is moving from the nearest pole, and subtractive when moving towards it.

Let us take the following example :

| AT NUT \triangle . \oplus PAGODA \triangle 360°. | | | | | |
|--|--------|----|----|-------------|-----------|
| Alt. | Times. | | | Hor. Angle. | |
| 39° | A.M. | h. | m. | s. | ° ' " |
| | | 8 | 20 | 14 | 15 05 30 |
| | " | 8 | 23 | 22 | 15 09 15 |
| | P.M. | 4 | 02 | 13 | 193 24 30 |
| | " | 4 | 05 | 20 | 193 28 45 |
| | | | | | Z. K. |

Lat. 30° N. Declination corrected to Greenwich time of a.m. observation 18° 14' N. Sun moving north.

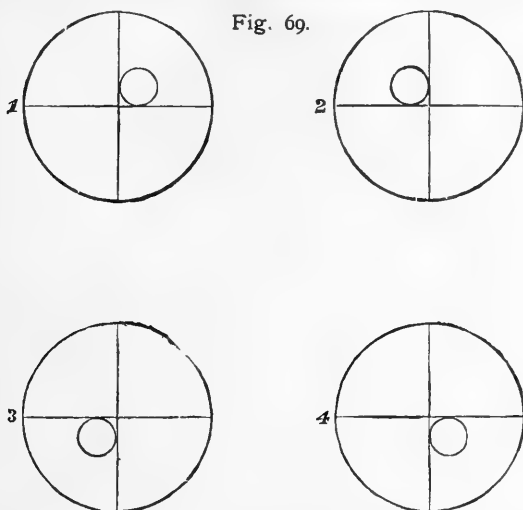
| | | | | | | | | | |
|----------------------------|----|--------|----|-------|--|-----|-----|-------|----------|
| Mean A.M. Times | .. | h. | m. | s. | A.M. angle | .. | ° | ' | " |
| P.M. | .. | 16 | 03 | 46 | P.M. | .. | 15 | 07 | 22 |
| Elapsed Time | .. | 7 | 41 | 58 | | | 193 | 26 | 37 |
| $\frac{1}{2}$ Elapsed Time | .. | 3 | 50 | 59 | Mean angle | .. | 208 | 33 | 59 |
| Var of dec. in 1 hour | | 37'' | 44 | | | | 104 | 16 | 59 |
| | | 3'85 | | | | | | | |
| | | 18720 | | | $\frac{c}{2}$ | .. | 2 | 15866 | |
| | | 29952 | | | $\operatorname{Cosec} \frac{E. T.}{2}$ | | | | 07279 |
| | | 11232 | | | $\operatorname{Sec} \text{ lat.}$ | | | | 06249 |
| | | | | | | | 2 | 29394 | .. 196'' |
| | | | | | $\frac{c}{2} = 144$ | 144 | | | |
| | | | | | Cor. | = | 3' | 17'' | |
| | | | | | Mean angle | .. | 104 | 16 | 59 |
| | | | | | | | - | 3 | 17 |
| Angle of South Point | .. | 104 | 13 | 42 | | | | | |
| Or bearing of Pagoda | .. | S. 104 | 13 | 42 E. | | | | | |

A number of similar sets, taken with different degrees as zero, will give a very correct result, and though all instrumental errors will not be eliminated, the majority of them will disappear.

In "single" observations, each set will consist of four Method by
Single
Altitudes contacts, in each of which the sun will be tangential to the vertical wire in a different quadrant of the field.

The mean of these will then be the angle to the sun's centre, corresponding to the mean of the four altitudes.

When the altitude is being taken by a sextant, it will only be necessary for the theodolite observer to be *very* exact with the contact of the side-limb of the sun; but his upper or lower limb, as the case may be, should be as nearly touching the horizontal wire as possible, to ensure the elimination of the wire error.



It is quite immaterial in which quadrant the observer commences; but whatever plan he adopts, he should always observe in the same manner, as it prevents confusion and mistakes. The sun will appear as in the diagrams in Fig. 69.

When taking the observation with the theodolite alone, it will of course be necessary to see that both the horizontal and vertical wires are truly tangential to the sun's limbs.

Six sets should give a very good bearing; but if the theodolite is a very small one, or is known to be badly graduated, more may be necessary.

Half the altitudes in the artificial horizon may be taken with upper limb and half with the lower; but this is not important, as if the observation be made when the sun is near the

prime vertical, a small error in the altitude will but slightly affect the azimuth.

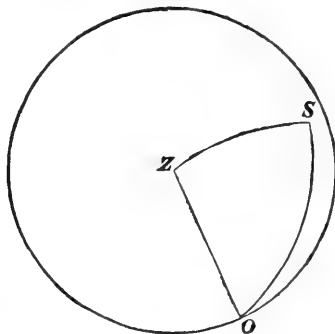
The azimuth of the sun having been obtained by the ordinary rule of nautical astronomy, the true bearing of the object is found by applying the mean of the theodolite angles of that set.

Method by
Sextant.

In finding a true bearing with sextant only, it will be more accurate if two observers are employed—one to take the altitude, the other to measure the angular distance at the same instant.

If only one observer is available, he must take altitude and angular distance alternately, taking care to end with the same

Fig. 70.



observation as that with which he begins, so that the mean of each kind will correspond as nearly as may be in time. Thus, if he begins with altitude, he must also end with altitude.

This method should not, however, ever be used when a theodolite is available, and is only adopted for true bearings from the ship, in an irregular survey.

It is essential to observe both the nearer and farther limbs of the sun and accept the mean; observations of one limb are liable to considerable error.

Calcula-
tion of
Horizon-
tal Angle.

In this instance we have to calculate the horizontal angle, which with the theodolite we obtained directly.

The object should be so chosen that the line joining it with the sun should not make a larger angle with the horizon than 20° , and the less the better, as any inaccuracies of observation will not then be much increased when the horizontal angle is deduced. If we take an object 90°

ERRATA

Page 356, line 10 from bottom :

Replace "essential" by "desirable."

Page 356, line 5 from bottom to line 2 on page 357.—Erase, and substitute:

The object should be chosen as near 90° as possible from the sun, and the altitude of the latter should not be more than 20° and near the prime vertical.

Page 357, line 4 from top :

Replace paragraph "First . . . mountain top" by "First, when the object whose bearing is desired has no appreciable altitude, and secondly, when it has a sensible altitude, as a mountain top, or has a depression due to the height of eye of the observer, as an object on the sea horizon."

Page 357, lines 9 and 10 from top :

Delete "on the horizon."

In margin amend to "**Object with zero altitude.**"

Page 357, Example, line 1 (in brackets):

Amend "on horizon" to "with zero altitude."

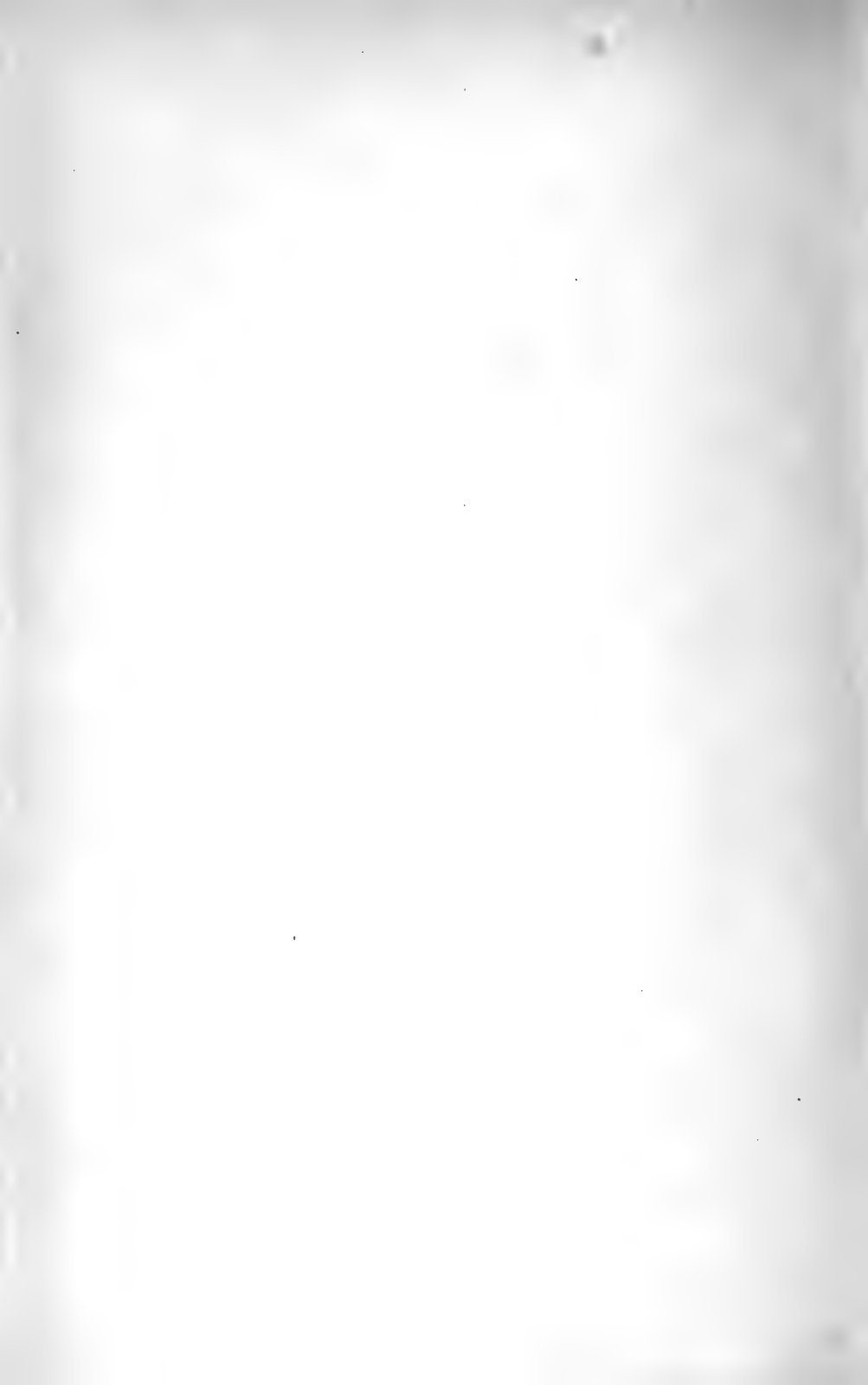
Page 357, Example, line 5:

After "Pine Δ " delete "on horizon."

Page 360, use of Polaris, line 2.—Add:

the most favourable conditions being a low latitude and object bearing nearly east or west.

Page 360, line 6.—Erase the word "large," and substitute "as nearly as possible 90° ."



or more from the sun, these conditions will be fulfilled, the sun being, of course, comparatively low, and near the prime vertical.

There are two separate cases :—

Two
Cases.

First, when the object whose bearing is desired is on the horizon ; and secondly, when it has a sensible altitude, as a mountain top.

In the first we have to solve a quadrantal triangle as shown in Fig. 70.

In this Fig. Z is zenith, S is sun, and O the object on the horizon. Object on
Horizon.

We have $ZO = 90^\circ$. ZS the apparent zenith distance, and OS the observed angular distance, to find OZS , the horizontal angle required, or

$$\text{Cos horiz. angle} = \text{Cos ang. dist.} \times \text{Sec app. alt.}$$

with the proper signs applied to the angles.

Example.

(Object on horizon, two observers with sextants and sea horizon.)

On June 1, 1881, at Ship IV. 7 hours 24 minutes A.M. mean time of place, observed altitude of \odot $30^\circ 13' 50''$, mean angular distance of \odot to Pine \triangle on horizon $84^\circ 26' 20''$, object right of \odot Lat. $40^\circ 26' 15''$ N., Long. $28^\circ 00'$ E. Index Errors $-35''$ and $0''$. H.E. 20 feet.

| | | | |
|------------------|---|------------------------|---|
| M. Time pl. .. | ^{h.} 7 ^{m.} 24 | \odot 's dec. 1st .. | [°] 22 ['] 6 ["] 59.6 N. |
| Long. in time .. | 1 52 | | 2 08 |
| Gr. Date 31st .. | 17.32 | Corrected dec. .. | 22 04 51.6 |
| „ 1st .. | -6.28 | Pol. dis. .. | 67 55 08 |
| Obd. alt. .. | [°] 30 ['] 13 ["] 50 | Var. .. | ["] 20.0 |
| Index error .. | -35 | | 6.4 |
| | <hr/> | | <hr/> |
| H. E. .. | [°] 30 ['] 13 ["] 15 | | 800 |
| | 4 15 | | 1200 |
| | <hr/> | | <hr/> |
| S. D. .. | [°] 30 ['] 09 ["] 00 | | 128'' 00 |
| | +15 48 | | <hr/> |
| App. alt. .. | [°] 30 ['] 24 ["] 48 | Obs. Ang. dist. .. | [°] 84 ['] 26 ["] 20 |
| Ref. .. | -1 31 | S. D. .. | 15 48 |
| | <hr/> | | <hr/> |
| T. alt. .. | [°] 30 ['] 23 ["] 17 | True Ang. dist. .. | [°] 84 ['] 42 ["] 08 |
| | <hr/> | | <hr/> |

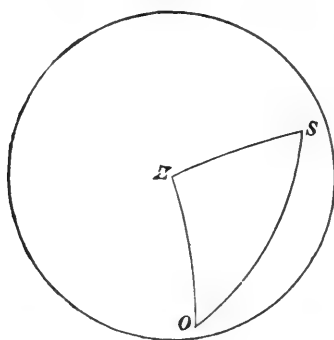
| | | | | | | | | | | | |
|----------------------|----|---|----|---|--------|----|----|--------------------|----|-------------------|-----------------|
| Lat. | .. | ° | 40 | ' | 26 | " | 15 | Sec. | .. | ° | 118550 |
| Alt. | .. | ° | 30 | ' | 23 | " | 17 | Sec. | .. | ° | 064181 |
| | | | | | | | | | | | |
| | | | 10 | | 02 | | 58 | | | | |
| P.D. | .. | | 67 | | 55 | | 08 | | | | |
| | | | | | | | | | | | |
| | | | 77 | | 58 | | 06 | $\frac{1}{2}$ Hav. | .. | | 4.798724 |
| | | | | | | | | | | | |
| | | | 57 | | 52 | | 10 | $\frac{1}{2}$ Hav. | .. | | 4.684677 |
| | | | | | | | | | | | |
| | | | | | | | | | | | <u>9.666132</u> |
| | | | | | | | | | | | |
| Azimuth of sun | .. | | | | | | | .. | | N. 85° 49' 25" E. | |
| Cos. true Ang. dist. | .. | | 8 | | 965353 | | | | | | |
| Sec. app. alt. | .. | | | | 064294 | | | | | | |
| | | | | | | | | | | | |
| Cos. Hor. Ang. | .. | | 9 | | 029647 | .. | 83 | ° | 51 | " | 14 Hor. ang. |
| | | | | | | | | | | | |
| Azimuth ☉ | .. | | | | | | N. | 85 | ° | 49 | " 25 E. |
| Hor. angle | .. | | | | | | | 83 | ° | 51 | " 14 |
| | | | | | | | | | | | |
| | | | | | | | N. | 169 | ° | 40 | " 39 E. |
| | | | | | | | | | | | |
| True bearing Pine △ | .. | | | | | | S. | 10 | ° | 19 | " 21 E. |

Object
elevated.

In the second case, we have a spherical triangle with three sides known, as in Fig. 71.

Here, Fig. 71, we have Z O, the zenith distance of the object

Fig. 71.



Z S, and O S, as before, the apparent zenith distance of sun and angular distance; to find O Z S, the horizontal angle

required, which can be done by any of the applications of the formula

cos O Z S = (cos OS - cos Z S . cos Z O) / (sin Z S . sin ZO)

Example.

(One observer with sextant, sea horizon, alternate observations, object elevated.)

At S. Ann's Δ, October 5, 1881, Lat. 5° 10' S., Long. 57° 14' E., the following observations were taken for true bearing of Snow Peak. Height of eye 10 feet, object right of ☉. M.T. place 8^h·00^m I.E. - 50".

| Alt. ☉ | | | Ang. Distance of Snow Peak ☉ | | | Elevation of Snow Peak. | | |
|--------|----|----|------------------------------|----|----|-------------------------|-----|-------|
| ° | ' | " | ° | ' | " | ° | ' | " |
| 30 | 06 | 10 | 94 | 14 | 40 | On arc .. | 1 | 26 10 |
| | 13 | 00 | | 16 | 10 | Off " .. | 1 | 24 30 |
| | 20 | 15 | | 18 | 30 | | | |
| | 28 | 00 | | 20 | 00 | | 1 | 25 20 |
| | 36 | 10 | | 21 | 20 | H. E. .. | - 3 | 07 |
| | 42 | 50 | | | | | 1 | 22 13 |

| | | | | | | | | | | | |
|----------------------|-------|-----|------|-------------------------|----|----|-------|--------|-----|----|----|
| 4th M.T. pl. .. | h. m. | 20 | 00 | ☉ dec. .. | 4 | 53 | 44 S. | Var .. | 57 | 7 | |
| Long. .. | | 3 | 49 | | | 7 | 30 | | | 7 | 8 |
| Gr. date 4th | | 16 | 11 | Corr. dec. .. | 4 | 46 | 14 | | | 46 | 16 |
| " " 5th | | - 7 | 49 | P. D. .. | 85 | 13 | 46 | | | 40 | 39 |
| | | | | | | | | 6) | 450 | 06 | |
| | | | | | | | | | | 7 | 30 |
| Mean. obs. alt. ☉ .. | ° | ' | " | Mean obs. ang. dist. .. | ° | ' | " | | | | |
| I. E. | 30 | 24 | 24 | I. E. | 94 | 18 | 08 | | | | |
| | | | - 50 | | | | - 50 | | | | |
| | | | 30 | | | | 94 | | | 17 | 18 |
| H. E. | | | - 3 | S. D. | | | + 16 | | | 02 | |
| | | | 30 | | | | 94 | | | 33 | 20 |
| S. D. | | | + 16 | Corr. ang. dist. .. | | | | | | | |
| | | | 30 | | | | | | | | |
| App. alt. | | | 30 | | | | | | | | |
| Ref. | | | - 1 | | | | | | | | |
| | | | 30 | | | | | | | | |
| Tr. alt. | | | 30 | | | | | | | | |

| | | | | | |
|---------|----|----|----|---------|--------|
| Lat. .. | 5 | 10 | 00 | Sec. .. | 001768 |
| Alt. .. | 30 | 34 | 59 | Sec. .. | 065052 |

| | | | |
|---------|----|----|----|
| | 25 | 24 | 59 |
| P.D. .. | 85 | 13 | 46 |

| | | | | | |
|--|-----|----|----|-----------------------|---------|
| | 110 | 38 | 45 | $\frac{1}{2}$ Hav. .. | 4915068 |
|--|-----|----|----|-----------------------|---------|

| | | | | | |
|--|----|----|----|-----------------------|---------|
| | 59 | 48 | 47 | $\frac{1}{2}$ Hav. .. | 4697741 |
|--|----|----|----|-----------------------|---------|

9.679629 S. 87° 30' 11" E. Azimuth ☉

| | | | | | |
|----------------|----|----|----|---------|--------|
| App. alt. ☉ .. | 30 | 36 | 29 | Sec. .. | 065163 |
| Alt. Snow Peak | 1 | 22 | 13 | Sec. .. | 000124 |

| | | | |
|---------------|----|----|----|
| | 29 | 14 | 16 |
| Ang. dist. .. | 94 | 33 | 20 |

| | | | | | |
|--|-----|----|----|-----------------------|---------|
| | 123 | 47 | 36 | $\frac{1}{2}$ Hav. .. | 4945517 |
|--|-----|----|----|-----------------------|---------|

| | | | | | |
|--|----|----|----|-----------------------|---------|
| | 65 | 19 | 04 | $\frac{1}{2}$ Hav. .. | 4732101 |
|--|----|----|----|-----------------------|---------|

9.742905

| | | | | |
|-----------------------|----|----|----|----|
| Horizontal angle. . . | .. | 96 | 06 | 40 |
|-----------------------|----|----|----|----|

| | | | | |
|-----------------|----|-------|----|-------|
| Azimuth ☉ | .. | S. 87 | 30 | 11 E. |
|-----------------|----|-------|----|-------|

| | | | |
|------------------------|------|----|-------|
| True bearing Snow Peak | S. 8 | 36 | 29 W. |
|------------------------|------|----|-------|

Use of
Polaris.

The Pole Star may be used in the Northern Hemisphere to obtain true bearings at night.

Circumstances under which this is useful are related at p. 213, which see.

The Greenwich time must be known, and the angle between the Pole Star and object whose bearing is required must be large.

Measure the angle and take the time.

Ascertaining the sidereal time of observation as in ordinary Pole Star calculation, add six hours to it for a second sidereal time.

Out of Table I. in "Nautical Almanac," take the correction with first sidereal time, which, applied with the reverse sign to the latitude, will give the altitude at the time.

Take out a second correction with second sidereal time, which will be the rectangular deviation of Polaris from the meridian.

To calculate the horizontal angle answering to this, the formula is

$$\sin \text{horizontal angle} = \sin \text{correction} \times \sec \text{alt.,}$$

which will give the true bearing of Polaris—east of meridian when first sidereal time is between 13 hours 20 minutes and 1 hour 20 minutes, west when otherwise.

Example.

August 10, 1881, Lat. $43^{\circ} 30' N.$, Long. $66^{\circ} 30' W.$, at $13^h 34^m$ G.M.T. Observed angle from Polaris to Seal Island light $80^{\circ} 10'$, right of Polaris.

| | | | |
|---|-----------------------------------|--|----------------------------------|
| G. M. T. .. | ^{h.} 13 ^{m.} 34 | Cor for 1st Sid. T. .. | [°] - ['] 0 17 |
| Long. .. | 4 26 | Latitude | 43 30 |
| <hr/> | | <hr/> | |
| M. T. ship .. | 9 08 | Altitude Polaris | 43 13 |
| Sid T. noon .. | 9 16 | | |
| Acceler. .. | 2 | | |
| <hr/> | | <hr/> | |
| 1st S. T. obs. | 18 26 | Sin. Corr. for 2nd S. T. | 8.35018 |
| | + 6 | Sec. Alt. | .13741 |
| <hr/> | | <hr/> | |
| 2nd „ „ .. | 0 26 | Sin. True Bear ^g . .. | 8.48759 |
| <hr/> | | <hr/> | |
| Corr. for 2nd S. T. .. | $1^{\circ} 17'$ | Polaris .. N. <u>$1^{\circ} 45' E.$</u> | |
| <hr/> | | | |
| Cos. ang. dist. | | 9.2324 | |
| Sec. Alt. | | 0.1374 | |
| <hr/> | | | |
| Cos. hor. ang. | | 9.3698 | |
| Hor. ang. | | $76^{\circ} 27'$ | |
| Polaris | | N. 1 45 E. | |
| <hr/> | | | |
| Seal I ^d . L ^t . .. | | N. <u>78 12</u> E. | |

VARIATION.

Accurate variations are very useful in all parts of the world, as from them the lines of equal variation shown on charts are drawn ; but to enable them to be so used, they must be trustworthy.

Variations obtained by swinging the ship carefully with a smooth sea, and in water of, say, over 50 fathoms, are most useful, as fear of local attraction is thereby removed. Sea Observations.

The bearing of the sun, or of an object sufficiently distant to maintain the same direction whilst steaming round, and of which the true bearing is obtained, should be observed on evenly distributed points. There is no necessity to observe more than on every other point, and good results will be obtained from the cardinal and quadrantal points.

In the mean of the total errors the deviation will be eliminated, and the result is the variation.

A full report of the observations should be transmitted home.

Shore

Observa-

tions for
Variation.

Shore variations are also of value, when taken on ground free from suspicion of local attraction, for the determination of the true variation, and, in other cases, for the information they afford on the amount of the local attraction as obtained by comparisons with the variation found from sea observations in the vicinity—a point of much interest, and often of practical importance.

The requirements for a good shore variation, that the Hydrographic Office can put confidence in, are as follows :

1. The true bearing of different points (about six) as equally distributed as possible round the circle, whose centre is the observation spot, must be well and accurately observed with a theodolite.

2. Bearings of all these points must be taken by the compass from the observation spot.

3. Different sets of observations must be made with different pivots and with different cards.

4. The spot on which the observation is made should be free from every suspicion of any iron in the vicinity, and the nature of the rock, or whatever the formation may be near the observation spot, should be mentioned in the return transmitted home.

Points 1 and 2 are necessary precautions against the errors of the card, caused either by bad graduation or from accidental bending of the card. In ascertaining the true bearings, it will only be necessary to observe one object, when theodolite angles to the others will give their difference of bearing.

As regards No. 3, all compass cards have an error caused by inaccurate affixing of the magnetic needles, and it is necessary to multiply observations, and make certain the card is working properly.

Variation
deduced
at Office.

Shore observations should be obtained at stations where the variation is already well known, when opportunity offers, as these will enable the Office to calculate the change of variation.

An example of observation for variation is appended.

Although the variation is here deduced to show the method, this would not be done in forwarding these observations to the Admiralty, as there are certain card errors to be applied first.

VARIATION.

| Date. | Observation Spot, Nature of ground, &c. | Nature of Observation. | Objects. | True Bearing. | Magnetic Bearings. | | | | |
|------------------|--|---|---|------------------|-------------------------|---------------------|---------------------|---------------------|---------------------|
| | | | | | Standard Compass B 154. | | | | Dover. H. O. |
| | | | | | Card A. | J. | Spare A. | A. | |
| 11 Dec. 1878. | West Base. | Single Observations Theodolite (6 in.) and Sextant. Artificial Horizon. | Mari. Mill . | N. 2 00 E. | N. 8 05 E. | N. 8 05 E. | N. 8 15 E. | N. 8 25 E. | N. 8 08 E. |
| | Pasha Liman I ^a . | | Chim . . . | 56 16 | 62 10 | 62 20 | 62 18 | 62 40 | 62 35 |
| | Sea of Marmara. | | Slope Mill . | 114 14 | 120 15 | 120 15 | 120 12 | 120 42 | 120 33 |
| | — | | Brush Δ . . | 179 54 | 186 20 | 186 10 | 186 18 | 186 10 | 186 15 |
| | Alluvial soil. | | Rok Δ . . . | 243 06 | 249 15 | 249 05 | 249 15 | 249 10 | 249 33 |
| | Lat. 40° 28' N. | | Araplar Hill . | 203 39 | 309 45 | 309 42 | 309 52 | 310 08 | 310 05 |
| | Long. 27° 34' E. | | Nest Δ . . | 342 37 | 348 50 | 348 40 | 348 42 | 349 05 | 348 50 |
| | | | Mean . . | 177 24 | 183 31 26 177 24 | 183 28 26 177 24 | 188 33 09 177 24 | 183 45 43 177 24 | 183 42 43 177 24 |
| | | | Rough Variation... (without card errors) | | W. 6 07 26 | 6 04 26 | 6 09 09 | 6 21 43 | 6 18 41 |

CHAPTER XVI

SEA OBSERVATIONS

Double Altitude — Sumner's Method — New Navigation — Short Equal Altitude—Circum-meridian Altitudes of Sun.

As regards Surveying Operations.

FOR surveying purposes, observations at sea are mainly required for fixing the ship's position when sounding banks, or looking for vigias.

We cannot hope to attain to any very great accuracy, and are much dependent on weather and the state of the sea and clearness of the horizon. As longitude must depend entirely on the chronometers, we must in cases where we require all the accuracy we can get, as in fixing the position of banks far away in mid-ocean, wait until we can again obtain Error and rates to give the final positions ; but with ordinarily good chronometers our daily positions obtained whilst sounding will be correct, comparatively one with the other, so that we can at once plot and delineate the shape of the banks, which is what we want at the time.

Refraction.

In all observations at sea it must be remembered that the horizon may be affected by abnormal refraction, and no dependence can be placed on a latitude or longitude deduced from altitudes observed on one side alone. In certain cases the error may amount to 2' or even 3'.

With a high sun at noon, when accuracy is aimed at, it is well to observe the opposite side of the horizon ; an awkward observation at first, which practice will render easy.

Position Early in the Day.

One great object when sounding or looking for banks is to obtain a position as early as possible in the day, after lying-to probably all night, as in the vicinity of banks currents are nearly always set up, and in variable directions, so that we cannot at all depend upon dead reckoning, or upon finding ourselves

where we laid-to the night before, and in many instances unless we know in what direction to go, it is useless to move at all.

In such cases observations should be taken throughout the night; for, though they will give but approximate results, latitudes by pairs of stars north and south of zenith by the same observer should give under favourable circumstances a position within five miles of the correct latitude, which will tell us if we are drifting much in the line of the meridian, and also affords us an approximate latitude to work longitude by, in the morning.

Star
Latitudes.

The worst of it is, that circumstances apparently favourable are often not really so, as the great source of error in night observations is the impossibility of being certain of the horizon. A false horizon will frequently look so well defined as to mislead the best observer, and will, of course, throw out the resulting latitude greatly. Thus we can put no great faith in latitude by stars, and none whatever in a single observation, or even in a single pair, as the horizon in one direction may be true and in another false. It is only in a series of pairs of stars through several hours that we can have any confidence, as if the results of these agree fairly, or steadily show movement in one direction (the effect of a current), we may then feel pretty sure of our position as far as latitude goes.

Night observations at sea for longitude are not of much use; but, under unusually good circumstances of horizon, the mean of two star chronometers, one east, the other west of meridian, may be better than nothing.

Star
Observations for
Longitude.

If, however, when the day has sufficiently broken to enable the horizon to be clearly seen, we can get observations of bright stars or planets on different bearings, we can obtain an excellent position by Sumner's method from which to start our day's work.

Stars at
Daybreak.

Or, if we can only get one daybreak star, as soon as we can get an observation of the sun, we can combine it with the daybreak observation.

The most satisfactory position is that obtained with a twilight horizon from the quadrilateral formed by the intersection of the lines of position derived from observations of a pair of stars on nearly opposite bearings, combined with observations of another pair of stars on bearings at right angles to the first pair. The patent log being read when each star is observed,

each line of position is brought up to the patent log reading at the time of observation of the last star of the series.

Selection
of Stars
for Obser-
vation.

The selection of stars sufficiently bright for observation is governed mainly by the angle at which the lines of position derived from one pair will cut those derived from the other. The accuracy of a line of position is independent of the hour angle, which need not be considered, when working by the "new navigation."

Identifica-
tion of
Stars.

A compass bearing of a star when its altitude is observed will enable it to be identified with certainty, either by means of the armillary sphere, or by the table in the pamphlet "What Star is it?" supplied in all chart boxes.

Venus by
Day.

The planet Venus can, when near quadrature, be observed all day. The altitude can be calculated, when she will easily be found in the field of the telescope. This observation is but too little used, as it is a most valuable one.

Different
Methods
of obtain-
ing Posi-
tion.

As observations must be carried on throughout the day, in order to get as many positions as we can, we now come to the different methods of obtaining latitude and longitude other than by the ordinary means of longitude by chronometer and latitude at noon.

There are three methods of finding latitude and longitude at the same time—viz., by Ivory's rule for double altitude; by Sumner's method; and by a short equal altitude; and for latitude only, we have circum-meridian altitudes. These are all of service under different circumstances, which will be hereafter described.

Daily
Mean
Error of
Standard
Chrono-
meter on
G.M.T.
derived
from Har-
bour
Rates of
all Chro-
nometers.
and Final
Correc-
tions to be
applied on
ascertain-
ing the
Rates.

When dealing with a large number of chronometers, after making the daily morning comparison with the standard, the error on G.M.T. of each chronometer should be worked up, using the harbour rates. Applying the daily comparisons, the performances of each separate chronometer are thus referred directly to the standard, and a mean error for the standard is adopted for each successive day, which is noted in red ink in the Comparison Book. On arrival in harbour, observations being obtained, the sea rate for each chronometer is found, and their errors on G.M.T. are worked up to the time of the daily comparisons for each day on the passage. Applying the daily comparisons to the errors of the different chronometers, a daily mean error for the standard is obtained depending on the sea rate of all the chronometers. The difference between

the daily mean errors of standard dependent on sea rates, and the error adopted as noted daily in the Comparison Book, represents the correction to be applied to the daily positions on passage found provisionally by using harbour rates.

DOUBLE ALTITUDE.

Ivory's rule for working a double altitude, with Riddle's extension, by which the longitude is also obtained, is too well known to require any special remarks. Ivory's
Double
Altitude

The condition requisite to make the position obtained by double altitude trustworthy is mainly that the sun should change in azimuth a fair amount; otherwise one of the triangles will be so ill-conditioned that a small error in either altitude or time will have a great effect on the result. Condi-
tions for
Good
Double
Altitude.

In the generality of cases we have this condition by allowing about two hours to elapse between the observations, and we can therefore get a fair position by about half-past nine or ten o'clock.

In low latitudes, however, when the declination and latitude are nearly the same, the sun will rise so nearly on a circle of altitude as to change the azimuth very slowly, and we must wait till nearly noon before we can put any confidence in the observation. Unless, then, we lose our meridian or circum-meridian observation, a double altitude is under these circumstances of little use to us. It is, however, more to be trusted than a Sumner, when change of azimuth is small; but it may be broadly stated that unless we are *very* ignorant (from lack of previous observations or other causes) of our position in latitude, we cannot under these circumstances do much before noon with observations of the sun alone; but we can get a good position by combining a daybreak observation of a star with one of the sun, as soon as it has sufficiently risen, by Sumner's method.

SUMNER'S METHOD.

Sumner's method of obtaining the latitude and longitude at any one moment, which is but too little used in ordinary navigation, depends upon the fact that a heavenly body at any

moment will be seen at an equal altitude from any part of a small circle of the earth, circumscribed round the spot where the body is vertical, with a radius equal to the zenith distance.

Knowing, therefore, the altitude of the sun, and the point on the earth whose zenith it is in, an observer will always be able to say that he is somewhere on the circumference of that circle. This alone would not give us much information, but it is seldom that we do not know our latitude to twenty or thirty miles. We shall know then that we are on that part of the circumference which includes these latitudes, and as when the sun is not very high the circle will be of large diameter, the portion of it within our limits may be, without much error, taken as a straight line.

In practice, then, having obtained an altitude of the sun, or any other body, we assume a latitude from our dead reckoning, and work out the longitude. By the aid of the azimuth tables we obtain the bearing of the body observed, and, having plotted the position obtained, we draw a line at right angles to the bearing, which is the "position line." We now know that we are somewhere on that line. We must know the Greenwich time, and therefore our positions will be as much dependent on the chronometers as any ordinary longitude.

The position line can be also obtained by working with two assumed latitudes and joining the positions resulting, but the above is shorter.

Waiting until the earth has revolved a sufficient amount to alter the bearing of the sun, we repeat the operation, and obtain another line, the direction of which will differ from the former by the difference in the azimuths of the sun at the two observations.

If we have been motionless in the interval, the intersection of these two lines will give us our exact position (always dependent on the chronometers); but if we have moved, we must so far trust our dead reckoning as to transfer the first line in the direction, and for the distance, we have run, when the intersection of the second position of the first line with the second line will be our position at the second observation.

H.M. ships are provided with a large sheet on which longitude is marked, leaving the navigator to complete the Mercator's

projection by measuring the meridional parts for the latitude he is in.

On this, or a similar sheet graduated on board, the Sumner lines will be plotted, as it will both spare the charts, and from the increased scale provided, will give a better position than if the plotting was done on the usual small-scale general chart of an ocean.

It is obvious that the value of a position will largely depend on the angle between the two lines, or, in other words, on the change in azimuth between the observations. Limits of Application.

In low latitudes, therefore, when declination and latitude are nearly alike, we cannot use this method with the sun alone early in the day, as the sun will rise nearly vertically from the horizon, and we want a change in azimuth of at least 20° to give us a trustworthy position; but observations close to noon, when the azimuth is changing rapidly, are valuable. The same circumstances will much detract from the value of a double altitude, as has been remarked, so that in such a case neither the one nor the other is much use as an absolute determination of position.

But Sumner's method has other resources. We can combine lines obtained from two or more stars, or a line obtained from the sun with one from the moon or other heavenly body, as, for instance, a star obtained at daybreak when the horizon is sufficiently defined for us to trust it, or Venus in daylight, as before remarked. All we need is that the bearings of the two bodies differ sufficiently to give a good intersection. Other Combinations by Sumner's Method.

By this means we can often get a good position early in the day, which we cannot get in any latitude with the sun alone, without a considerable interval of time elapsing.

This combination, therefore, of stars and the sun affords us the best and earliest opportunity of determining our position, and we should always endeavour to obtain it.

Should we be able to get a good meridian or circum-meridian altitude of a star, we shall of course use the resulting latitude. Advantages over Double Altitude.

A sun-Sumner requires the same circumstances and observations as a double altitude, but it has several advantages over the latter.

In the first place, the first half of the observation can be worked out at once, by which means we not only obtain the

line on which we know we must be, and so have an approximation to our position at once, but also, having worked half of the calculation, it will not require many minutes after the second observation is taken to complete it and obtain the true position.

Secondly, errors of calculation are less likely to be made in a Sumner, as it involves merely the ordinary "chronometer" problem.

Thirdly, the fact of obtaining a line of position is of great value in many cases, as we can always tell roughly in what direction to go to shorten our distance to any given point, unless it should fall on or near the line, and when searching for a vigia this knowledge, early in the day after a night's lying-to, will be invaluable.

Fourthly, we can repeat the observations a third time, and so check our first position with but little labour of calculation long before noon, especially in the case where we have combined a star with the sun, and are, perhaps, doubtful of the star observation, either from faintness of the star or indistinctness of the horizon.

Bearing of
Land and
Sumner
Line.

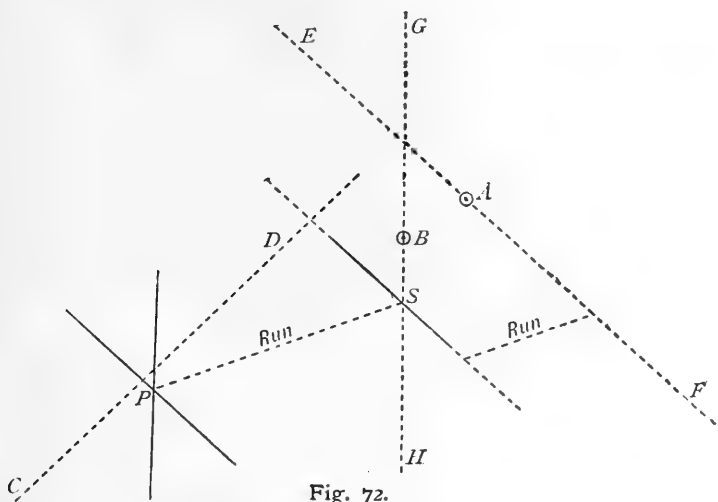
The true bearing of a distant mountain whose position is known will also give a position by combination with a Sumner line, if its direction is such as to make a good cut with the latter.

Example
of Sumner.

In Fig. 72 let us suppose A to be the position found by assuming a latitude and working out the altitude of a star obtained at daybreak. Drawing a line at right angles to the bearing, we get our first Sumner line E F, and we know we are somewhere on it. Having run west by south 6.2 miles, we get an altitude of the sun; and assuming in this case a latitude a little south, we get another position, B, and draw another position line, G H. To project the run, we draw a line in the required direction, and for the distance run, from any part of the line A, and draw another line parallel to the line E F, through the end of the run line. The position S, where this last intersects line G H, is the position of ship at second observation.

Running on in the same direction for 12 miles, we get another altitude of the sun, and another resulting Sumner line, C D. Transferring the two first lines by the run as before, we now have three lines intersecting, or nearly so, at P, and by their coincidence or not we can measure the accuracy of our former

positions—to a certain extent, that is, for it must be remembered that as the intersection of our lines is governed by the run allowed, a current, or constant error in calculating the run, might give an apparently good position which may really be



considerably in error, even when the third intersection is obtained, with certain arrangements of the lines and the run.

Sumner's method is, in fact, the means by which all individual observations can be combined, and is from every point of view invaluable.

NEW NAVIGATION.

The method of obtaining positions known by the name of "New Navigation" is particularly applicable when the bearing of the heavenly body lies between 20° and 70° from the meridian. It gives a shorter line of position, thus coinciding more accurately with the arc of a circle of equal altitude, than a line of position drawn from a point found by the chronometer method.

"New Navigation" is now becoming so well known that it will be sufficient to mention that it consists in assuming a latitude and longitude, and in calculating a correction to this assumed position. A line of position can then be drawn through the true position thus obtained, and its results combined with

other observations by Sumner's method. The calculation is given in the book quoted in the footnote.*

The chronometer method is better for observations near the prime vertical, and the rigorous ex-meridian method for observations within the limits applicable to it. It must be remembered that in the latter case the line of position is not the parallel of latitude, but is the line at right angles to the bearing, drawn through the point found.

Weir's diagram is a very convenient method of obtaining the azimuth of a star.

SHORT EQUAL ALTITUDE.

In low latitudes, where the motion of the sun in altitude is rapid nearly to the time of transit, a very good longitude may be obtained at noon, by a short equal altitude, taking observations about twenty minutes before and after noon. The change of declination in this short interval will not affect the time, so that the middle time between the observations as shown by the watch can be taken for the time by the watch at apparent noon. All we have to do, therefore, is to take the difference between mean time of apparent noon and the Greenwich time, as shown by our chronometer, which gives us longitude directly.

CIRCUM-MERIDIAN ALTITUDES OF SUN.

These are of great value, as, when the observations are within the limits of time from noon, the resulting latitude is as correct as from a meridian observation, which may be lost from clouds. They should be worked in precisely the same manner as the shore observations of the same description, and should be obtained as near noon as possible. If more than four or five minutes have to be added to the observed altitude, they will not be of much value.

If Raper's most valuable book is at hand, a short and correct rule, in connection with two of his tables, will be found at p. 232 of the thirteenth edition, which will give the reduction as nearly as requisite for sea work.

* "Ex-Meridian Altitude Tables and New Navigation," by C. Brent, R.N., A. F. Walton, R.N., and G. Williams, R.N. Geo. Philip & Son, London, 1886.

CHAPTER XVII

THE COMPLETED CHART

Fair Chart—Reducing Plans—Delineation—Symbols—Colouring—
Graduation.

THE work is sent home to be published in several ways, according to circumstances. Trans-
mission
Home.

When the detail, as it proceeds, is inked on the original sheet itself, it may be necessary to transmit a portion home before the survey is all complete, and a tracing is often used for this purpose, as the original sheet, with the “points” still accumulating, must be retained on board; but, if possible, it is better to send work home on drawing-paper, which is not liable to so many accidents from tearing, etc., can be more fully worked up as regards detail, and can be better kept as a record, though the originals will in the end be transmitted to the Admiralty in any case.

When the detail is placed directly on the original sheet, it is very difficult to keep it clean enough for everything to be clear and distinct, as straight-edges, protractors, etc., will be constantly placed on the chart over the completed part, and lines must be often drawn over it. Original
Chart. It *can* be kept clean enough for transmission home as a finished chart, and by doing so, all errors arising from imperfect transferring will be avoided; but the surface of the paper must get so rubbed by constant cleanings, that, if a large sheet, it is seldom satisfactory. Several hands may have been employed in it, and the chart will then bear a piecemeal look. If this original sheet is not sent home, a copy has to be made on another sheet of paper, which will be the fair chart.

The usual mode of making this is to place the new sheet under the old one, and prick the “points” through the latter, Fair
Chart.

on to the former. A careful tracing having been made of the working sheet, it is placed on to the fair sheet, so that the points all correspond, and by means of transfer paper is traced on to the fair sheet, and inked in.

Great care is requisite, in transferring in this manner, that the tracing does not move from its proper position, and heavy weights must be used to prevent it from doing so. Errors have often crept in from careless transferring and want of proper examination and comparison afterwards.

In working with the method recommended by the writer—viz., each assistant's work plotted and inked on to his own separate board, and all then placed on one tracing—the final sheet can either be the original on which all the points have been plotted, if that has been kept clean enough; or a sheet may be pricked through, as mentioned above, for the purpose; but in either case only one complete chart will be made, the general tracing sufficing to show whether the work of different assistants has met, and what is wanted to complete.

This, or these (as in a large sheet there will be several tracings for different parts), will be the tracing used for making the final chart in this case.

These tracings should not be too large, as they are apt to get distorted. For fine work it is desirable to make small tracings on paper for the special work of transferring.

This chart will also be the work of one hand, who will, after transferring outline, soundings, etc., from the general tracings, have the original little bits before him while inking in; these little bits having been taken off their boards, and so reduced, by having superfluous paper cut off, as to be handy to lay on the sheet.

Original
Field
Sheets.

By washing off the field boards, the paper will have become distorted and contracted, but not to a sufficient degree to interfere with the small detail of sinuosity of the coast, which is what we mainly want for them. Everything will have been traced on the general tracings before the paper has been removed, and care must be taken that this is so, as it cannot be done afterwards.

"Points"
to be
Shown.

In whatever manner the final chart is sent in to the office, all "points" must be distinctly marked on it, especially main points. These latter are often distinguished by the triangle which means theodolite station, and in surveys where the

sextant has also been employed in triangulation, should certainly be so. The "points" are necessary to join one chart to another, and also, in case of future revision of the chart, they afford means to the reviser of measuring the accuracy of his predecessor's groundwork.

Plans sent home by officers in general service ships often lose much of their value from neglect of this. The existence of the "points," and their proper position, will at once give a confidence in the detail of the plan, that it is impossible to accord to the work of an officer, however zealous, of whom nothing is known as to his hydrographical capability, and who fails to give any indication in his chart of how it has been constructed.

REDUCING PLANS.

In a survey of an extensive nature, bays, harbours, etc., will often be done on a larger scale than the rest of the sheet. These must be either left blank on the coast sheet, or else reduced from the large-scale plans.

It may sometimes happen that a portion of an anchorage is surveyed on the small scale before it is decided to make a large plan of it, on discovering it to be worth while to do so. This must not appear, however, on the completed chart; it must be all reduced from the larger scale.

Instruments for reducing—*e.g.*, eidographs—are not supplied, and the reduction is accomplished by "squaring." Reduction
by Squar-
ing.

This consists of ruling similar lines on both sheets, forming squares and diagonals all over the part to be reduced.

The two stations farthest apart on the plan, which must also be plotted on a small-scale chart, are joined by a line on both sheets, the "directing line." Then, taking the smaller first, divide this line into as many equal parts as is thought necessary. These parts will be from $\frac{1}{4}$ to $\frac{1}{8}$ inch long. Set off lines at right angles to the directing line from each point measured, and then lines parallel to the directing line, at the same distance apart as the others. The portion of the sheet required is now covered with squares. Rule also the diagonals. These will check the correctness of the squares, as they should, of course, pass exactly through each corner.

Now do the same for the large scale, making *an equal number* of squares.

It will be seen that nothing is measured, everything being done by subdivision of the directing line.

Great care is necessary to rule all these lines truly rectangular and equidistant.

Number the lines on each plan, to prevent mistakes, giving the same number to similar lines. Letters may be put to one set of lines, and numbers to those at right angles.

Then, taking proportional compasses, set them to the difference of the scale as ascertained by measuring the distances apart of similar lines. The distance of each little detail of the plan, from the nearest lines, can be put down by the same distance from the similar lines on the small scale.

Reducing is an operation demanding even more patience and trouble than usual, and it is better to leave the space blank than to reduce it carelessly.

REDUCING PLANS BY PHOTOGRAPHY.

Photography has recently been employed with great advantage for this purpose. The method adopted by the Hydrographical Department is to obtain a blue print of a photograph of the original drawing, and to ink in such details as are considered necessary to be shown on the plan when reduced, omitting those that are unsuitable for the smaller scale. The soundings are selected with a view to avoiding overcrowding, but the selection should be judicious in order to show appropriately the undulations of the bottom, so far as the reduced scale will admit; particular care being taken that the shoaler soundings are not omitted in places where it is essential to indicate them. The soundings so selected should be inked in, in type of such character that, when reduced to the required scale, they shall be of suitable size. Names, etc., should be similarly treated. A line should be drawn joining two stations, as far apart as possible, on the blue print, and the distance in inches of these stations from each other as they will appear on the reduced plan should be furnished to the photographer. The blue print inked in as described above is then brought down to the required scale by photography, and an ordinary

black print supplied. Blue being non-actinic, only those parts that have been inked in will now appear.

If a blue print is not available, a tracing of the original drawing should be prepared, omitting details unsuitable to the smaller scale to which the original is to be reduced, and regulating the size of the figures, lettering, etc., as described in the case of the blue print. The tracing is then reduced by photography to the required scale.

The lenses of the present day have been brought to great perfection, and experience shows that they produce no distortion even near the edges of a large photograph where it is most likely to occur. The only sources of error lie in the possibility of the board upon which the drawing is pinned for photographing not being perfectly flat, or not being placed at right angles to the axis of the lens. Both these points require attention on the part of the photographer. By the aid of a few large squares on the original drawing and similar squares on the reduced photograph, drawn in the usual manner, an error due to any cause is quickly detected. All reductions are tested in this manner before being accepted.

Reference to plates A, B, C, D will facilitate the preparation of tracings and blue prints for reduction in scale in proportions varying from one-half to one-fourteenth of the scale of the original drawing, intermediate proportions being regulated accordingly. From these plates a tolerably good idea can be formed as to the amount of detail necessary to be shown in each case, and other matters, such as the spacing of the soundings, size of the figures, lettering, thickness of coast-line, etc.

In the case of small reductions in scale it is unnecessary to use photography. Large squares being drawn on the original document, and similar squares on the required scale on tracing-paper, the details can be traced by fitting each square on the tracing-paper over its corresponding square on the original. The difference in scale being small, the several squares will correspond so closely that no practical error is introduced. The smaller the squares the more nearly will this be the case, and their size should depend on the proportion that one scale bears to the other; the smaller the difference in scale the fewer squares being required.

Sources of Error.

Plates illustrating the Preparation of Blue Prints for Reduction in Scale by Photography.

Reduction by Method of Equal Squares.

DELINEATION, SYMBOLS, AND COLOURING.

The annexed specimen chart, taken from the "Admiralty Manual," shows the method of delineation employed in fair chart-work.

The following symbols are in use in surveying, in field-books, and rough charts.

Signs for
Week
Days.

The days of the week are thus symbolized by the astronomical signs of the planets.

| | | |
|-----------------|--------------------|------------|
| Sunday | Sun's Day | Sun .. ☉ |
| Monday | Moon's Day | Moon .. ☾ |
| Tuesday | Teut's Day | Mars .. ♂ |
| Wednesday | Woden's Day | Mercury ☿ |
| Thursday | Thor's Day | Jupiter ♃ |
| Friday | Friga's Day | Venus .. ♀ |
| Saturday | Saturn's Day | Saturn ♄ |

Other
Symbols.

The following signs are useful in the field-books.

| | |
|---|-------|
| Objects in line, called transit | ☐ |
| Station, where angles are taken | △ |
| Zero, from which angles are measured | ⊕ |
| Single altitude Sun's lower limb | ☉ |
| " " " upper " " " " " " " " | ☉ |
| Double " " " lower limb in artificial horizon | ☉ |
| " " " upper " " " " " " " " | ☉ |
| Sun's right limb | ☉ |
| " left " " " " " " " " | ☉ |
| Sun's centre | ⊙ |
| Right extreme, or tangent, as of an island .. | → |
| Left " " " " " " " " | ← |
| Zero correct " " " " " " " " | Z. K. |
| Windmill " " " " " " " " | ⊗ |
| Water-level | w. l. |
| Whitewash | w. w. |

Some charts are worked up by indian ink alone in all parts ; in others, colour is used to assist the delineation of the different parts, indian ink being always used over the colour, in exactly the same manner as if there was no groundwork.

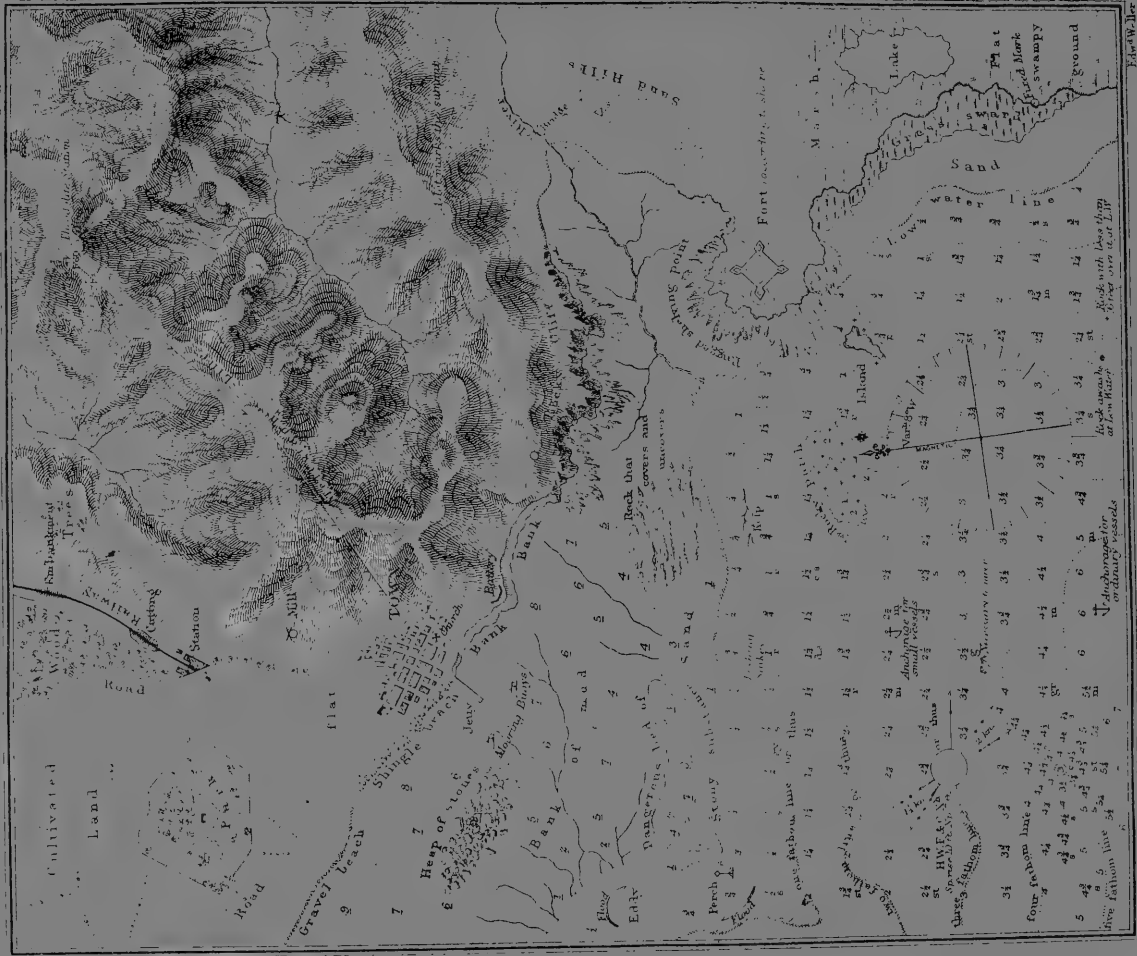
Colouring.

A wash of some colour on the land helps to throw it up very much ; but care is very necessary in giving this edging that it be not too deep, and that too much water is not used,

course of day & dark time 85, 90, 95
 2 ground in mud, 3 rock & stones

Figure A. To illustrate Hydrographic delineation.

Bayes 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000



Figures underlined thus 2 show heights in feet above low water Springs, of banks which cover and uncover.
 Tide arrows - Flood - - - - - Ebb - - - - - dots on them thus - - - - - the 2nd and 3rd hour of the tide. Current arrows thus - - - - -



or the paper will distort, and the tracing will not fit. Also in drying, that it does so gradually and generally, not allowing a streak of sunlight, for example, to fall across one part of the sheet.

If using colour, the following tints should be used for the different parts :

| | | | | | |
|---------------------------------|----|----|----|----|--|
| Towns and Buildings | .. | .. | .. | .. | Carmine. |
| Hills | .. | .. | .. | .. | Payne's Gray. |
| Cliffs | .. | .. | .. | .. | Black. |
| Roads | .. | .. | .. | .. | Burnt Sienna. |
| Rivers and Lakes | .. | .. | .. | .. | Prussian Blue. |
| Sand, Sand Banks, Sand Hills or | | | | | } Gamboge, dots black, Carmine dots for low water round edge. |
| Sandy Islets | .. | .. | .. | .. | |
| Shingle | .. | .. | .. | .. | Raw Sienna. |
| Coral | .. | .. | .. | .. | Carmine and Burnt Sienna. |
| Low Water Rocks | .. | .. | .. | .. | Burnt Sienna. |
| Mud | .. | .. | .. | .. | } Payne's Gray, edge of fine black dots. |
| Mangroves, Cultivated Ground, | | | | | |
| Grass, Meadows, Trees.. | .. | .. | .. | .. | } Prussian Green. |
| Swamps and Marshy Land | .. | .. | .. | .. | |
| | | | | | Prussian Blue. |

The three and five fathom lines should be coloured with cobalt—the former with a light tint all over the space included between it and low water, and the latter with a narrow edge inside the fathom line.

N.B.—To make indian ink perfectly black, mix a little indigo with it.

When the country is mountainous, no general wash, but only a local green in the valleys, and on flat ground, has a good effect.

As hills in most large-scale charts are now engraved in con- Hills.
tours, it is best to use this system in the fair chart.

Shading of indian ink, put on with a brush, is done quickly, and shows up very well.

Simple contour lines will enable the chart to be engraved almost as well as the other modes, but does not look so well.

A specimen of this method of delineating hills is shown in Fig. 73, and it is the most suitable for reduction in scale by photography.

In charts issued by the British Admiralty the shading is put on hills as though it were a raised map with the light coming from the north-west.

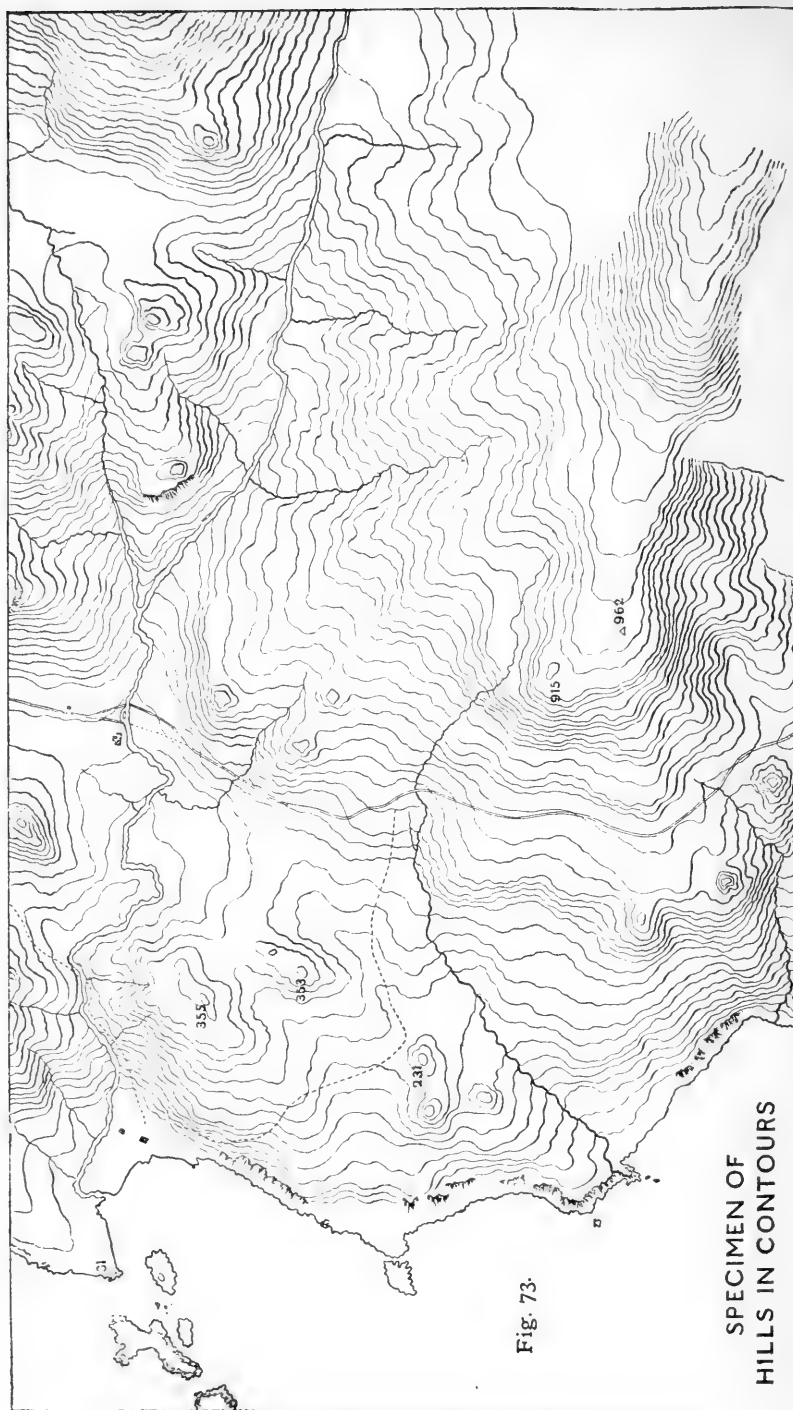


Fig. 73.

SPECIMEN OF
HILLS IN CONTOURS

In inserting the names, care should be taken that no letters Names. are upside down. Thus, it is often necessary to write a name in nearly a meridional direction, and it will depend upon whether the trend of the name is east or west of the meridian, whether it is written from south to north or north to south.

Thus in the two instances given here, Fig. 74, if the names had been written in the opposite direction, part of them would have been upside down. All names should be readable by turning the head, without the necessity of moving the chart.*

All names of capes, etc., should be as much on the land as possible. The soundings being the most important part of a chart, they should be kept as clear and distinct as practicable.

Different characters should be used for the names of different classes of objects—thus, one style for bays, another for points, another for shoals, and so on.

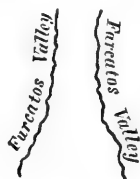


Fig. 74.

If native names cannot be ascertained, other names should be given by the surveyor to prominent points, islands, shoals, etc. It is difficult to write sailing directions unless there are a sufficient number of names to refer to. It is also inconvenient for the navigator using the chart to be unable to record concisely such points of land, etc., of which he is likely to take bearings.

Photography being now largely used to assist in the rapid production of the surveyors' work after its receipt in office, colour washes should be avoided, as it is found that they prevent a faithful reproduction of the work by this means. Drawing of Fair Sheets.

The following colours are most suitable to depict the various

* *Vide* "Instructions for Hydrographic Surveyors" for useful hints.

features of a chart in order to distinguish them more clearly than by using black alone :

| | | |
|------------------------|-------|--|
| Towns and Buildings | | Carminc tint. |
| Lakes | | Cobalt tint round the edge |
| Sand and Shingle | | Indian Ink pen-work. |
| Hill Contour Lines | | Faint Indian Ink. |
| Heights and Stations | | Carminc. |
| Sandy Islets | | Black dots. |
| Sand-banks that dry | | Gamboge dots. |
| Coral | | Carminc and Burnt Sienna. |
| Roads | | Burnt Sienna. |
| Low-water Rocks | | Burnt Sienna pen-work. |
| Mud | | Payne's Gray, edge of fine black dots. |
| Mangroves | | |
| Cultivated Ground | } .. | Prussian Green pen-work. |
| Grass and Meadow Land | | |
| Trees | | |
| Swampy and Marshy Land | | Prussian Blue pen-work. |
| Current Arrows | | Indian Ink. |
| One-Fathom Line | | Black dots. |
| Other Fathom Lines | | Carminc contour lines. |

The three and five fathom lines should be coloured with cobalt—the former with a light tint all over the space included between it and low water, and the latter with a narrow edge inside the fathom line.

Too great elaboration in working up sand or mud flats, low-water rocks, and coral, should be avoided, as this is as bad for photographic purposes as a colour wash ; this is especially the case where the low-water line approaches the high line. In the latter case care should be taken to leave a clear space between the low and high water lines.

Hills are better left unshaded, the topography being represented by contour lines alone, as shown in Fig. 73.

Side for
Scale.

The scale of the chart is got from the longest calculated distance on it. This will, in cases of plans, generally be the same as that we originally plotted from, in which case we already know our scale. But if we were obliged to plot from a short side, and have since obtained data which will enable us to calculate a longer distance, we must measure the distance between the two points on our chart, and dividing this number of inches and decimals by the distance as calculated, we shall get the true scale.

It is well to indicate the two stations from which the scale is derived, by drawing a red line between them, and writing, either against it, or elsewhere on the chart, the calculated distance and bearing. If a long distance, this last should be the Mercatorial bearing.

In the case of extended surveys, or when there is no regular triangulation, the scale will depend upon the distance obtained between two stations by astronomical observations. This distance being calculated, the scale will be obtained as before.

The soundings in the chart sent home should be as thick as possible, without sacrificing legibility. There is always a great temptation to thin them out, so as to look better ; but that is the work of the Hydrographic Office, and will probably have to be done again there in any case, as the scale on which the chart is published is usually smaller than that on which it is constructed, and if so, will not permit all soundings in the original to appear. Soundings Thick.

The natural scale, or the proportion which our chart lineally bears to the actual size of the portion of the globe it represents, is obtained by dividing the number of inches corresponding to one mile on our chart, obtained as above, by the number of inches in the nautical mile at the latitude. It is given in the form of a fraction, whose numerator is one. Natural Scale

Thus, supposing our scale is found to be 1·8 inches to a mile, in latitude 3°, we divide 72,552 (the number of inches in a mile) by 1·8.

This gives $\frac{1}{40,306}$ as the natural scale.

This natural scale should be noted on all sheets that are not graduated.

When the chart includes a considerable extent of coast-line that is intended to form part of a navigational sheet, it will have eventually to be redrawn on Mercator's projection, as it is on that projection all charts are published.

To do this, the sheet must be graduated—*i.e.*, have the meridians and parallels placed upon it, as it is by means of them that a chart on one projection is redrawn on another.

GRADUATION OF THE SHEET.

We have before said that a chart constructed by drawing right lines from one object to another, when graduated, has to be considered as being on the gnomonic projection, and the general features of this projection have been explained.* It now remains to consider how to graduate such a chart. Gnomonic Projection.

* Page 98.

A sheet may be graduated either before or after the chart is drawn on it. The methods are substantially the same, and will differ only in some preparatory work necessary in the latter case.

We will first consider the case of graduation after the chart is complete, and to do this we must suppose our observations to be obtained, and that we know the latitudes and longitudes of two stations on our chart as far apart as possible in opposite corners of the chart.

We require, first of all, the reciprocal bearings of each from the other, and the distance between them.



Fig. 75.

In Fig. 75 let A and B be two stations whose latitudes and longitudes we have obtained ; P is the pole. Add to the diff. long. the spheroidal correction, and use this corrected diff. long. in the calculations. Calculate by spherical trigonometry the bearing of each station from the other.

We have P B and P A the co-latitudes, and B P A the diff. longitude. P B A and B A P are the angles required. The latter subtracted from 180° will give us B A Q, and the difference between P A B and B A Q is the convergency. Find also the distance A B, to get the scale.

Now in Fig. 76 let A B be these same stations plotted on our chart. Required to graduate it.

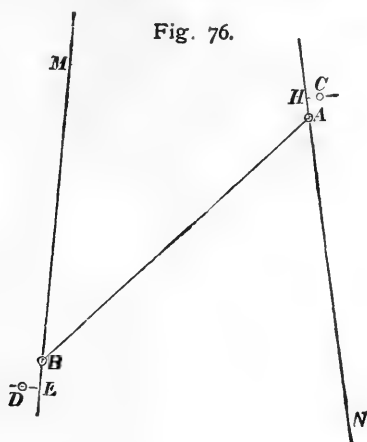
Join A B, and from A and B lay off (by chords) the reciprocal bearings of one another, ascertained as above, as A N, B M, which will be meridians passing through those points.

From A and B measure, on the meridians, A H, B E, the

distance, according to scale, to the nearest even minute of latitude (as 1', 5', 10', etc., as convenient).

At H and E lay off short perpendiculars to the meridians, and on these measure the distances H C, E D, the lengths of departure, according to scale, to the nearest even minutes of longitude that may be convenient.

In high latitudes and large scales, if the even meridian required is many miles distant, error will be introduced by this latter operation. It will only be correct for short distances, as the curve of the parallel, on which we ought to



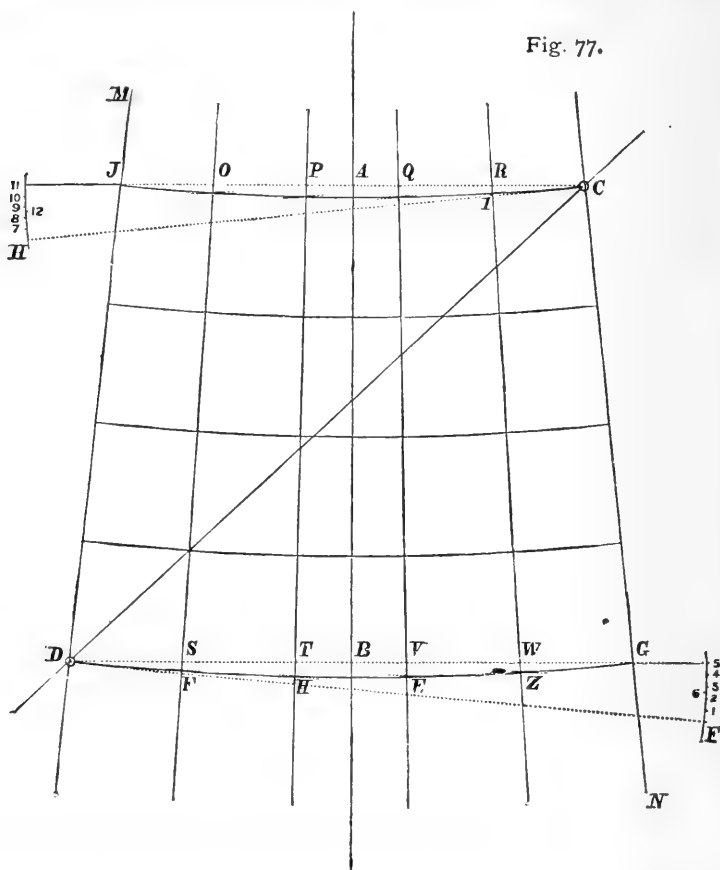
measure this departure, will not coincide with the perpendicular to the meridian for more than a mile or two in such a case.

We have now C and D, two stations on even meridians and even parallels, which we shall take as our points for graduation. This is exactly the case when we wish to graduate the sheet first, so that henceforward the methods are identical. In the case of after-graduation, when these even points have been obtained, we can rub out on our chart all lines already ruled, to prevent confusion, and we will take a new figure for the similar purpose of facilitating comprehension.

In Fig. 77 let C and D be the positions for graduation. Calculate spherically, as before, the bearings of C and D from one another, and lay off the meridians C N, D M.

From C and D lay off the perpendiculars C H, D F, and

from these perpendiculars lay off, on the side of the pole, half the convergency calculated for the difference of longitude in the latitude of C and D respectively, as C 11, D 5, cutting the opposite meridians respectively in J and G. Then J will be on



the same parallel as C, and G as D, and J D, C G, should be equal.*

To get the central meridian of the chart, bisect J C and D G in A and B, and join them.

Then J G joined should intersect C D in the central meridian. This is a capital check for our correctness so far.

* See Appendix B.

To get other meridians, divide J C and D G as many times as there are meridians required, and join them, as O S, P T, Q V, etc.

To get the parallels, which it will be remembered are curves, divide the half convergency chord, already measured, into as many parts as we have meridians.

In our figure we want five meridians from D to G, therefore we divide the chord into five parts, as 1, 2, 3, 4. Draw a small portion of D 4, cutting R W in Z. Z will then be the position on R W of the parallel of D G. By similarly drawing D 3 to cut Q V in E, D 2 to cut P T in H, and D 1 to cut O S in F, we obtain a series of points on the meridians, which, connected together, will form the curve of the parallel D G required. In high latitudes we want more meridians, to draw the curve exactly, than in low, and we must therefore be guided by circumstances as to the number of them.

Similarly, we obtain the curve of the parallel J C.

To draw more parallels, divide each meridian between the parallels obtained into as many parts as required, and join them.

This process demands considerable care and accuracy in drawing every line, and should be checked wherever practicable.

The margin of the chart is marked by subdividing the distance between each parallel or degree to the unit required.

There are other ways of drawing this graduation, all founded on the same principle. As this is, in the writer's opinion, the best of them, it is here given.

All charts should be graduated, but in the case of plans of comparatively small extent, it will be sufficient to graduate their margins in skeleton form, giving a scale of latitude and distance, and also a scale of longitude. The graduation must necessarily be on the gnomonic projection, except in the case of a Mercator's chart, to which points have been transferred by their latitudes and longitudes. It is needless to say that it is impossible to graduate on Mercator's projection a sheet on which points have been plotted by angles and distances.

The skeleton graduation of plans is carried out by ruling the true meridian through one of the stations forming the long side of the plan, and graduating on a central meridian on the principles described on the following page.

Graduating a
Sheet on
Central
Meridian.

A meridian having been drawn through one of the \triangle^s , a short perpendicular is erected at each end. Knowing approximately the latitude of the spots at which the perpendiculars are ruled, calculate the departure in each latitude corresponding to the difference of longitude between the meridian already drawn and a central meridian passing through the nearest exact 5' or 10' of longitude. Ascertain very accurately the scale of the chart from points as widely separated from each other as possible; the distance between them being calculated in the triangulation or obtained from their astronomical positions.

Turn the departures, as found above, at the latitudes of the two perpendiculars into inches according to scale of chart, and set them off on these lines. Join the points thus found, giving a central meridian passing through an exact 5' or 10' of longitude. From the \triangle through which the original meridian is drawn, draw a long line perpendicular to that meridian; lay off from the perpendicular in direction of the pole half the convergency due to the difference of longitude between the original meridian and the central meridian. The point at which the line cuts the latter is in the same latitude as the \triangle . From this point on central meridian measure off N. and S. the difference of latitude to the required margin on the scale already found. Through these extreme N. and S. points draw lines perpendicular to the central meridian extending both ways.

Calculate departure in both marginal latitudes corresponding to the difference of longitude between central meridian and margin of chart E. and W., and also the convergency for those departures in both latitudes.

At the N. and S. marginal points on the central meridian lay off from the perpendicular to the meridian half the respective convergencies towards the pole, and on those lines measure off the respective departures found above according to scale of chart. The four corners of the chart are now fixed, and the intermediate meridians are found by subdividing the perpendiculars between the central meridian and margin of chart and joining the points so found at the N. and S. ends of the chart. The parallels of latitude are found by subdividing the half convergencies into as many parts as there are meridians,

and forming the curve in the manner described on p. 387 when graduating in the ordinary way. The extension of the graduation of a chart is carried out on similar principles.

All graduation should be done in one forenoon, so as to permit the paper to alter as little as possible during the operations.

The meridians and parallels being ruled at, say, every 10', the distance in inches is measured N. or S. from the \triangle to the nearest parallel; 10' of difference of latitude being measured on the brass scale, the ratio which the first measurement bears to the second gives the proportion of 10' corresponding to the difference of latitude of the \triangle N. or S. of the parallel used. The distance of the \triangle in inches should then be measured E. or W. of the nearest meridian; the distance between two adjacent meridians in the latitude of the \triangle being also measured on the brass scale. The proportion of 10' represented by the ratio of the two measurements is the difference of longitude of the \triangle E. or W. of the meridian used.

To take off Latitude and Longitude of a \triangle from a Sheet graduated on the Gnomonic Projection.

Besides a comprehensive title in the usual form, in which should be given the names of all officers who took part in the survey and the months of the year during which the soundings were obtained, every original chart must have a memoir written on it containing the following information, to enable the authorities at home to put the proper value on the work :

Memoir of Construction and Title.

1. The method of construction, with the base used, and how measured.

2. References to triangulation sheet, and returns of latitudes and meridian distances which may affect the sheet, stating the latitudes, meridian distances, and true bearings as actually observed, together with any correction that may have been applied to them to obtain the finally accepted result, and the number of feet per mile by which the distances by triangulation have been corrected to agree with the accepted positions.

3. In a graduated chart, the latitudes and longitudes adopted for the graduating points, with the secondary meridians on which they depend.

4. A table giving the latitudes and longitudes of various points on the chart, as calculated in the triangulation and as taken from the graduation.

5. The longest calculated side, with its Mercatorial bearing and half convergency when the latter is appreciable.

6. Whether the topography has been sketched on the ground or from the ship.

7. Reference to tidal observations, the datum or level to which the soundings have been reduced, referred to a permanent mark on shore wherever this is possible, together with any similar information that may be useful as a record.

In home waters the connection of the datum for soundings with the Ordnance Survey datum should be stated.

8. A list of the most conspicuous objects recommended for use in fixing, in order that they may be suitably delineated on the chart.

The name of the officer who has drawn the chart should always be stated in the lower left-hand corner.

Trans-
ferring to
Merca-
tor's Pro-
jection.

It is scarcely necessary to describe the construction of a Mercator's chart, as every naval officer learns it as part of his education.

To redraw a survey on Mercator's projection similar meridians and parallels must be drawn on both charts, and enough of them to make the parallelograms formed by them small enough to reduce the discrepancy between the shape of any parallelograms on either chart to as little as possible. The soundings, coast-line, etc., in each parallelogram of the gnomonic chart are then transferred to the same parallelogram of the Mercator, by latitude and longitude of each detail.

CHAPTER XVIII

DEEP-SEA SOUNDINGS*

Wire Sounding—Dredging—Nature of Ocean Bottom—Submarine Sentry.

IN the first edition of this work the method of deep-sea sounding with a hempen line was alone described. Hemp has now been entirely superseded by wire, and therefore the machines employed in wire-sounding and the methods of using wire will alone be treated of.

Besides the advantage of weight, greater compactness of the apparatus, the celerity with which the weight descends, and the greater speed at which it can be reeled in, wire, from its small size and the smoothness of its surface, enables in many cases soundings of greater accuracy to be obtained. In sounding in a surface current with hemp, the line was carried along with the current, and it was impossible to keep the ship over the lead. The result was that when the lead reached the bottom the ship was a long way astern, and an empirical correction had to be made to arrive at the vertical depth. With the fine wire now used the friction is so slight that the ship can be kept over the lead without the wire getting under the bottom, and the length of wire out is the depth.

Advantage of Wire.

Soundings with wire can be carried out at night with equal facility as in daytime, and in almost any circumstances of wind and weather short of a fresh gale, against which the ship could not steam or face the sea.

The machine used in surveying vessels for wire deep-sea sounding is that devised by Mr. Lucas, of the Telegraphic Construction and Maintenance Company, and has undergone several modifications.

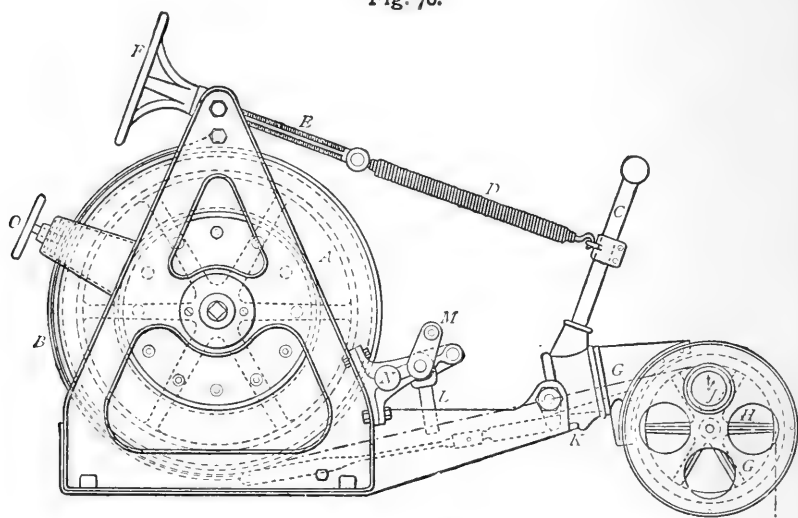
Lucas Machine.

* This chapter is entirely from information supplied by Captain A. M. Field, R.N., supplemented by notes from Captain W. U. Moore, R.N.

The large machine now supplied holds over 5,000 fathoms of 20-gauge wire, and is very compact. It is fitted with two brakes : one a screw brake for holding the reel when required, the other an automatic brake for stopping the reel when the weights strike the bottom. A guider for the purpose of winding the wire uniformly on to the reel is also attached, and is worked by a small handle.

After leaving the reel the wire passes over a registering

Fig. 78.



Automatic Sounding Machine to carry 6,000 Fathoms of Wire.

- | | | |
|---------------------|---------------------|-------------------------|
| A Reel or drum. | F Hand wheel. | L Wire-guiding rollers. |
| B Brake. | G Swivelling frame. | M Handle for working |
| C Brake lever. | H Measuring wheel. | roller. |
| D Springs. | J Indicator. | N Bolt. |
| E Regulating screw. | K Stop. | O Screw brake. |

wheel, the dial of which indicates the amount of wire run out, no matter how little or how much wire is on the reel.

A machine of smaller size, but very similar in type, is supplied for use in boats for soundings of, say, more than 15 fathoms, and is also useful from the ship for serial temperatures and other purposes.

Heaving in is accomplished by means of a hemp "swifter" or driving-belt, which conveys the motion of the drum of a donkey engine to the drum carrying the wire of the sounding machine.

It being impracticable to regulate the speed of the engine by hand according to the heave of the ship, in order to obviate the sudden and excessive strains on the wire so caused, an ingenious mechanical arrangement has been fitted to machines of recent pattern, by which frictional discs, geared by cog-wheels and capable of adjustment, are interposed on the axle connecting the grooved wheel actuated by the hemp swifter and the revolving drum carrying the wire.

By this arrangement the latter can be controlled as desired, both in speed and direction of motion, by means of a lever regulating a strap on the frictional discs, which may be set by experiment to act at any given tension of the wire. As the tension approaches this limit, the velocity of revolution of the drum is automatically checked; and if the tension further increases, the motion of the drum is actually reversed, thus causing the wire to run out, until the tension is relieved sufficiently to allow the frictional discs again to act in the direction of heaving in.

The drum may be stopped instantly by moving the lever in the proper direction to throw the apparatus out of gear.

The large machine is represented in Fig. 78, but further detailed description will not be given, as the type may be further altered.

The wire used is galvanised steel wire of 20-gauge. It is **Wire.** supplied on drums in 5,000-fathom lengths, which are sometimes in one piece, but often have a splice in them. The drums are in hermetically-sealed tins. Though galvanised, the wire requires looking after. The galvanising process is not perfect, and it may be thin in some places, and even actually bare spots may occur. The wire should therefore be passed through an oily rag as often as possible, and oily cloths kept on the machine to protect the outer layers from damp air. After a long sounding cruise it is probably safer to condemn the wire on the machine.

A fortnight's continuous use is about the limit to the trustworthiness of any kind of wire; no amount of care will prevent it from becoming brittle; and directly it can be snapped by twisting in the hand, it should be condemned and passed on to the boat's machines. A magnifying-glass will assist in examining its condition.

The wire is liable to cut grooves in the interior of the swivelling frame. A file must constantly be applied to smooth them down, or they will rip the splices. A roller of hard steel, underneath which the wire passes, and placed in rear of the swivelling frame, obviates this to a great extent.

The wire when new has a breaking strain of 240 pounds.

Smaller wire of 21-gauge has also been supplied, for the purpose of allowing a sufficient length to be on the reel for very deep soundings, but with the larger machines now supplied will probably be discontinued. Its breaking strain is 190 pounds.

Splices. Splices are made about 5 feet in length, one wire being laid round the other in a long spiral of about one turn an inch. The ends are soldered, and a seizing of fine wire laid over the end and for 2 or 3 inches up the splice. No end must project. Solder is then applied along the whole length of the splice. A third seizing can be placed in the centre.

Splices are the weakest points of the wire. They should be frequently examined, and their positions noted, so that, both in running out and heaving in, they may be eased round the wheels, with the guider nearly in the centre to avoid tearing.

**Sounding
Rods and
Sinkers.** For depths of 1,000 fathoms and under, the lead can be recovered, and no detaching rod is necessary. A lead of 30 to 40 pounds weight is suitable.

At a little risk to the wire, when sounding from astern up to that depth, the ship may go ahead directly bottom is struck, increasing her speed as the wire comes in; the great saving in time thus effected will often justify the increased risk of parting the wire.

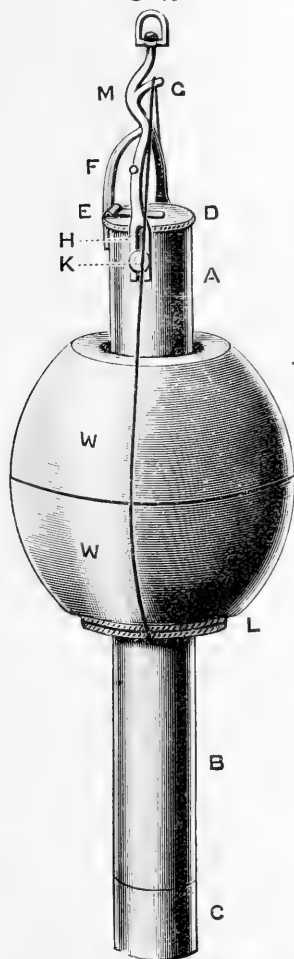
For greater depths two kinds of rods for slipping sinkers are supplied, the "Baillie" and the "Driver." Both are fitted with tubes to bring up a specimen of the bottom, and the same sinkers fit them both.

The construction of the "Driver" is easier than that of the "Baillie" rod, and with a piece of gas-piping cut to the proper length the ship's blacksmith can make one in a day; it is shown in Fig. 79.

A B C is a tube about 2 feet in length, fitted at the top with a flap valve, D, working on a hinge at E. The lower part of the

tube C screws on and off, and contains a double-flap valve to retain the bottom specimen. The sinkers W W, each 25 pounds in weight, conical in form, and pierced with a cylindrical hole through which the Driver rod passes loosely, are slung by wire

Fig. 79.



or cod line secured to a flat ring or grummet shown at L and passing over the stud G. A stud, K, on each side of the tube fits loosely into the slot H in the lower part of the slipping lever M H.

The weight of the apparatus being taken by the sounding

wire, the sinker remains suspended ; but on striking the bottom, the wire slackens, and the weight of the sinker drags the slipping lever down till the stud K bears against the upper part of the slot H. By this action the point M of the slipping lever is brought to bear against the upper end of the standard E F, being thereby forced outwards sufficiently to ensure that the weight acting at the point G will tilt the slipping lever right over, and thus disengage the sling. The tube being then drawn up, the sinkers are left behind. In descending, the valves at top and bottom, opening upwards, allow the water to pass through freely ; but on drawing up they are closed, thus retaining the plug of sand or mud with which the tube is filled.

For water under 2,000 fathoms, two conical weights are sufficient. In deeper water, a third cylindrical weight should be put between them.

Splicing
Wire into
Hemp.

It is important to have a piece of hemp line, some 10 fathoms long, interposed between the end of the wire and the lead or rod. This is for the purpose of preventing the wire from kinking when the lead strikes. A piece of sheet lead about 1 pound in weight wrapped round the hemp just below the junction keeps the wire taut, while the hemp slacks. To splice the hemp to the wire, lay the wire in the lay of the hemp for about 6 feet, putting on a good racking seizing of well-waxed twine at about every foot. Test this splice well.

Before splicing, the wire must be led from the reel of the machine, between the jaws of the guiding lever, through the hollow spindle of the swivelling frame, and over the registering wheel.

Winding
the Wire.

The wire must be carefully transferred from the drum on which it is supplied to the reel of the machine by mounting the former on a temporary spindle, and fitting a brake, by which the wire can be kept taut. Winding must be even, the wire passing through a piece of greased canvas in a man's hand.

Wire
Stoppers.

Small brass screw stoppers are provided for holding the wire, if necessary, during a sounding. These should be fitted with a hempen tail to make fast to a cleat or other fixture.

For the greater depths it is usual to sound from forward,

but some officers have successfully accomplished it from aft in bad weather. A projecting platform is fitted on the forecastle, to which the machine is bolted so as to plumb the water, being pointed in a direction slightly on the bow. An endless hemp swifter of 2-inch rope connects the deck engine and sounding machine. This is led through blocks to the forecastle, and so to the machine. One or two turns are taken round the drum of the deck engine, and the bight passes through a leading block with a jigger attached, which is placed abaft the deck engine. By means of the jigger the swifter can be kept to the requisite amount of tautness. The details of this arrangement will of course vary in different ships, and with individual tastes. Specially-made sister-blocks for guiding the swifter are now supplied.

Method of
working
Machine.

As the wire runs out, the regulating screw of the brake must be gradually screwed up, so as to increase the power of the brake in proportion to the amount of wire out. The regulating screw is marked for each 500 fathoms. In fairly smooth water the brake will at once act when the weights strike the bottom, and the reel stops.

When sounding in depths of less than 3,000 fathoms it is best to use only one spring, but beyond that depth two springs are required. The marks on the regulating screw are only intended as a guide; the real test is that the brake is just on the balance so as to act when the strain lessens, which may be known by the swivelling frame being just lifted off the stop. As the wire weighs $7\frac{1}{2}$ pounds for each 500 fathoms, the 500-fathom mark on the screw should be at the position in which the screw has to be to sustain a weight of $7\frac{1}{2}$ pounds; the 1,000-fathom mark 15 pounds, and so on. This can be tested and the marks verified.

A spring balance is supplied for attachment to the brake lever when heaving in, by which the amount of strain can be seen, and the deck engine worked accordingly.

It is necessary to establish some system of signals by which the officer on the forecastle, who is carrying out the sounding, can control the helm, main engines, and deck engine, both by day and night.

Signals

The signals given in Figs. 80 and 81 have been used with success for regulating the helm and engines for day work :

By night.

Green light for starboard helm.

Red light for port helm.

No light for amidships.

If lights are waved, hard over.

Fig. 80.

Helm. Red Flag.

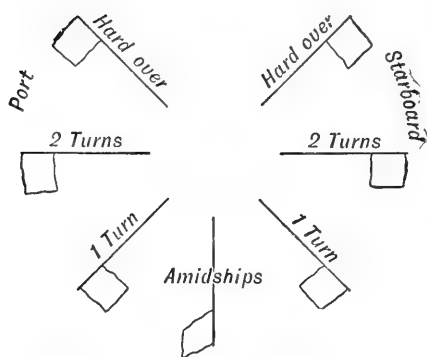
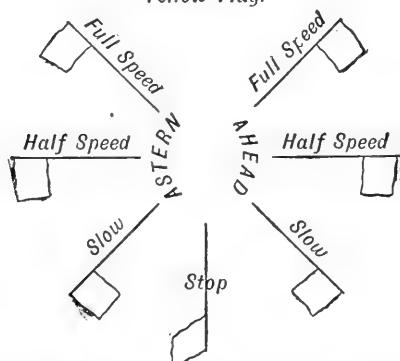


Fig. 81 gives signals for main engines :

Fig. 81.

Main Engines.

Yellow Flag.



By night, a white light in starboard fore rigging for ahead, and a white light in port fore rigging for astern ; the height indicates the speed.

For Deck Engine.

Blue flag held vertically downwards — Ordinary speed for heaving in.

Blue flag held horizontally—Slow.

Blue flag held over man's head—Stop.

By night, pass the word or arrange a bell near the deck engine.

See head-rails cleared away as necessary.

Have ready wire-stoppers, weights, sounding-rod, grummet or ring, and sling, oil-can and spanners, dish and spoon for collecting bottom specimen.

Preparing
for a
Sounding.

See that sounding machine is properly placed, and that the swifter runs fair, and is put round both deck engine and sounding machine wheel in the right directions ; jigger in place, but not taut. Place indicator at zero.

Hook the springs downward to brake lever, and see regulating screw set to zero, and screw brake screwed down. Wire guiding rollers must be turned back, by taking out the bolt, and slewing the rollers clear, so as to allow the wire to run clear. All parts of the machine should be well oiled, and the winch handles unshipped.

See that the wire is evenly wound, and taut.

Ease down the weights, using a stick with hook at the end to prevent a jerk, as the strain comes on the machine.

Letting
go.

Attend the screw brake, and ease down gently and carefully to the first 100 fathoms or so, according to the weather, after which the screw brake is no longer necessary, and may be lifted clear of the rim of the reel.

As the wire runs out, screw up the regulating screw of the brake.

When the bottom is reached the springs come into action, the reel will stop, and the depth can be read on the indicator. The length of stray hempen line, less the drift to the water's edge, must be added.

When bottom is struck, ship one of the winch handles ; then press the brake lever outwards to free the reel, and reel up some 10 or 20 fathoms as quickly as possible to get the rod off the bottom. If the sinkers have not detached, the effort required to reel up by hand will indicate that they are still on,

Heaving
up.

and the operation of letting go and heaving up by hand must be repeated until it is certain that the weights are off, using the screw brake each time for letting go.

When the weights are gone, screw down the screw brake, and holding the brake lever by hand, run out the regulating screw, disconnect the springs and hook the spring balance, stopping the legs of the springs loosely to the balance to keep them out of the way.

See the guiding lever in gear.

Bring to and set up the swifter, which may be ready in position round the grooved wheel, the latter being secured to the shaft by means of the T head screw when required to connect.

Unship the winch handle, and heave in by the deck engine, taking care not to heave in so fast as to bring a greater strain, as shown by the spring balance, than 100 or 130 pounds. In smooth water and depth less than 2,000 fathoms the strain may be as little as 70 or 80 pounds with the wire coming in at a good pace.

When the indicator shows 30 fathoms, stop the deck engine, and heave the remainder in by hand.

When the hemp appears at the surface, heave very slowly, and when up to the machine secure the reel by screwing down the screw brake, and lift the rod in by hand.

It is very important to guide the wire on to the reel carefully and evenly by the guiding apparatus. Wire badly wound is sure to develop slack turns on running out, and will probably kink and snap.

When the rod is up disconnect the guiding gear, and get everything ready for another cast.

During the whole of the operation the ship must be carefully looked after, in order to keep the wire up and down, or as nearly up and down as possible.

Manage-
ment of
Ship.

Though perfectly simple to effect after a little practice, inexperienced officers frequently part the wire or get erroneous soundings, and the following notes may be of service until experience is gained.

Sounding from forward, when there is little or no current, all sails are furled except the spanker, which should be set with the sheet to windward.

Keep the wind slightly on that bow on which the sounding

machine is fixed ; and never let the ship get more than two points off the wind unless a weather tide necessitates it.

Always endeavour to keep the position by small changes of speed and helm, avoiding high speed. This demands the closest watch, and the moment the wire is seen to be getting out of the vertical, apply the brake and steam her up with the helm in the necessary position. Bear in mind that the helm is of little use, even with slight headway, unless the screw is working.

If you have failed to catch her in time, act decisively with engines and helm before matters get too bad, using high speed if necessary. A few quick revolutions of the screw with helm hard over is the best plan, checking the headway thus unavoidably given by a back turn when she has sufficient swing on, remembering that going astern always causes the ship's head to fall off from the wind, but to a less extent if the wind is on that side to which the ship's head naturally turns in a calm on reversing the engines.

The worst position is when the wire gets under the bottom, as it may catch the copper, and everything must be done to prevent this occurring. It will generally happen from allowing the wind to get on the wrong bow. In such a case it will probably be necessary to steam her rapidly round till the wind is on the right bow, and then let her drop down until the wire is again clear.

Should the wire foul the copper, or, as it may do, the gangway wire used for serial temperatures, it is sometimes possible to clear it by getting out the lower boom and using long hook ropes of ordinary sounding line with weighted smooth hooks.

A surface current across the wind complicates matters considerably. In order to get an up and down sounding, the ship must be moved against the current, and the wind must then be more or less on the beam ; with a weatherly current the wind may be nearly aft. Under such conditions it requires a practised eye and hand to manage the ship.

It must be understood that unless the wire is nearly up and down, it may be very difficult to say when the bottom is reached, and the depth, as given by the wire out, will not be accurate. If wire is run out after the lead has reached the bottom, kinks will result, and the wire will part.

It not infrequently happens that the wire parts for no apparent cause, but this may be due to a kink in the wire from a previous sounding.

Unless conditions of current, as above mentioned, necessitate it, it is generally fatal to let the ship fall off broadside to the wind. Should she be allowed to get round with the wind aft, there is probably no remedy but to heave in again, and commence afresh.

Care is necessary in heaving in the last 50 fathoms, so as to stop the deck engine in time.

Time
occupied.

The time interval with wire, when not pitching heavily, up to depths of between 2,000 and 3,000 fathoms, is about one minute per 100 fathoms. Reeling in may be accomplished at nearly the same rate.

A sounding of 1,000 fathoms may be obtained in 25 minutes from the time the weight is lowered to the time the order is given to put the ship on her course. 2,000 fathoms will require 45 minutes, and 3,000 fathoms 75 minutes. Beyond that depth, much greater caution being required, the time occupied is correspondingly increased, and reeling in must be done very deliberately. The deepest sounding hitherto obtained is 5,269 fathoms. Soundings at such depths may occupy as long as five or six hours.

Though the time of running out each hundred fathoms is no longer required, as with hemp, for ascertaining when the sinkers strike the bottom, it is well to take the intervals, as they assist in the regulation of the brake.

Serial
Tempera-
tures.

If a second wire machine is available (a boat's machine will do), serial temperatures can be conveniently taken from the gangway whilst the sounding is being obtained forward, thus gaining time.

A 30-pound sinker is attached to the end of the wire, and the thermometers are secured to the wire by the metal clips at the back of the cases, at the required distances. See that the indices are down before attaching the thermometers.

There is a certain amount of extra risk in thus working from the gangway while the other wire is over, as the two wires may foul deep down, when the fact of the thermometers acting as toggles may make them difficult to clear. The time saved, however, justifies it in fine weather, and when

experience in sounding is gained. To avoid heavy loss, however, not more than four thermometers should be on the wire.

The temperature of the water is usually taken at intervals of 100 fathoms down to a depth of 1,000 fathoms, and at closer intervals in the first 100 fathoms.

Deep-sea soundings on every voyage are now a recognised Sounding
on
Voyages. part of a surveying ship's routine. It is only in this way that depths so useful for submarine cable, as well as for scientific purposes, can be accumulated without the expenditure of time involved in special sounding cruises, though those are occasionally necessary. As a rough rule, a sounding after daybreak, and before sunset, should be obtained daily, when observation can be got.

Connected with deep-sea sounding, though not such a common Dredging. part of a surveyor's duty, is dredging, on which a few words may be useful.

The dredge consists of a strong iron frame, the sides forming lips, which are connected at each end by an iron bar, and are chamfered off to fairly fine edges. These edges slightly incline outwards, as seen in the sketch. On the iron bars arms are fitted, and to the eye at the extremity of one of them the dredging hawser is bent, the eye of the other arm being seized to it to form a span of such a strength that the seizing will carry away if the dredge catches.

Attached by wire seizings to holes in the lower part of each lip is a stout canvas bag, perforated with holes in its lower portion to permit the water to flow through.

A stout iron bar, to which three long swabs are secured, is suspended by ropes from the iron end bars immediately below the canvas bag.

Dredges are of various dimensions, but a convenient size is as follows, as illustrated by the sketch Fig. 82 :

A and B, arms, $2\frac{1}{2}$ feet long.

C, hawser.

D E and F G, lips of the dredge, $2\frac{1}{2}$ feet long.

H, holes to which bag is laced.

I I, perforations in the canvas bag.

K K, swab bar.

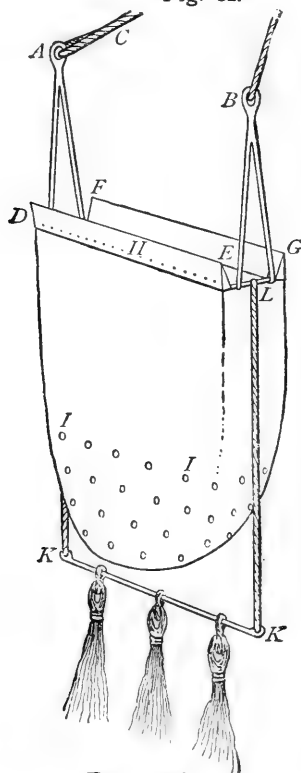
L, bar of dredge mouth, 6 inches.

The hawser is weighted with about 60 pounds, at 5 fathoms from the dredge.

On a sandy bottom, a net bag is substituted for the canvas bag, which gets full of sand.

On a rough bottom an iron triangle carrying swabs only is used, the arms being stopped lightly together so as to carry

Fig. 82.



The Dredge.

away, if caught ; or even one large swab at the end of a rope, weighted 2 feet above it to keep it down. These are especially useful on coral banks, where a regular dredge may very likely be lost.

To dredge, turn the ship away from the wind or current, and drop the dredge from aft with slight headway on, taking care that the bag and swabs do not capsize and foul the mouth of

the dredge. Ease out the hawser to about three times the depth of water, and let the ship drift for about 20 or 30 minutes. The dredge can be hoisted in by a burton from the mizzen gaff.

NATURE OF OCEAN BOTTOM.

The deposits on the floor of the ocean may be classed under the following heads :

1. *Shore Deposits*.—Within a distance of about 200 miles from land the deposits partake of the nature of the coast, thus : Around volcanic islands the deposits are grey or black in colour, and consist chiefly of pumice, scoriæ, and volcanic sand. Around coral islands the deposits are white, and consist of the detritus of the neighbouring reefs ; whilst in the vicinity of land which is not volcanic or coral the deposits are usually blue or green muds, and consist chiefly of the detritus of rivers and washings of the coast. These latter contain frequently some surface shells and diatoms. The green muds are especially interesting, as they generally contain some beautiful internal casts of carbonate of lime organisms in glauconite.

2. *Globigerina Ooze* is widely distributed over the bed of the ocean. It is of a white or light brown colour and sticky nature, and consists chiefly of minute globular shells of carbonate of lime, called globigerina. It is easily known by its appearance under the microscope, and from the fact of its effervescing strongly when treated with dilute hydrochloric acid. It has occasionally been found at a depth of 2,800 fathoms, but is usually purest at 2,000 fathoms.

3. *Pteropod Ooze* is somewhat similar to globigerina ooze, but consists of the shells of animals (Pteropods), which can be seen without artificial aid. It also effervesces when treated with dilute acid, but generally will be found at depths under 1,200 fathoms when little or no land débris exists.

4. *Diatom Ooze* is of a white or rose colour, and is chiefly composed of the silicious casts of minute plants (Diatoms). It effervesces but slightly, if at all, when treated with acid, and has hitherto been found only in the Antarctic Ocean.

5. *Radiolarian Ooze* may be white or brown in colour, and is composed chiefly of the skeletons of minute animals (Radio-

laria), intermixed occasionally with a few globigerina shells. When so intermixed it effervesces slightly with dilute hydrochloric acid, but when pure it does not effervesce. The skeletons of these radiolaria are beautiful objects under the microscope. This deposit has hitherto been found only in the Pacific and Indian Oceans, at depths exceeding 2,300 fathoms.

6. *Red, Grey, or Chocolate Clays* are widely distributed over the floor of the ocean at depths exceeding 2,100 fathoms. The red colour is due to the presence of oxide of iron, and the chocolate colour to peroxide of manganese. Pumice stone, manganese nodules, sharks' teeth, and the ear-bones of whales, have frequently been found embedded in the clays, which do not effervesce when treated with acid, unless, which is occasionally the case, they have a slight intermixture of globigerina ooze.

General Remarks.—All oceanic deposits contain small black, red, and yellow magnetic particles, but these are not abundant in the clays. Should the sounding tube come up empty, as it does occasionally, though rarely, it should be carefully examined on the outside for black-brown streaks, as these indicate the presence of oxide of manganese at the bottom in the form of nodules or stones too large for the tube to bring up.

As it is important to describe the substances brought up in the sounding tube by terms readily recognisable, and as the foregoing were adopted after much consideration by the scientific staff of the *Challenger*, it is recommended that surveyors employed in deep-sea sounding should endeavour so far as is practicable to follow this nomenclature.

SUBMARINE SENTRY.

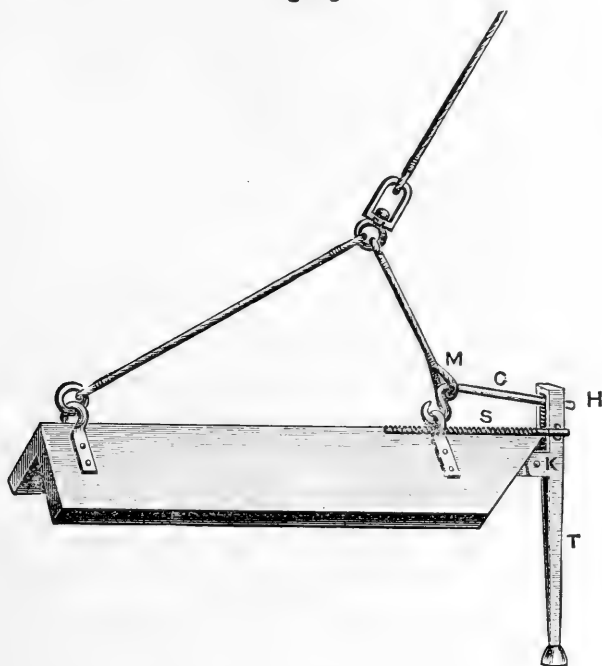
The use of the submarine sentry has already been referred to on p. 216, in connection with searching for vigias. It supplies an automatic warning of the approach of a ship to shallow water, and a description of the apparatus is here appended.

By means of a single stout wire, the sinker, an inverted kite, called the "sentry," can be towed steadily for any length of

time, at any required vertical depth down to 40 fathoms with the red kite, and 30 fathoms with the black kite. Should it strike the bottom, through the water shallowing to less than the set depth, it will at once free itself and rise to the surface, simultaneously sounding an alarm on board, thus giving instant warning.

The vertical depth at which the sentry sets itself when a given length of wire is paid out is not changed by any variation of speed between 5 and 13 knots, and is read off on

Fig. 83.



the graduated dial-plate on the winch. One set of graduations on the dial indicates the amount of wire out; the other two sets refer to the red and black kites respectively, and show the depth at which the sentry is towing.

By this machine single soundings down to 40 fathoms can be taken at any time while the ship is under way, the sentry being let down slowly. The gong will indicate when the bottom is touched, and the dial corresponding to the kite used will show

at once the vertical depth at the place where the sentry struck. The winch is secured to the deck a short distance from the stern ; the towing wire passes from the drum under a roller-fairlead at the foot of the winch, thence through an iron block with sheaves of large diameter, suspended from a short davit on the stern rail, and secured to the sling of the sentry.

The dial being set to zero, with the sentry at the water's edge, the ship's speed is reduced to 8 or 9 knots, and the wire paid out freely until the kite is fairly in the water, when the brake should be applied steadily and without jerking, veering slowly until the required depth is attained, when the pawl is put on the ratchet wheel and the speed increased to 12 knots, if desired, when using the black kite, or 10 knots with the red kite.

The kite in its position when being towed is indicated in Fig. 83. The point of the catch C, passing through a thimble M in the short leg of the sling, is slipped into the hole at the top of the trigger T, which is hinged at K and kept in its place by the spring S attached to the hook H. On the trigger striking the bottom the catch is released, the short leg of the sling slips off, and the sentry, which then rises to the surface, is left towing by the long leg.

The winch is fitted with two handles for heaving in the wire ; one gives great power and slow speed, and the other, acting on the drum spindle direct, winds in quickly. The wire supplied with the machine has a steady breaking strain of about 1,600 pounds. Using the black kite at a speed of 7 knots, the strain on the wire is about 150 pounds, and at 10 knots about 300 pounds. The red kite increases the strain largely.

CHAPTER XIX

MISCELLANEOUS

Distortion of Printed Charts—Sailing Directions—Observations on Under-
Currents—Pillsbury Current Meter—Exploring a River—Swinging Ship.

IN printing charts from an engraved plate, the paper has to be damped. This results in distortion on the sheet drying, and angles laid off on a published sheet will never be found to agree exactly, especially if the sheet is large. This must always be borne in mind, in trying angles on a published chart.

Distortion
in Printed
Charts.

For this reason, when a published plan is to be examined, a dry "proof" is supplied to the surveyor from the Admiralty. This is an impression "pulled," as it is termed, on to a dry sheet. It is much fainter than a damp-pulled copy, and would not do for ordinary use; but being an exact facsimile of the copper plate, all angles, bearings, etc., should agree precisely, if the original survey is correct.

This fact of the distortion of published charts is not generally known, and many reports of so-called inaccuracies have been made in ignorance of it. The amount of it varies with the goodness of the paper, and the trouble bestowed by the printer in damping his paper uniformly. It is a fact much to be deplored, and the man who invents a means of obviating it will bestow a great boon on cartography.

No survey is complete unless accompanied by sailing directions. Notes for these should be continually accumulating during the course of the survey, and the directions themselves should be completed immediately after the work in the field, when every essential point is fresh in the memory. The instructions for writing sailing directions issued by the Hydrographic Department, Admiralty, should be consulted.

Sailing
Direc-
tions.

OBSERVATIONS ON UNDER-CURRENTS.

Though not in the ordinary run of surveying operations, a slight description of the method of discovering the direction and approximate rate of under-currents may be useful.

To ascertain these satisfactorily, special gear is necessary.

General
Principle.

The general principle is to expose a large surface to the action of the under-current, and to support this in the water by a floating buoy which will present as small a surface as possible to the action of the surface stream.

The experiments must be carried on from boats, and therefore the gear must be as light as possible, for easy handling.

A series of observations on the under-currents in the Bosphorus and Dardanelles resulted in the author's adopting the following :*

Appar-
atus used.

A light, flat wooden board, 6 feet square, with a wing 2 feet in length, at right angles to the rest of the frame, was used as the submerged drag (Fig. 84).

To the extremities of the wing the sling, *a a*, was made fast, and to this sling the supporting line to the buoy was bent, at such a point as kept the surface of the drag vertical when the strain came on.

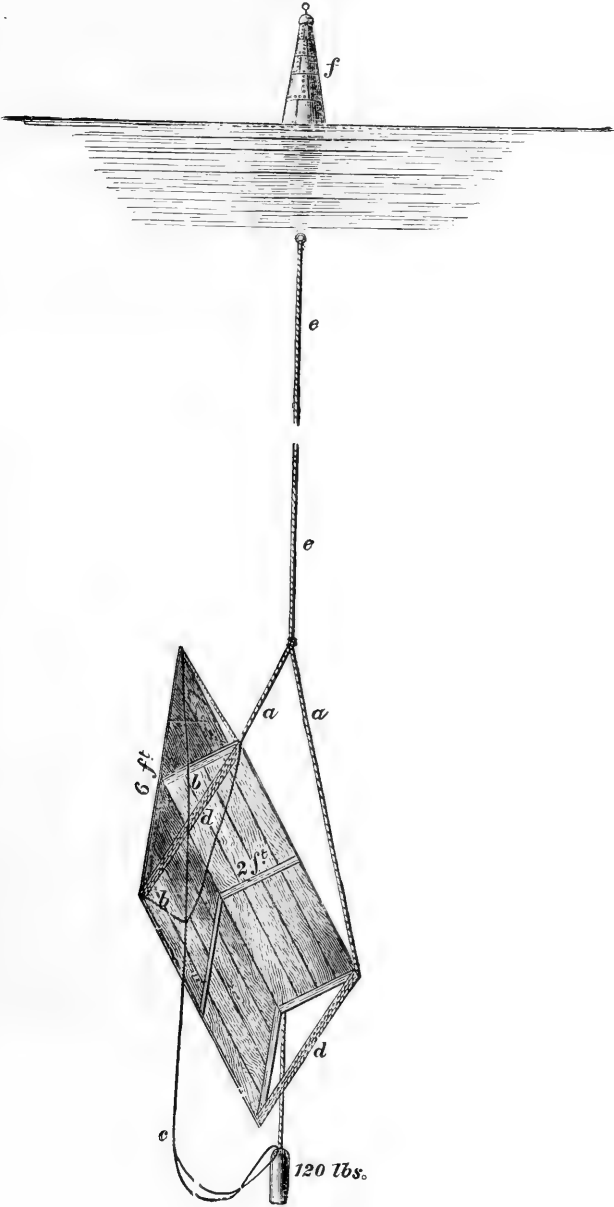
It weighed 70 pounds in air, and took 120 pounds of lead to sink it satisfactorily. These leads were made fast with a little drift, and another line, *c*, was bent, both to them and to the lifting sling, *b b*, so that the weight of the leads could be taken off the drag, when pulled up to the surface, before finally hoisting it into the boat.

An iron buoy, 1 foot in diameter and 5 feet long, supported this structure well when the surface current was not very strong, and only presented an area of less than 1 square foot to pull through the water.

When the surface current was swift, other buoys had to be added, attached in line to the upper end of the first, for under these circumstances the single buoy was dragged under water, and its motion could not be followed. Several disappeared in this way, some to reappear when the apparatus got into slacker water, some for good and all.

* "Observation of Currents in Dardanelles and Bosphorus."

Fig. 84.



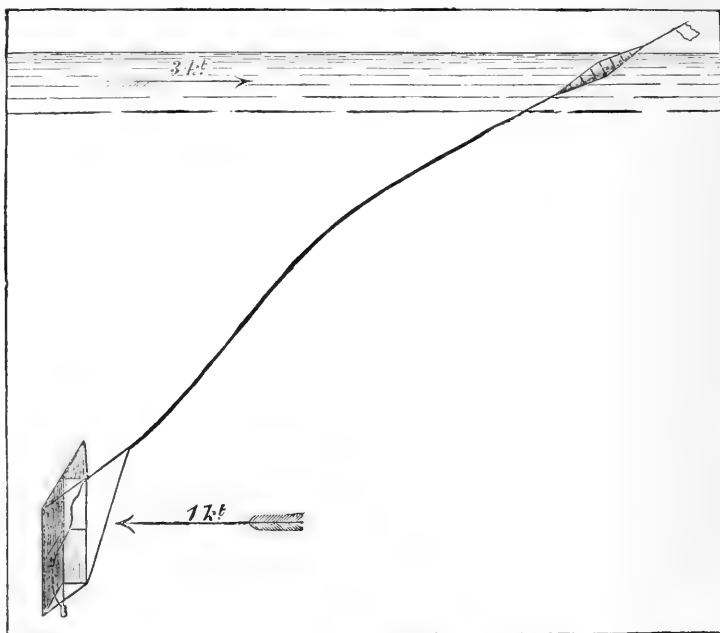
Method of
ascertain-
ing Rate
and Direc-
tion.

To ascertain the movement of the floating buoy, and therefore the direction in which the drag was carried by the under-current, a fix was taken to shore objects, and plotted on a large-scale sheet of points, when the drag was let go free from the boat. Subsequent fixes and times taken enabled the course and distance of the buoy in the intervals to be recorded on this sheet.

Surface
Current.

A small buoy, weighted so as to float awash, put into the

Fig. 85.



water at the same spot and time, and followed by another boat, afforded means of ascertaining the surface current.

Defects.

This arrangement worked very satisfactorily altogether, but there are several defects in it.

The depth of the submerged drag will not be the length of the line allowed, but some unknown quantity less, as will be seen by the accompanying sketch (Fig. 85.)

This must be estimated.

Rate less
than True
Rate.

The force expended in dragging the buoy through the surface water, and overcoming the friction of the suspending line,

is also an unknown quantity, but will always have the effect of retarding the motion of the submerged drag. The rate therefore recorded, by the movement of the surface buoy, will always be *less* than the true rate of the under-current.

We do not imagine that the apparatus described above may not be much improved upon, but we give it as a starting-point for any officer employed in future investigations of a similar character.

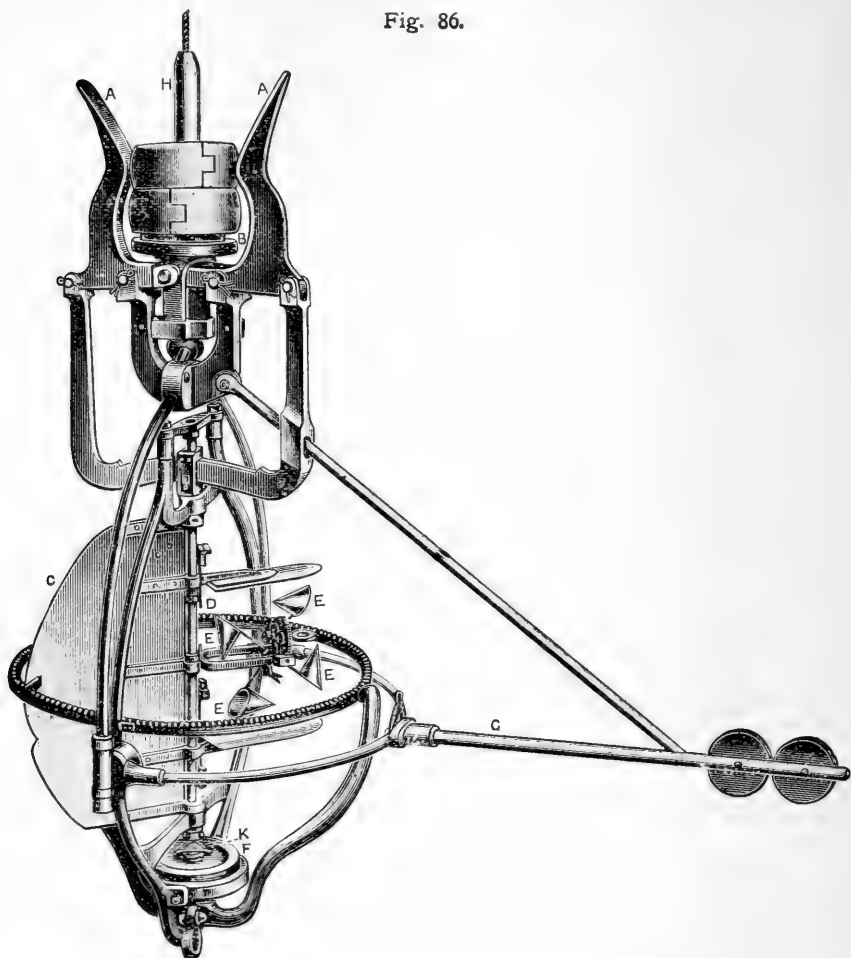
Several instrument makers now turn out "Current Meters" of various forms. Doubtless these could be, with a little ingenuity, adapted to sea-work, at least to show the true *rate* of an under-current. Current
Meter.

A deep-sea current meter, devised (1876) by Lieutenant Pillsbury, U.S.N., has, with several modifications, been used with success on many occasions, notably by the U.S. Coast and Geodetic Survey steamer *Blake* in the investigation of the Gulf Stream. The instrument is first lowered to the required depth, and when ready is put into action by means of a heavy weight, or messenger, travelling down the supporting line and striking on a metal plate, thus closing the jaws of the levers and enabling the instrument to begin working. The rudder is then free to revolve inside the framework and take up the direction of the current; the small cones can revolve on their axis and register the number of revolutions, while the compass needle is released and free to take up the north and south line. On the despatch of a second messenger, which strikes on the top of the first and forces the jaws of the levers open, every part of the machine is simultaneously locked. Having noted the exact time of starting each of the messengers, the time during which the instrument has been working at the required depth is known, and from this the velocity of the current can be calculated, the number of revolutions having been recorded, while the direction is shown by the angle between the compass needle and the direction of the rudder. Pillsbury
Current
Meter.

The instrument is shown in Fig. 68. A A are the jaws of the levers through which the first messenger passes and strikes on the metal plate B. The force of the blow is sufficient to press B down, thus bringing the jaws as close together as possible, and putting the meter into action. The second messenger, falling on the first, opens the levers again, and prevents

their closing, thus keeping all parts of the machine locked. C is the rudder which takes up the direction of the current when the levers are unlocked. D is a set of small levers on the rudder in connection with A A. The outer end of the tail

Fig. 86.



of the rudder fits into the notches on the outer ring of the frame when the machine is locked, and thus keeps the rudder fixed; but when the first messenger has started the machine by pressing down B and opening the levers A A, this small lever is raised, and the rudder can revolve freely.

E E are four small cones, which revolve on their axis in a vertical plane similar to an anemometer. The axis is connected by a worm-screw to geared wheels, which register the number of revolutions up to 5,000, corresponding to about four nautical miles. There is a small lever in connection with A A which prevents the cones from revolving when the machine is locked, but allows them to revolve freely when the meter is in action. Below the rudder-post is a compass-bowl F, which is hung in gimbals, and capable of removal. The needle is so arranged that it can be lifted off the pivot by means of a lever in connection with A A. When the meter is in action the needle swings freely on its pivot, but when the levers are locked it is raised off its pivot by the inverted cup-piece K placed inside the triple claws on the top of the compass, and screwed to the lever, thus locking the needle without chance of moving. The compass-bowl should be filled with fresh water before lowering the instrument into the sea, and the top screwed home tightly. The needle should be removed and carefully dried after use, to prevent corrosion. The long arm G is to keep the machine steady in one direction. It works up and down a jackstay, which passes between two sheaves at the extremity of the long arm. This also assists to keep the machine in as upright a position as possible, and prevents it from being drifted astern with the current. A weight of as much as 8 or 10 hundredweight is required at the bottom of the jackstay in a very strong current. An elongated weight of from 60 to 80 pounds must be suspended from the eye at the bottom of the meter to help to keep it as vertical as possible. On the outer part of the horizontal notched ring forming the frame, and placed on the side of the machine opposite to the projecting arm G, it has been found necessary to bolt a short arm supported by stays from above, from which is suspended a leaden counterpoise weight to assist in keeping the apparatus upright. This additional fitting is not shown in Fig. 86. A three-quarter-inch phosphor-bronze wire rope is used for lowering the machine. It is rove through a metal sheave H and india-rubber washer, and spliced round a heart which is attached to the metal plate B. The messengers are fitted with a hinged joint to enable them to be placed round the wire rope, and secured with a screw bolt. To obtain the exact value of a revolution of the small cones,

it is necessary to make experiments when the actual speed of the current is known, by immersing the meter just below the surface and taking careful observations of the surface current by means of a current log or weighted pole. From the number of revolutions registered by the meter in a certain number of minutes, and taking the mean of several observations, a very fair value for a revolution can be deduced. On every occasion of using the meter for under-current observations, the value of a revolution should be redetermined, as it is apt to vary owing to small differences in the friction caused by want of oil or the presence of dust or grit; the force of the current is probably another important factor in determining the value of a revolution.

Rough
Estima-
tion of
Under-
Current.

In smooth water, a very fair approximation of the strength of an under-current may be arrived at by the aid of a steam-boat and wire sounding machine. The lead lowered to different depths being acted on by the under-current, whilst the surface current has little or no effect on the wire, the rate of the boat steaming to keep the wire up and down, as compared with the rate at which the surface current is running, will enable the under-current to be estimated.

Tempera-
ture and
Density.

The observations are made more complete by ascertaining the temperature and density of the water at the depths experimented on.

EXPLORING A RIVER.

Running
Survey.

Narrow rivers, navigable for boats, will generally be sufficiently laid down on a marine chart by a sketch survey, made from the boat (a steam pinnace, if possible), while passing up and down. Patent log and compass will be the instruments mainly used for putting down the direction and length of each reach; though if we have objects that we can use for a sextant fix, we shall of course use them in preference, at any rate from time to time. We must endeavour in every case to get a good fix at our furthest point, and the course of the river, as mapped by patent log and compass, will then be squared in on that, and the fixed points at the entrance and any other fixes we may have got. Any elevated points near, which we can ascend and fix, and from them get angles to bends and

reaches of the river, will much assist us, especially when, which is so often the case, the river is thickly lined with trees and jungle.

The patent log will be fitted, as already described, with the dial on the gunwale and the fan towing astern. Theodolite legs standing in the stern-sheets make an excellent stand for a prismatic compass, and enable us to get a better bearing than by holding it in the hand.

It is in rounding the bends that the greatest error in mapping a river is introduced, as the distance run over while gradually altering course must be estimated by eye, which requires considerable experience at judging distances.

Current must be taken into consideration, and may be obtained, if time allows, by anchoring the boat for half an hour, and reading the patent log, or in shorter time, by heaving the current log. Current.

In a river where the tide extends some distance up, and where the land is low and jungly, as in so many mangrove rivers, our difficulties are much increased, as the velocity of the current will be constantly varying, and we cannot hope to obtain any sextant fixes to check our position. In cases of this kind, if it is desired to have any degree of accuracy in the sketch, the only way is to run over the work again, making an independent map, and squaring in afterwards a mean of the two.

It is best always to plot as we go. Mistakes are thus rendered less likely, and the vexed question of the bends can best be solved by placing their shape on the paper at once. We can also look at our work on the way down again, and correct little inaccuracies more readily. Plotting
in Boat.

If it is desired to make a large-scale plan of a river of greater width, the best method is to employ several boats at once, four if possible, which will triangulate their way up, two on either side. Starting from two fixed points at the mouth of the river, two boats will remain there while the other two go up to convenient positions, whence they can see the boats remaining at the first points. Angles will then be taken from all, to everything conspicuous, and to one another, and the lower boats will, leaving marks at their old stations, move up to two new positions above the other boats, when the angles will be repeated, and so on, the lower boats moving on each time. Survey on
larger
Scale.

The shore line can either be sketched by the boats as they go up, or done afterwards more correctly when the marks are all up and fixed. Soundings, in the same way, can either be taken from the boats as they move from station to station, in which case they would cross over each time so as to get a diagonal line across the channel, or can be more regularly taken afterwards, as the circumstances of the case may require.

Everything must be plotted afterwards, and communication between the boats as they pass one another, when names can be given and objects pointed out for mutual observation, will greatly facilitate the comprehension of one another's angles, when putting down the points.

SWINGING SHIP.

Though the compass is but little employed in surveying, it is occasionally unavoidably brought into use.

As deviation varies with lapse of time and change of latitude, it must be constantly ascertained by swinging ship.

The methods in use in swinging ship are well known, but perhaps a repetition may not be thrown away.

They are two in number :

One, by observing the compass-bearing of a distant object whose true magnetic bearing is known.

The other, by reciprocal bearings of the compass on board, and another on shore.

By distant
Object.

The first is the best and simplest when the magnetic bearing of the distant object can be well determined.

The object should be, at the least, six miles distant, and the more the better.

Its bearing can be obtained from observations on shore from a spot in line with the ship, or by true bearing with known variation applied.

Objects for this purpose are sometimes indicated on the charts or in the sailing directions, and the bearings given.

The deviation, for each position of ship's head, is then the difference between this fixed magnetic bearing and the observed bearing by compass.

The ship can either be hauled round with hawsers, at anchor,

or, if the object be far enough off, can be steamed round a circle small enough to make no difference in the bearing.

If steamed round, it is well to repeat the operation, turning in the opposite direction, as the compass may partake of the swing of the ship, which will introduce error. The mean of the two will then be the bearing to use.

The second method is perhaps the one generally employed, and is very convenient with a theodolite at hand. By reciprocal Bearings.

An officer is landed with azimuth compass and theodolite.

He obtains the bearings with the compass of some well-defined object, and setting up his theodolite, takes it for his zero.

In arranging the theodolite on zero, it saves calculation to point the degree and minute of the magnetic bearing to the zero instead of 360° . Thus, if the zero bears by compass S. $44^\circ 20'$ E. (supposed to be unaffected), set the vernier to $135^\circ 40'$. The angles read to the ship will then be the angle east of the magnetic north.

A flag on a long staff is held behind the theodolite, when all is ready.

The ship, under steam, and with a flag placed exactly over the standard compass, steams slowly round, hoisting a large flag close up to the masthead just before the ship's head comes to each point, which is dipped at the moment of observation, when the bearing of the shore station is taken.

The flag on shore is dipped, to show that the angle of the flag over the compass has been obtained by the theodolite, and is again shown as a response, when the flag is mastheaded for the next observation.

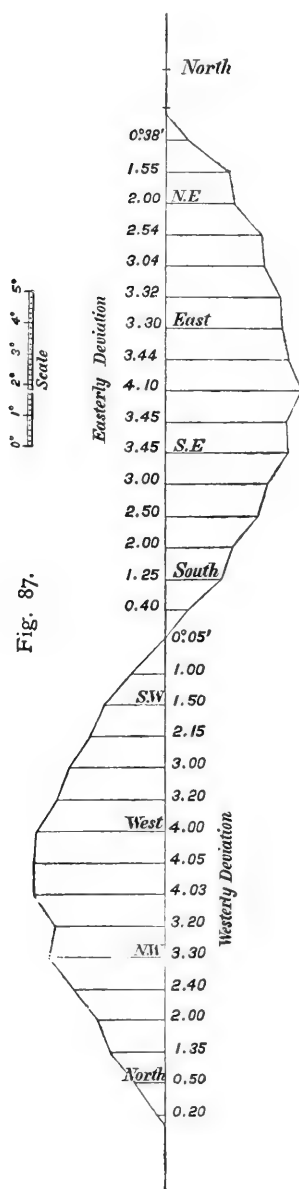
The time of each observation is taken by previously compared watches.

In this case, too, the ship should be swung in the opposite direction, if it is deemed necessary.

The difference of the reciprocal bearings is the deviation at each observation.

If more than one observation at any or all points has been obtained, the results are meaned for the accepted deviation.

It is usual to observe at every point of the compass, for the ship's head, but in some vessels it may be necessary to subdivide this.



The readiest way of examining the results of our observations is by use of the Graphic method (Fig. 87).

Drawing a long line, measure off equal parts along it, for the points of ship's head, and at each point on this normal lay off, at right angles, a line equal to the degrees and minutes of the deviation, on any scale we choose—easterly deviation to the right, westerly to the left of the normal.

Lines drawn through the extremities of these abscissæ will denote the curve of deviation observed.

By the irregularities of this curve, we can judge of the correctness of the observations very fairly ; and for our final table of deviation, we can draw a mean curve, if there are many irregularities, and measure to that for the amount of deviation for each point.

The valuable results for variation obtainable from swinging have already been mentioned at p. 361.

SUPPLEMENTAL

SIMPLIFIED METHOD OF SHIP SOUNDING.

Fig. 88.

Fig. 88.

Depth line, $\frac{3}{4}$ " Extra special flexible galvanized wire

Shackle 3" 2052

Thimble Patt. 679 welded

2 parts $\frac{3}{4}$ "

Thimble Patt. 673 welded

Shackle 3" 2052

Thimble 678 welded

112 lb Lead

Shackle 3" 2052

Swivel $3\frac{1}{2}$ " brass (647 B)

Fore and after 1" wire

422

The other end of the fore and after, 100 fathoms long, is rove through leading blocks on the lower boom and at the masthead, and secured to the sounding winch, on which it is reeled up. The other end of the depth-line is rove through a leading block on the sounding spar, through another block on a davit placed a short distance before or abaft the sounding spar, through a block shackled to a counterpoise 112-pound lead, through a block on the submarine sentry davit, placed as most convenient, and reeled up on the submarine sentry winch.

The leading blocks on the lower boom and sounding spar should project at about equal distances from the fore and aft line of the ship.

The general arrangement is shown in Fig. 89.

In depths exceeding 20 fathoms, and at a speed of $3\frac{1}{2}$ knots, it is necessary to heave the lead up to the lower boom: in lesser depths it need not be hauled so far forward.

The lead being hove sufficiently forward to "let go," the winch is revolved rapidly in the direction of "heaving out"; the leadsman, assisted by the action of the counterpoise lead, gathers in the slack of the depth-line, and calls the depth as the lead, resting on the bottom, passes him with the depth-line up and down and taut.

The effect of the counterpoise is to make the working of the depth-line practically automatic.

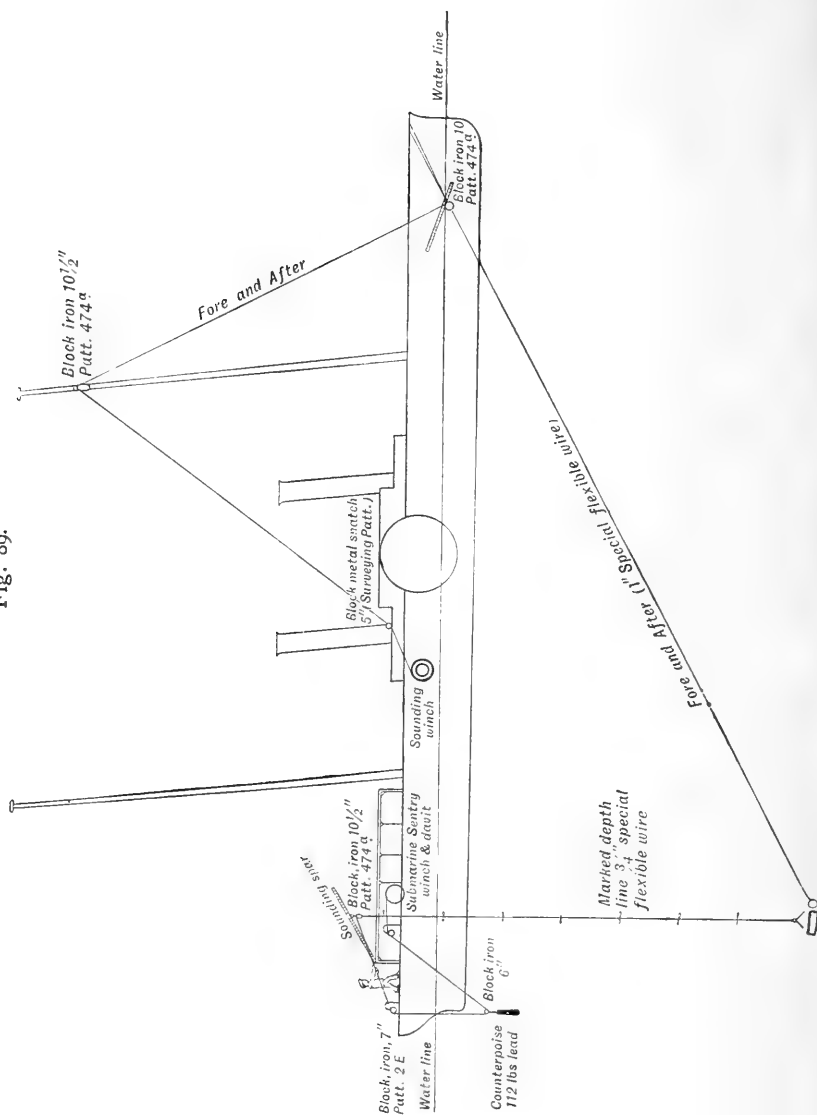
It is necessary, when the depths change much on a line of soundings, to adjust the position of the counterpoise accordingly; this is easily and quickly done by means of the submarine sentry winch, on which the depth-line is reeled up for that purpose.

In the *Research*, with no screw propeller to consider, the counterpoise hangs over the stern; in a screw vessel it would probably be desirable to take the counterpoise sufficiently far forward to avoid risk of fouling the propeller.

It is claimed that the rapidity with which soundings can be obtained by this method renders profitable the work of sounding from the ship on large scale work such as the 6.9 inch scale common in English waters.

The nature of the bottom must be obtained by other means, such as a boat's sounding machine with "snapper" lead.

Fig. 89.



ROLLER SOUNDING.

BY CAPTAIN B. T. SOMERVILLE, R.N., H.M. SURVEYING SHIP
"RESEARCH."

This method of obtaining soundings, by which the depth is continuously recorded, has been found to be a valuable one so long as its natural limitations are not exceeded. These include:

1. A speed of ship not exceeding $3\frac{1}{2}$ knots.
2. A depth not exceeding 20 fathoms.
3. A smooth sand or mud bottom.
4. A tidal stream, or wind of such a force that the ship will not have her head directed more than about 25° off the course she is making good.

It would prove most valuable, for example, in the sounding of banks in river estuaries, or on the East Coast of England; and provided the bottom was sufficiently smooth, and the tidal stream and weather suitable, would be ideal for sectional sounding work.

A roller, of the form of an ordinary garden roller, is towed along on the bottom by a bridle consisting of two wire ropes, coming from blocks near the ends of the two lower booms. General Description (see Fig. 90).

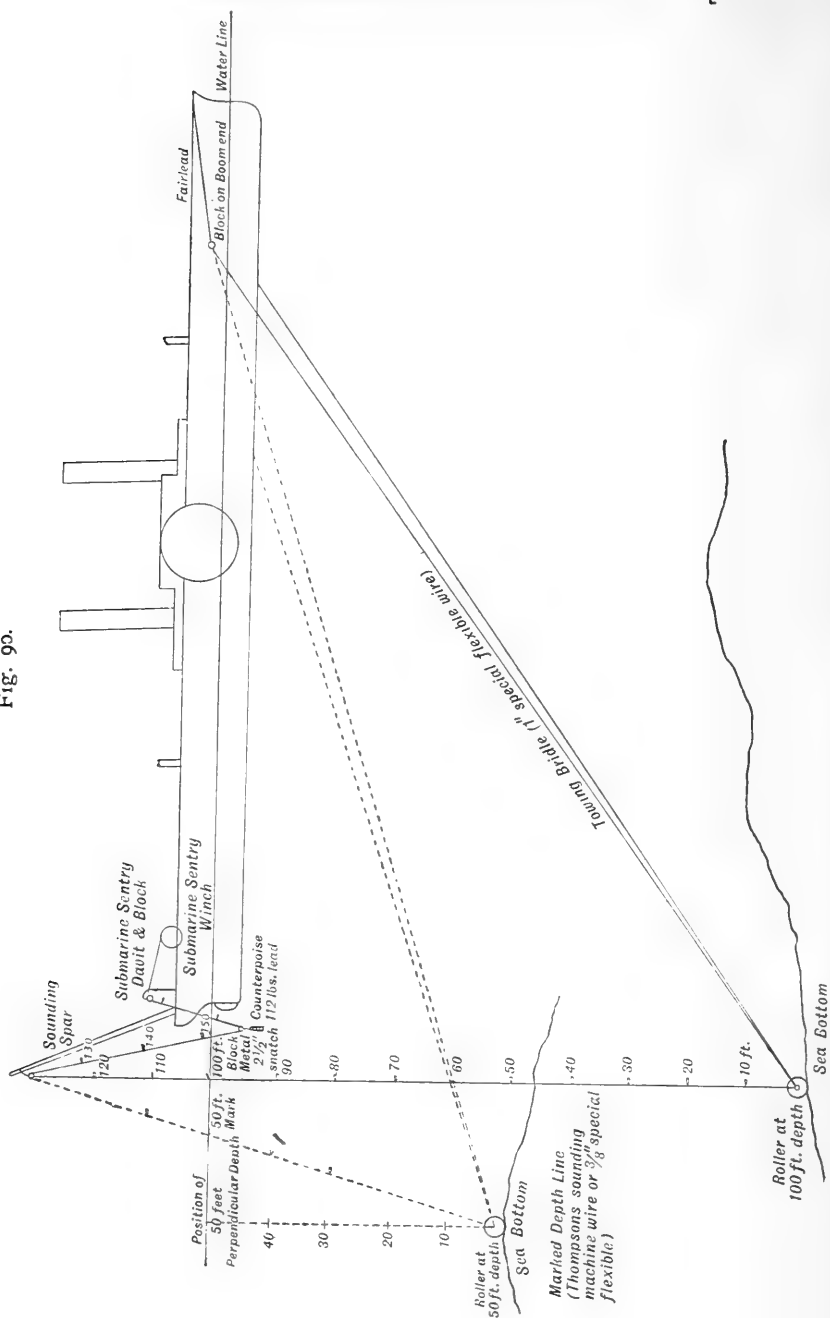
A marked depth-line, of fine wire, is attached to the roller, and comes from there to the surface, being rove through a block at the head of a spar over the counter of the ship, and kept taut by means of a counterpoise weight.

The bridle is arranged of such a length that the roller shall be perpendicularly beneath the head of the sounding spar aft when the depth is 100 feet. As the towing lines are secured at this permanent length, it follows that for all depths less than 100 feet the roller will be abaft the perpendicular, and for all depths over 100 feet will be before it.

The depth-line is, therefore, at a certain angle from the perpendicular for each particular depth other than 100 feet; and a calculation is thus necessary to find the position for the marks on the depth-line so that it may show the perpendicular depth over the roller. Fig. 91 shows the problem of the calculation, which, as will be seen, is a simple one. It is readily solved by protraction from calculating paper, with the conditions drawn on it to scale.

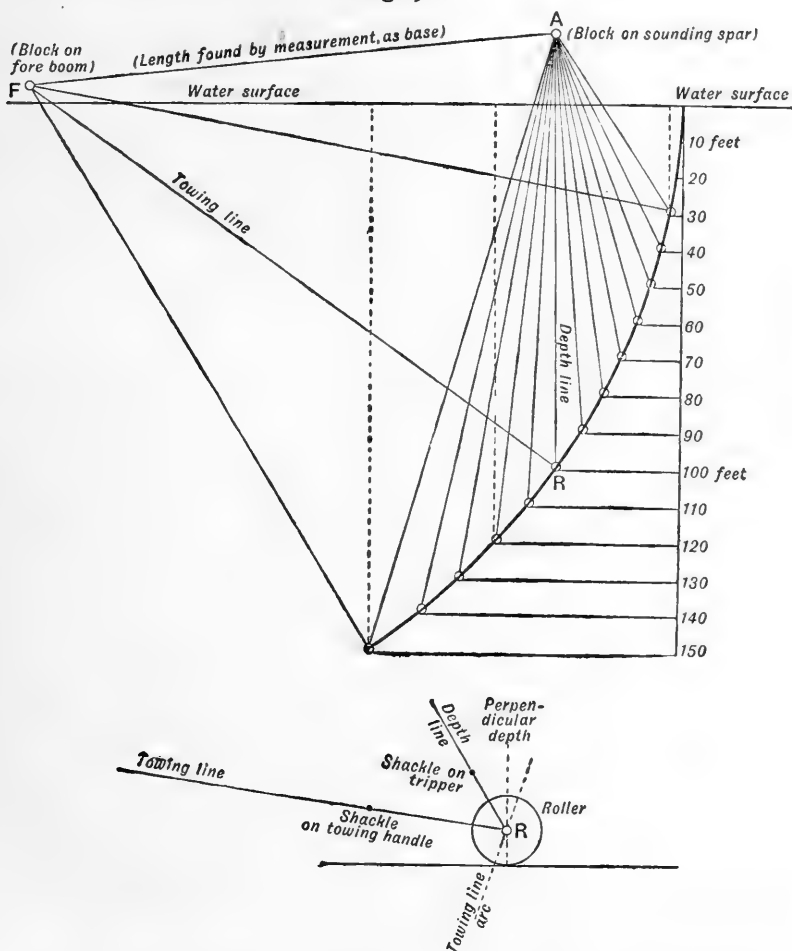
It will be observed that it is better to have the blocks on

Fig. 90.



the lower booms as low as possible, and the block for the depth-line as high as possible, in order that the horizontal travel of the roller from the 100-foot perpendicular position may be as small as possible.

Fig. 91.



If required, this can, however, be allowed for in plotting when the scale of the work renders it necessary.

As fitted in the *Research*, when the depth is 30 feet, the roller is at a horizontal distance of 29 feet abaft the perpendicular,

and is 16 feet before it when the depth is 120 feet: the correction is not, therefore, noticeable, except on a very large scale of plotting.

Method.

The roller (see Fig. 92), with both towing lines attached, is hoisted out with a burton, on a slip toggle; the ship is started at slowest speed, and the roller is lowered from one lower boom by its bridle, the opposite bridle being eased simultaneously, until both bridles are at the marks indicating that the roller is at its proper position, and are then secured permanently. The depth-wire requires careful attention as the roller comes aft,

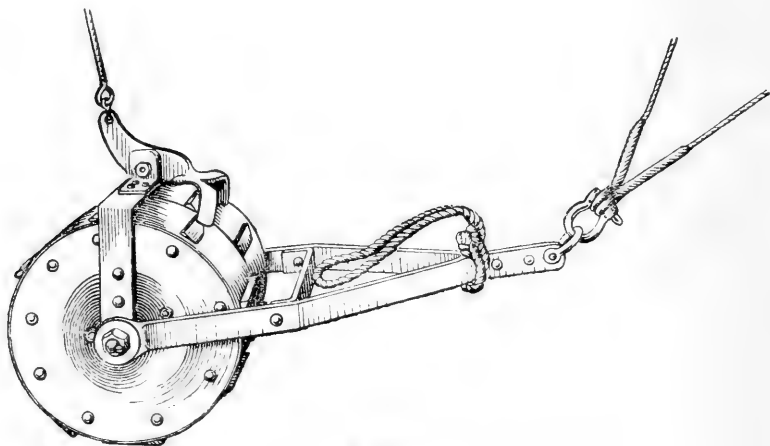


Fig. 92.

and the counterpoise must be put over as soon as possible, in order to keep the wire taut all the time.

When the roller is towing in place, a "leadsman" is stationed to read the depths shown on the depth-wire: and is ordered to hail them at even numbers of seconds apart, as desired—about every twenty to thirty seconds is a convenient distance for plotting—and also when any sudden shoaling or irregularity in depths is observed, in between hails.

NOTE.—It will be remarked that every sounding thus taken can be correctly plotted; for the spacing between fixes for each sounding will be exact. This is seldom the case in ordinary sounding, when it takes considerably longer to obtain a deep sounding than a shoal one.

If the ship's course has been straight between fixes, it may be said that each roller-sounding is accurately fixed, by time, and not merely those at which sextant angles have been taken. This is a great advantage on large-scale work; and the result is very apparent where lines of soundings cross one another; these rarely showing any discrepancies.

The roller is of the form of an ordinary garden-roller, in two halves, to permit of easy turning when altering course. It is made of mild steel, the ends of both sections being covered, and the interior filled with lead to bring it to a weight of 500 pounds, which is found to be a convenient size for manipulation. It requires to be strongly constructed to stand the rolling over the sea bottom, especially when unexpected rocky ground comes in its path. The towing-lines are shackled directly to the eye in the end of the towing-handle, and the depth-line to the eye of the tripper.

**Descrip-
tion of
Fittings
(see
Fig. 93).
Roller.**

The main objection to the system of towing the depth-indicator to obtain soundings is the difficulty of knowing whether or not the object towed—in this case the roller—is actually on the bottom. This has been overcome by a fitting, consisting of a tripper arrangement on the roller, to which the depth-wire is attached, and of a series of ribs, or battens, riveted to the surface of the roller, arranged so that, as the roller revolves, the toes of the tripper, passing over the battens, which are of triangular section, give a series of small jerks to the depth-wire, which shall be observable above the water. It has been found in practice that these jerks are scarcely ever observable to sight, because the irregularities of the bottom cause greater jerks to the line than those caused by the battens on the tripper, thus rendering the latter imperceptible; but it has also been found that they sometimes can be distinguished by feeling the wire, and, more certainly than in either of these methods, if the ear be placed against the foot of the spar which carries the depth-line block, the continuous and even rattling of the tripper passing successively over the battens on each section of the roller can easily be distinguished—the sound ceasing directly the speed of the ship is increased sufficiently for the roller to tow clear of the bottom, and thus to cease revolving.

These are lengths of 1-inch steel-wire rope, and require no fitting, beyond a thimble turned into the end for shackling to

Tripper.

**Towing-
Lines.**

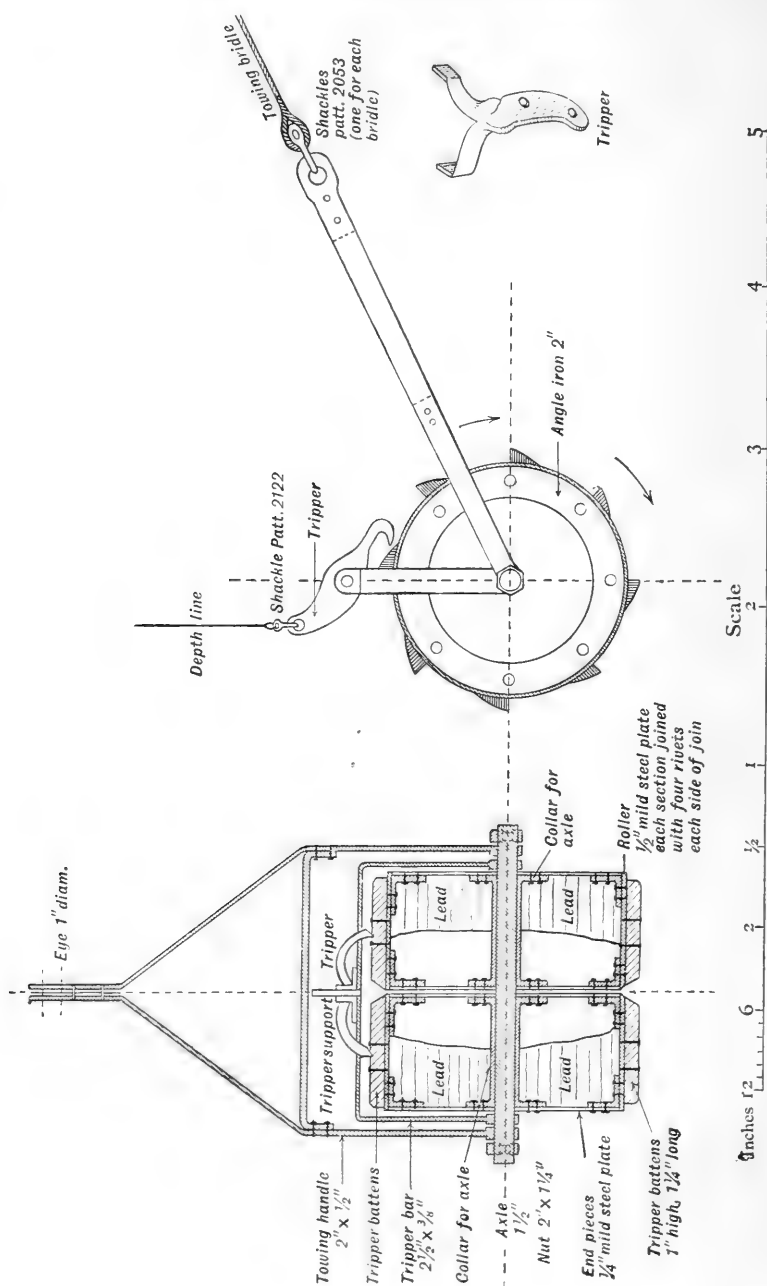


Fig. 93.

the roller towing-handle. They require to be carefully measured, and to have marks tuckd into them, to show when the roller is towing at the required position, according to the calculations derived from the ship's measurements.

It is advisable to have two marks in each line; one to be at the block on the boom end, the other at some position in the ship (*e.g.*, the fairlead in the knightheads), so that it may be seen that the boom is itself at its proper altitude to suit the calculated lengths.

Since the marking of the depth-wire depends on everything being at the exact positions in which they were measured for the original calculations, it is essential that these towing-line marks shall be carefully adhered to. The boom guys and topping lifts should be marked similarly.

This is made of $\frac{3}{8}$ -inch special flexible steel-wire, and is marked as a lead-line in feet, according to the calculated positions for the marks; so that (for example) when the 40-foot mark is at the surface, it indicates that there is 40 feet perpendicular depth over the roller—all depths, of course, being measured from the bottom of the roller. Depth-Line.

A thimble is turned into the lower end for shackling to the tripper on the roller; and the wire is then rove through the block at the head of the sounding spar, then through the block on the counterpoise, and the end brought inboard.

As it is necessary to alter the position of the counterpoise as the depths alter, to prevent it touching the bottom, and as the depth-wire is of too small a size for proper manipulation by hand, a convenient disposition of the inboard end has been found in reeving it, after it has passed through the counterpoise block, through the block on the submarine sentry davit, and securing it to the drum of the sentry winch. It can then be eased out, or hauled in on this drum, as required without difficulty or loss of time, by the winch-handle.

A 112-pound lead is used for this, with a $2\frac{1}{2}$ -inch metal snatch block (surveying pattern) secured to the ring at the top for the depth-line. It is advisable to have a preventer line made fast to the counterpoise, so that it shall not be lost in case of the depth-line carrying away. Should the roller encounter a rocky patch on the bottom, the sudden rises and falls that take place when passing over it naturally cause equally sudden falls Counterpoise.

and rises of the counterpoise. These falls have occasionally been found sufficient to cause the depth-wire to part, and require guarding against in arranging a preventer line.

To employ a heavier depth-line is not recommended on account of the sag that would be occasioned by its greater surface of resistance; nor is the accident sufficiently common to warrant it.

Lower
Booms.

The ship's ordinary lower booms have been found equal to the work entailed on them as spreaders of the towing-lines.

The blocks for taking the wires are 10½-inch iron blocks (pattern 474A), shackled to the band for the guys and topping lifts.

The fore-guy should be rove double for greater support. The booms should be dropped to the height above the water required by the calculations (in *Research*, 3 feet), and both guys and topping lift marked for this position.

Remarks.

This method of sounding has been used considerably, in suitable places, during the last two seasons in H.M.S. *Research*. Its readings have constantly been checked by soundings taken in the ordinary manner, and have always proved to be exactly the same in all depths up to about 20 fathoms.

It was first carried out by means of a "sledge," in place of a roller, also weighing 500 pounds; a roller was substituted as, being in two sections on the same axle, it follows more satisfactorily the path of the ship, according to the alterations of helm, or to the sagging due to tide, or wind: also, it tows more lightly than the sledge. Perfectly good results are, however, obtainable with the latter: and it has the merit of being easily constructed on board the ship, which a roller scarcely could be.

The necessary gear is easy to rig and unrig, and has hitherto stood well all the wear and tear of various descriptions of sea bottom, without mishap (except once or twice to the depth-line, as mentioned above), including unexpected patches of rock.

It is not considered, however, that it should, generally speaking, be employed on known rocky or coral grounds, as the strain brought on the booms, and the probability of the bight of the towing bridle catching in a pinnacle or coral boulder, would almost certainly bring about an accident.

The best results are obtained when the depths are between 30 and 80 feet, and if it is desired carefully to delineate the upward swing of a bank of sand or mud, from a level ocean

floor, no more certain method can be adopted. When the roller is towing, it is most interesting to see on the depth-line how every small irregularity of the bottom is exhibited, as well as the larger and more important elevations and depressions. The Tarbert Bank, off the west coast of Jura, was almost entirely sounded, and its 10-fathom limit line delineated by this method, the whole of the roller fittings (with the exception of the two actual sections of the roller) having been made on board by the ship's blacksmith.

As regards the question of speed, it is found that a 500-pound weight, as described, leaves the bottom at 15 fathoms directly the speed is increased above $3\frac{1}{2}$ knots; and in 20 fathoms depth, the speed must not be much above $2\frac{1}{2}$ knots. It is considered that, on account of sag, the speed should not, even in 5 fathoms depth, be above 4 knots, though the roller still remains on the bottom in those conditions.

As regards the question of a tidal stream or wind acting on the ship in any other direction than right ahead, or right astern, so that she has to have her head directed at an angle to the line it is desired to run over the ground, it will be noticed that this angle may be so great that the roller will assume a position at some distance off on the quarter, instead of dead astern; and that, consequently, the depth-line, coming to the surface at a greater angle than calculated for the astern position, will give an incorrect reading of the depth.

The size of the angle to which the ship may thus be laid off her course-made-good, without error of depth-line reading, is governed by the proportion which the distance between the blocks on the boom-ends, through which the towing bridles lead, bears to the length, calculated along the centre line of the ship, at which the roller is towing.

In Fig. 94, if R be the position of the roller, B the towing block on the boom-end, AR the centre line of the ship, then the roller will tow dead astern so long as the ship is not off her course-made-good at a greater angle than ARB.

In the *Research*, for example, this angle, when the roller is at a depth of 50 feet, is $12^{\circ} 20'$; but as a further 10° off her course only makes an incorrectness in the depth-line reading of 3 inches, it may be said that the ship may be about 25° off her course-made-good without sensible error in the soundings as read.

The error, however, rapidly increases beyond that limit; so that it would be necessary to wait for slack water of tidal streams, or better weather, in the case of wind, should a larger angle be necessary to keep the ship on the line desired to be run. These conditions, of course, vary with every ship, according to the lengths of her towing fittings, as well as to the depth in which the roller may be; in the latter case, the error is less in greater depths, and more in lesser depths.

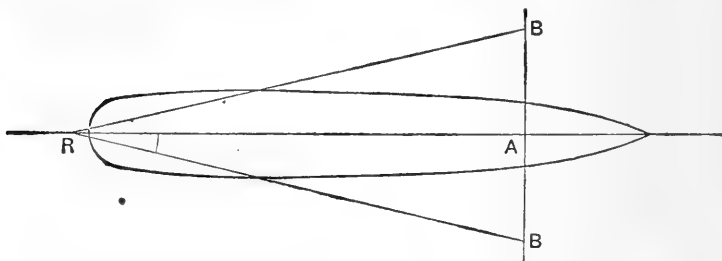


Fig. 94.

It may be remarked that a fitting of this nature could probably be adapted for boats; for example, by a steam cutter towing a roller or sledge along the bottom, at a convenient distance, with a small boat perpendicularly over it, carrying the depth-line, and the officers fixing the position. The readings might even be automatically recorded, as in a tide-gauge. In the case of a boat, however, a towing bridle would have to be dispensed with, and a straight course carefully preserved by a transit, or otherwise.

DOUGLAS-SCHÄFER SOUNDING TRAVELLER.

This is a contrivance devised to obtain soundings rapidly from a ship under way, and is the result of the ingenuity of Commander H. P. Douglas, R.N., and Lieutenant J. S. Schäfer, R.N.

Continuous soundings have been obtained with it, in depths of from 20 to 40 fathoms, at an average rate of one sounding per minute.

It is used in conjunction with a wire jackstay running from the head of the sounding spar to the end of the lower boom and is supplied to all H.M. surveying vessels. The following

abbreviated description is taken from a pamphlet issued by the Hydrographic Department.

The sounding spar is secured in position by guys and topping lift of wire, set up by bottle screws. At the head of the spar there is a collar with four lugs, one to take topping lift, two to take the guys, and the fourth has shackled to it a 5-inch swivel block, through which is rove the jackstay of $1\frac{1}{4}$ -inch wire. This lug is bent in a forward direction, so that the sounding line will clear it when being hove in.

Shackled to the tail of the 5-inch block there is a small snatch block, through which is rove the hauling aft line attached to the traveller. An iron davit, carrying a 12-inch swivel block, through which the lead-line is rove, is clamped to the head of the sounding spar, from which it projects in such a position that the head is inclined forward so that the lead-line when up and down is in the same vertical plane as the jackstay.

The diameter of the spar at its head should be $6\frac{1}{2}$ inches, and it should be stepped at such an angle that its head is vertically 2 feet outside the outboard end of the sounding platform; the length of the spar is governed by the angle of inclination of the jackstay, which should be about 10° . The lower boom, at right angles to the ship's side, and horizontal, or at any angle from the horizontal, has a block at its outer end through which the jackstay is rove and shackled to a wire pendant secured to a bolt in the ship's side in such a position that when taut the pendant forms a martingale to keep the boom from topping.

The lead-line, to which is attached an iron ball, is rove through the 12-inch swivel block at the end of the davit on the spar, and the lead is suspended from the ball by a piece of chain 2 feet in length.

The traveller, fitted with two projecting carrier horns, being placed on the jackstay, the latter is set up by an Axallwazo pulley; the traveller is then hauled aft by the hauling aft line. The lead and line are in position for commencing sounding when the ball is close up or near to the block on the davit. The carrier horns of the traveller being under the ball and above the lead, when the lead-line is released the ball falls into the carrier horns. A cone-shaped releasing buffer is attached to the jackstay, on which it slides with the cone pointing aft, and a line

is secured to the shackle or the band of the buffer and brought inboard. By means of this line the buffer may be controlled and secured at any desired position on the jackstay.

On releasing the hauling aft line attached to the traveller, the lead-line being clear for running, the traveller slides down the jackstay, and on impact with the releasing buffer the tripper will be raised, releasing the tongue; the weight of the lead will revolve the tumbler and the ball will be dropped through the carrier horns, thus releasing the lead. As soon as the lead becomes disengaged the traveller will automatically reset itself by means of the counterpoise weight. The sounding is obtained by the leadsman in the usual way by hauling the inboard part of the sounding line taut when the outboard part is up and down. The lead is then hove up and the traveller hauled aft again ready for engaging.

SWEEPING.

The ever-increasing draught and value of modern shipping renders necessary, in certain areas, more minute and thorough search for hidden dangers than can be afforded by the use of the lead alone.

Various methods of sweeping have been devised and carried out in recent years by H.M. surveying vessels, of which the following pages give some account.

It is necessarily a laborious process, and the object yet to be attained is to cover a sufficiently wide front on a single line of sweep.

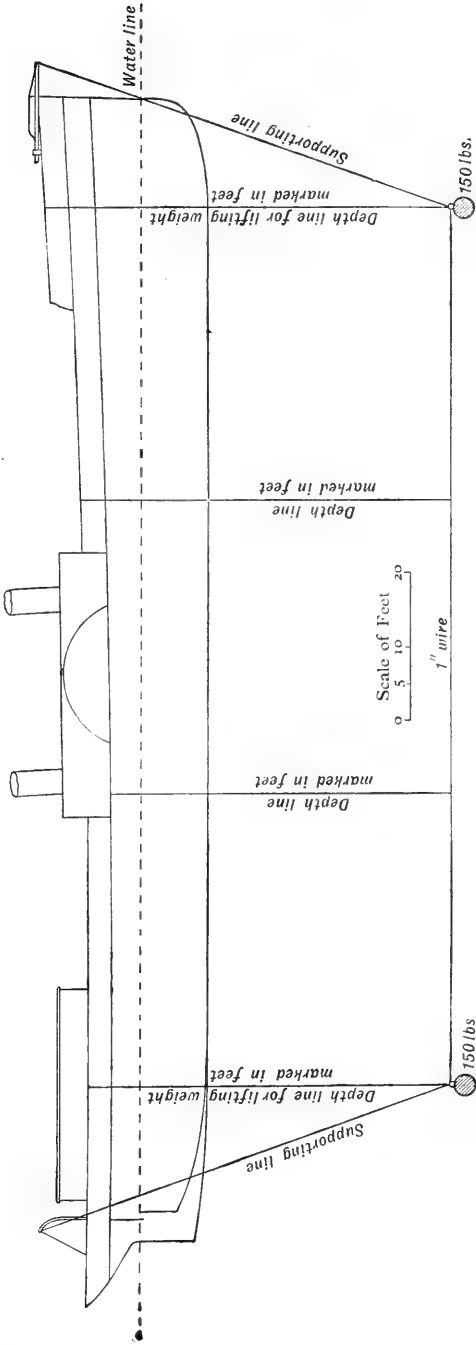
Sweeping
by means
of the
Ship.

(Captain F. C. C. Pasco, R.N., when commanding H.M.S. *Research*, used the ship drifting broadside to the current, with a sweep consisting of 1-inch wire rope weighted at each end with 150 pounds, and supported at each end with 3-inch hemp rove through convenient blocks, the distance between the supporting blocks being 40 feet greater than the length of sweep. This gives tension sufficient to keep the sweep taut. Up and down lines, marked in feet, support the sweep at equidistant intervals.

The arrangement is shown in Fig. 95.

Under favourable conditions, in calm weather with from 1 to $1\frac{1}{2}$ knots tide, the action of the sweep touching the bottom

Fig. 95.



is detected at once; but with a fresh breeze the drifting of the ship through the water causes the sweep to grow out from the ship, and renders it difficult to judge if it touches anything. The intermediate depth-lines should be kept in hand.

The negative results given by the sweep may be advantageously supplemented by stationing leadsmen at intervals along the ship's side, keeping their leads up and down on the bottom, and using heavy leads with $\frac{1}{4}$ -inch wire rope, and calling soundings as required.

In sweeping by drifting, it must be remembered that tidal streams splitting on a rock have a tendency to carry the ship clear of it.

It is not easy to manœuvre the ship so as to pass over fresh ground at each sweep. Tidal streams vary in direction even at short intervals, and it cannot be depended upon that the ship will make a sweep regularly parallel to any preceding sweep, and angular "holidays" thus appear between the sweeps. Nor does the ship take a straight course when making a sweep for any considerable distance.

Suitable weather for drifting does not often occur, and then only a portion of each day is available.

It is useless to attempt it if the wind is contrary to the tide or obliquely directed. If the fore and aft line of the ship does not remain at right angles to her general track through the water, only a narrow area is covered by the sweep-line or by the soundings; nor does the sweep get a fair chance of catching in any obstructions, owing to the oblique direction of its movement.

Alterna-
tive
Method.

In the method above described the sweep wire should be theoretically taut, but in practice this does not always appear to be the case.

In order to obviate this defect, Commander F. A. Rayne used six steel angle bars, $2\frac{1}{2}$ inches \times $2\frac{1}{2}$ inches \times $\frac{1}{2}$ inch, each 25 feet in length, and weighing about 187 pounds, to take the place of the sweep wire.

With these bars slung from each extremity point uppermost (see Fig. 96), there was no sag in the centre of the sweep thus formed.



Fig. 96.

Holes were bored in the extremities of each bar, through which a wire strop was passed, and to which a shackle, with the hemp lowering line and wire depth-line on it, was secured. Except at the extremes of the sweep, the shackle was passed through the strops at the ends of the bars adjoining each other, and thus the six bars were loosely joined into one flexible length. The bars were slung over the ship's side from convenient davits or spars specially rigged out, so that the sweep hung in a straight line parallel to the keel.

The lowering lines were used solely for raising or lowering the sweep; the wire depth-lines were only used to regulate its depth, thus placing no undue strain on them.

This method proved highly satisfactory: the sweep on touching the smallest shoal patch grew away from the ship's side, and swung back to its vertical position when the shoal patch had been passed.

Owing to the sweep bars being joined together, if a shoal patch was touched, it caused those bars next to it to draw out from the vertical as well, thus giving an erroneous impression of the size of the shoal. This could be obviated by slinging each bar separately from both its extremities; but as, in this case, twelve depth and twelve lowering lines would be needed, many more hands would be necessary to work the sweep. With the current setting about 2 points from the direction of the length of an area swept, it was found possible to keep the ship at right angles to the direction of the drift down the area. A twin-screw ship with out-turning screws has a great advantage in this respect; the effect of wind on the lie of the ship may be corrected by an awning triced up aft, or a staysail forward (according to the direction of the wind), which materially assists the action of the screw.

The iron bars used by Commander Rayne are very much lighter than railway iron, and he considers they might be used satisfactorily in sweeping over areas when the bottom is of soft mud. There is also less likelihood of missing a smooth rock, as is sometimes the case when sweeping with wire. When sweeping in difficult areas, such as narrow channels between dangers, and in cases where the current runs across the length of a long narrow area, where it would take a long time to sweep a succession of short lines with the current across that area,

it might be advantageous to hang the sweep under the ship athwartships, its extremes being slung from the outer ends of the lower booms, on which chains are rigged for the sounders. The ship might then be steamed slowly ahead against the current without any appreciable slant from the vertical being given to the sweep. This has been tried successfully. The objection to this method is that, should the sweep foul the bottom, the lower booms are liable to be carried away unless one end of the sweep is slipped at once, and that a shallow patch is not so easily detected if it occurs in the central part of the sweep.

SWEEPING BY BOATS.

"Sea-lark"
Sweep.

This sweep, designed by Lieutenant J. M. Jackson, R.N., consists of 50 yards of three-stranded wire, which is kept taut and at any convenient depth by two submarine kites towed by two steamboats (see Fig. 97).

The kites are towed from sweeping machines (see Figs. 98

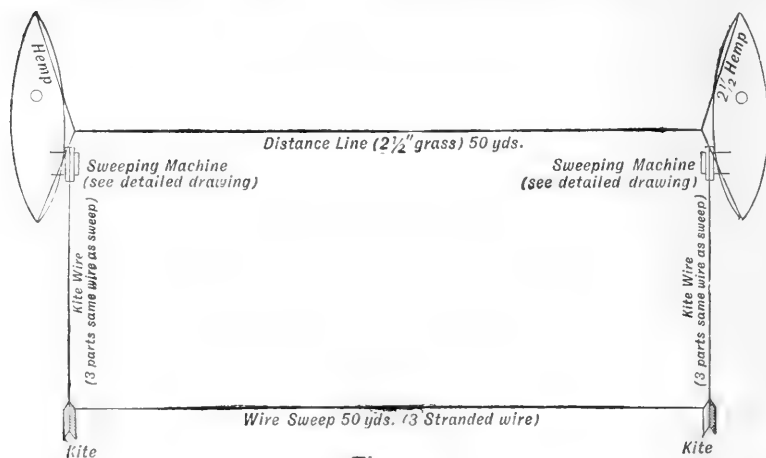


Fig. 97.

and 99) mounted on a sounding machine platform in each boat; the kite wire, consisting of three parts of the same wire as that used for the sweep, laid up together. The platforms are strengthened to take the strain of the kites.

The boats are connected by a distance line of 2 1/2-inch grass

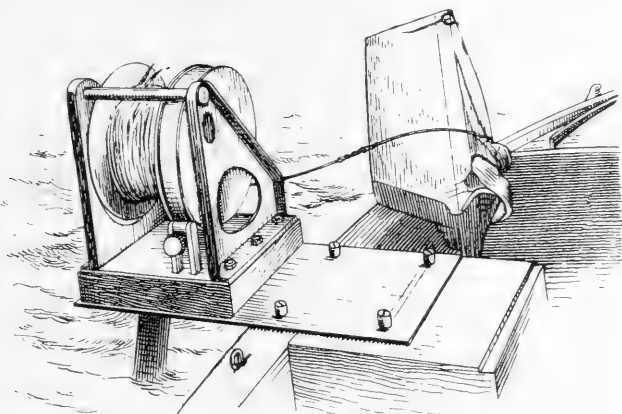


Fig. 98.

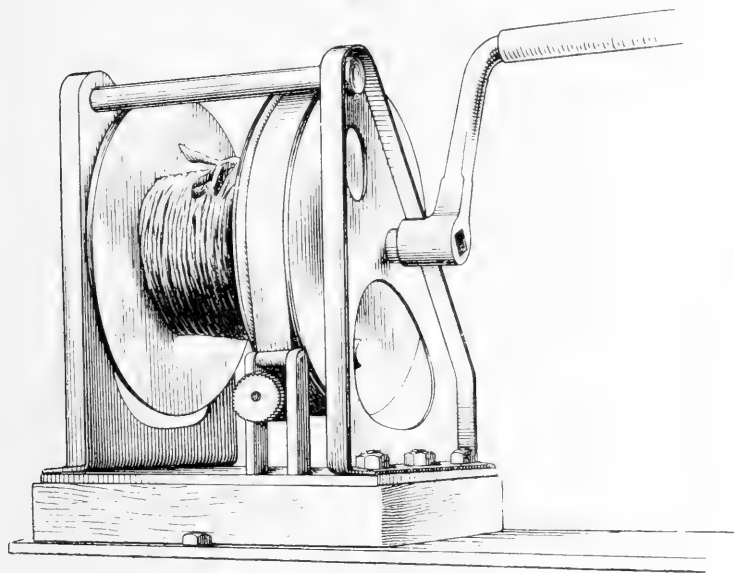


Fig. 99.

line, secured to a hemp span between the bow and stern of each boat.

The length of the distance line is exactly equal to that of the sweep wire; one boat runs the required transit, and the other keeps the distance line taut.

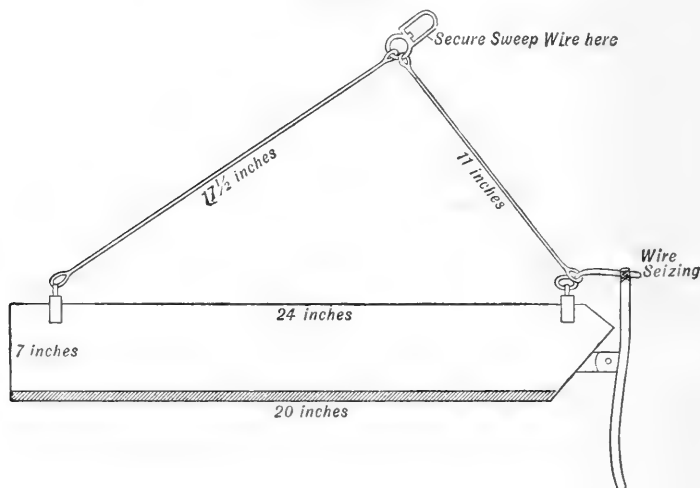
The kite wires are marked in fathoms, and the actual depth

at which the sweep is set can be seen from the following table, which is applicable to the old pattern red kite of James submarine sentry (see Fig. 100), and used with the particular description of wire above stated. The dimensions of the kite are given in Fig. 100.

| <i>Kite Wire Out.</i> | | | | <i>Depth of Sweep.</i> |
|-----------------------|----|----|----|------------------------|
| 6 fathoms | .. | .. | .. | 26 feet |
| 8 " | .. | .. | .. | 34 " |
| 10 " | .. | .. | .. | 45 " |
| 12 " | .. | .. | .. | 56 " |

The general construction of the sweeping machines may be seen by inspection of Figs. 98, 99, and 101; the framework, plates, and drums being made of mild steel, the band brake (*b*) and its adjusting gear of brass, and the brake drum (enclosed between the two plates *a'* and *a*) of wood.

Fig. 100.



The kites will carry the 50 yard wire sweep and still remain exactly astern of these respective boats at any speed between 3 and 6 knots: the distance line and wire sweep being of equal length, if the former is taut, the sweep will be taut also.

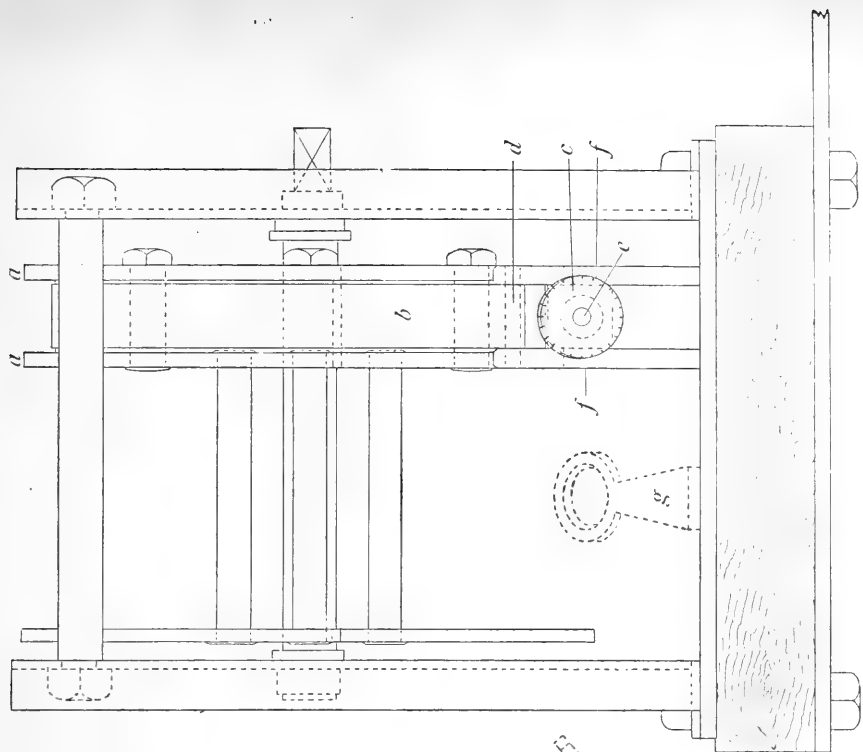
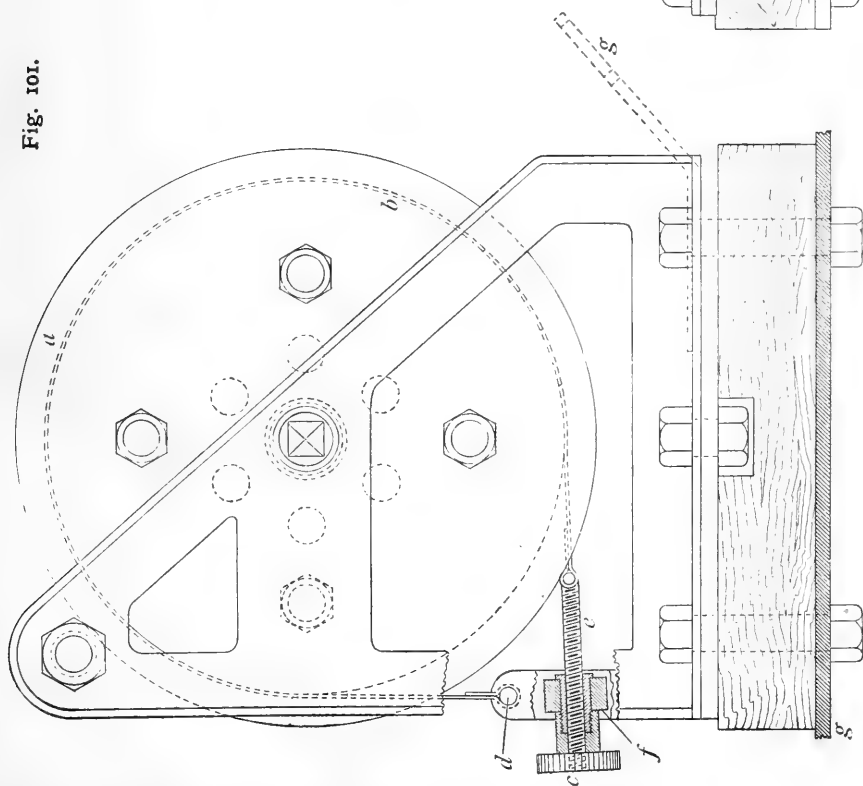


Fig. 101.



It is safe to sweep at the rate of 5 knots. One man should be stationed at each machine, with one hand on the thumb-screw controlling the brake, and the other on the wire abaft the machine.

The instant the sweep wire fouls a rock the kites converge; the brake must be at once eased, letting out wire till the boats have gathered sternway.

If the sweep wire is badly foul of a rock, there is no danger of losing either of the kites, as by bringing a heavy strain on the kite wire, the sweep wire will carry away first, its strength being only one-third of that of the former. The kite being released, a spare sweep wire can be quickly fitted.

Sweeping
by One
Boat.
The
"Otter"
Sweep.

This sweep was also designed by Lieutenant J. M. Jackson, R.N. It is carried out by one boat, the width of the sweep being 42 yards, and "otter boards" the means employed to hold the sweep wire taut (see Figs. 102 and 103). The same machines and kite wires as previously described are used with two slight additions: first, a fairlead of mild steel fitted for leading the wire to drum (see *g* in Fig. 101); second, at least 40 fathoms of wire must be on each drum.

The sweep can be used for sweeping at any required depth, alterations in depth being obtained by altering position of "otter boards" on kite wires.

To sweep at a depth of 6 fathoms with 42 yards wire sweep—

1. Attach one end of sweep wire to one kite (say, starboard), put starboard kite over *going slow ahead* and paying out sweep wire at same time till 8 fathoms of kite wire are out.

2. From depth scale: 8 fathoms kite wire = 5 fathoms 4 feet depth of sweep; so attach otter board to kite wire at $8\frac{1}{2}$ fathoms; put over otter board and pay out kite wire and sweep wire together till about 20 fathoms of kite wire are out. (NOTE.—In putting otter board over, one hand should tend its lower tail wire to start it on right tack, or it may foul screw.)

3. Then secure other end of sweep wire to port kite, let out kite and 8 fathoms of kite wire and secure port otter board at same distance above kite as starboard, and pay out to about 20 fathoms kite wire.

4. Increase to half speed, pay out both kite wires together to a length of 29 fathoms, and put band brakes hard on.

5. Increase to full speed (6-knot boat) when otter boards

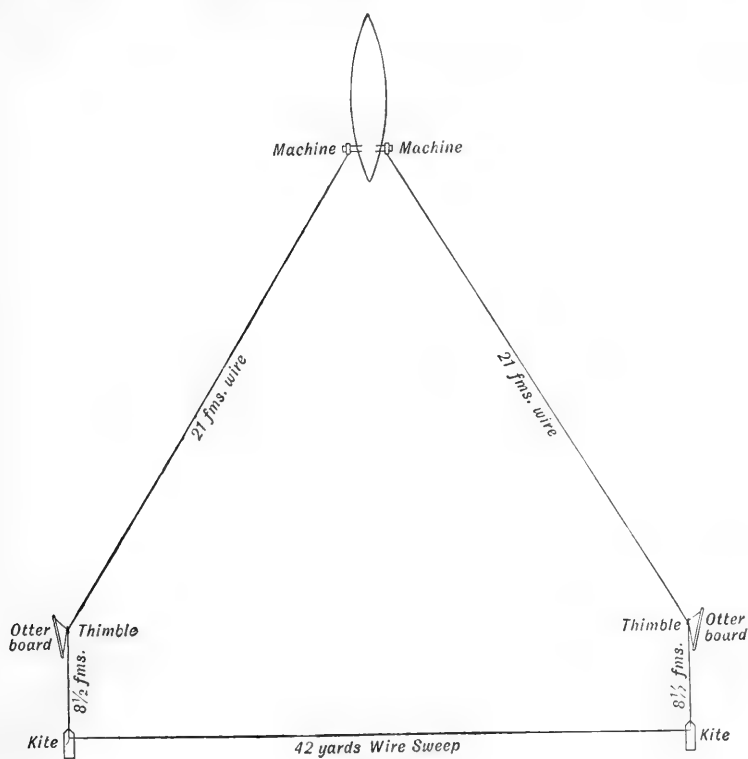


Fig. 102.

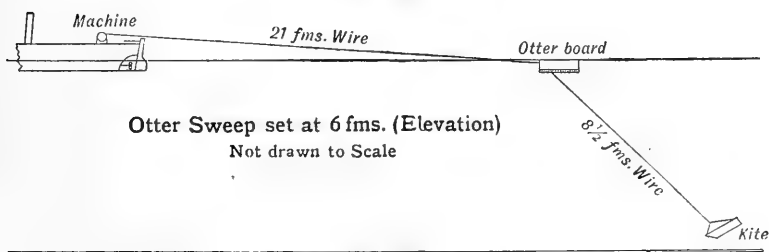


Fig. 103.

separate to 45 yards apart, rise to and remain on (or within a foot of) surface, thus keeping kites beneath them at proper distance apart and 42 yards wire sweep taut at depth of 6 fathoms.

If otters separate too widely, which can easily be seen by

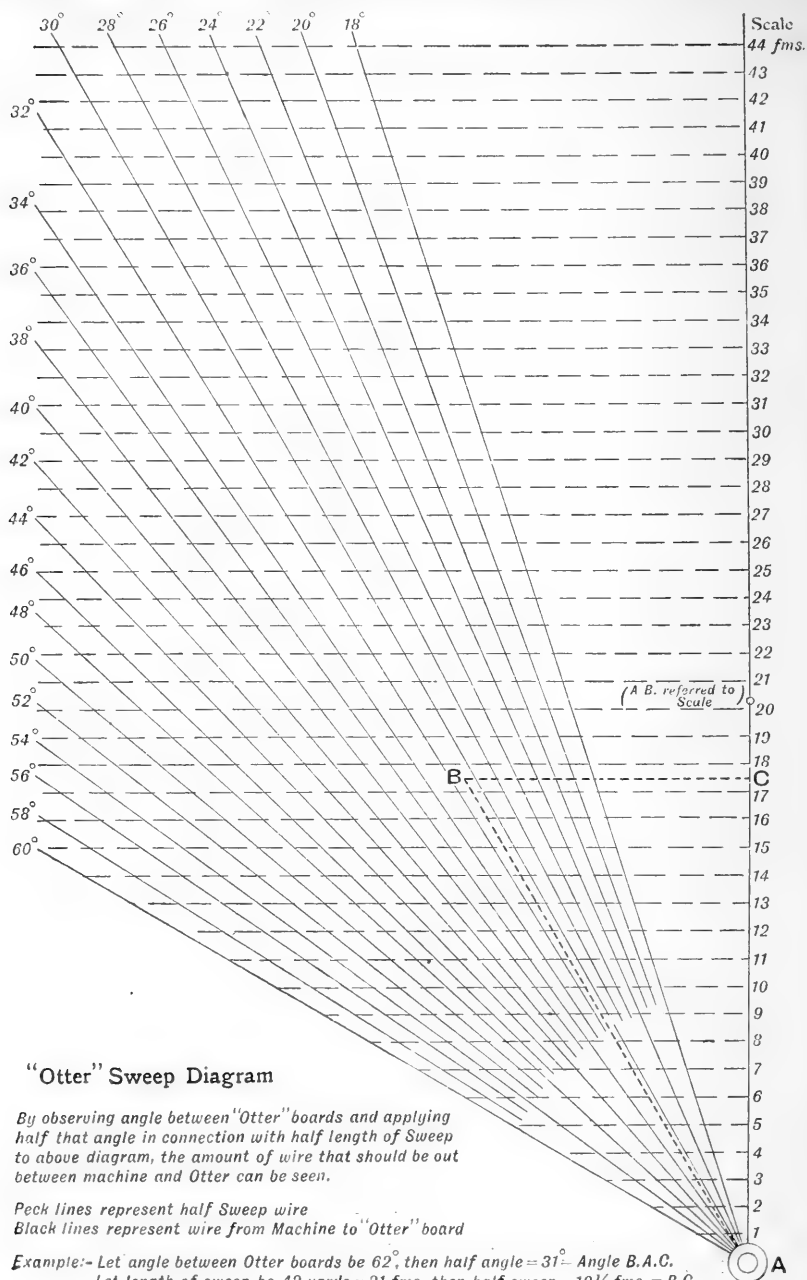


Fig. 104.

sextant angle between them, and applying angle (see Fig. 104), the head wires of otters should be slightly shortened, or, if preferred, the speed of boat slightly reduced, which will have the effect of making them close, though reduction of speed will also cause sweep to be slightly deeper owing to otters then being towed slightly below surface.

A man is stationed at each machine watching otters, with one hand on wire abaft machine and the other on thumb-screw of band brake. Immediately sweep fouls a rock the otters converge and dive; the machine man eases brake and lets out

Plan of Otter Board

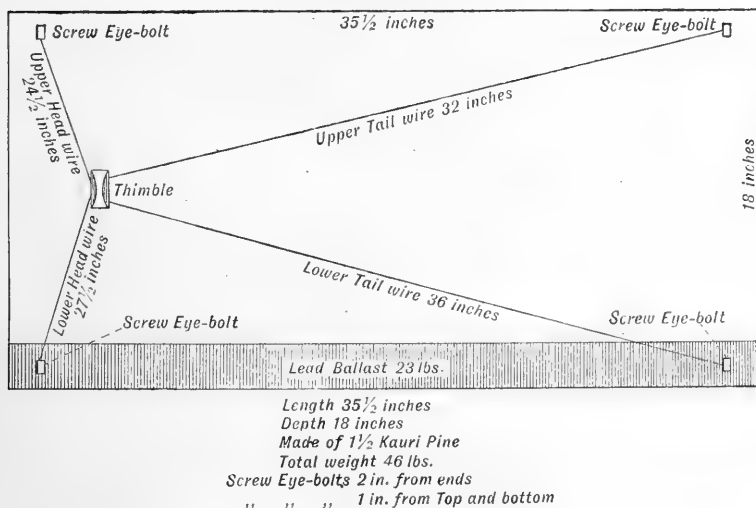


Fig. 105.

wire to ease strain till boat has gathered sternway; the kite wires are then hove in, otter boards taken off, and sweep cleared.

If sweep is badly foul of a rock, as is likely to occur with coral, by heaving in kite wires, which have over three times breaking strain of sweep wire, sweep wire can be parted, kites hauled in, and new sweep wire fitted very quickly.

Otter boards may be secured to kite wires either by stoppers similar to but stronger than those supplied for sounding wire, or by a wire seizing through otter's thimble, round kite wire,

and passed through parts of kite wire to prevent slipping. If wire stoppers are used, a wire "preventer" should be fitted, as otter boards sink if free.

The head and tail wires of otter boards should not be at first secured permanently to the thimble, as, however carefully otter boards are made and wires adjusted (so that to all appearances otters are identical except that wires are secured in each case to inner side of board), some slight difference will probably be found either in their surface keeping or station keeping on the quarter, which can only be remedied by experimental adjustment of wires. These adjustments should be carried out with kite attached to otter.

A lighter otter board than the one shown in drawing (Fig. 105) is not satisfactory, not being steady.

METHOD OF SWEEPING IN THE ENTRANCE TO ST. JOHN'S HARBOUR, NEWFOUNDLAND, USING BOATS ONLY.

By CAPTAIN J. W. F. COMBE, R.N., ADMIRALTY SURVEYING
VESSEL "ELLINOR."

In adopting a method for sweeping an area in the entrance to the harbour of St. John's, through the Narrows (1,250 yards in length on the leading marks, and 110 yards wide), for rocks or any other obstructions, a spar 66 feet long was obtained, though a longer one would have been used if procurable (see sketch).

From each end a weight of 200 pounds was suspended by a $1\frac{1}{2}$ -inch manilla rope, the weights connected below by 50 feet of $\frac{3}{8}$ -inch wire, called the "bottom line," the horizontal depth of same being regulated by a measured line vertically over each weight and attached to the spar.

At distances of 16 and 17 feet along on the bottom line two light lines were attached, made fast above to small floats by length of line corresponding to depth of bottom line; these floats helped to indicate, by their change of position on the surface relatively with the spar, when the bottom line touched any obstruction.

According to the depth required to sweep below the sea sur-

face, occasioned by the rise and fall of tide, so were the weights lowered or raised and the attending lines adjusted.

Two boats were employed to tow the spar broadside on, tow-lines being attached to where the lead-lines were made fast to the spar, thereby making it 50 feet between the boats—one boat acting as the directing one, working on transits, allowing each successive sweep to slightly overlay the previous one, fixes being frequently obtained to plot progress of work as on a line of soundings. Weights of 200 pounds being found too large for convenient handling in boats, those of 100 pounds were used; in the first instance, 100 pounds was lowered with the bottom line between, the second 100 pounds being lowered on the suspending lines, the whole weight then allowed to come upon the same.

It was calculated that the tension on the bottom line, between the weights, at a depth of 30 feet and using 200 pounds at each end, was 52 pounds; if the spar has good buoyancy, greater weights could be used to increase the tension between them on the bottom line, thus assuming the character of an iron bar.

As a test of this method of sweeping, the Merlin Rock, with 29 feet over it at low water, was swept for, and was found at once, the sweep having been adjusted to about one foot below the level of the rock.

REPORT ON THE SWEEPING OF SHEERNESS BAR.

By LIEUTENANT-COMMANDER L. D. PENFOLD,
H.M. SURVEYING SHIP "TRITON."

A lighter was specially fitted with wooden spars, rigged up over the stern, as shown in Fig. 106. From the spars mentioned a railway iron 30 feet long was slung athwartships by two wire pennants about 6 fathoms long, shackled on near each end of the iron, enabling the bar to be raised or lowered as required by means of winches. These pennants were marked in feet from 25 to 32 feet with spun yarn for graduating the depths of the bar.

The lighter was moored over the area swept by four light anchors on 3½-inch hemp hawsers, which were spliced so as to be continuous each side of the deck (Fig. 107).

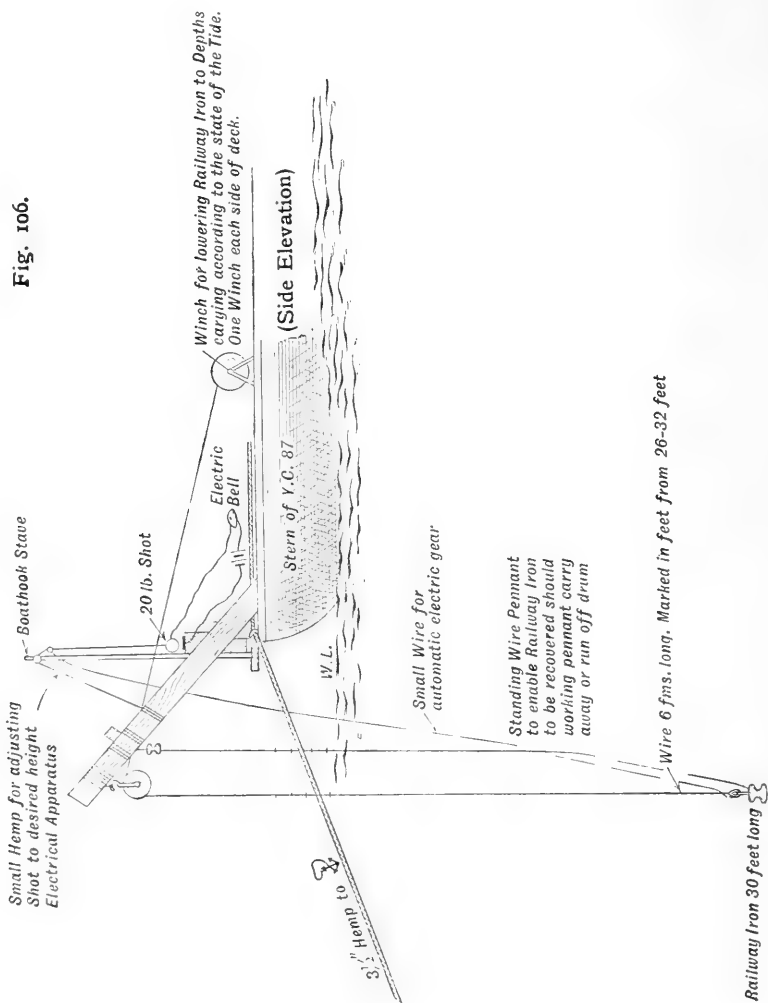
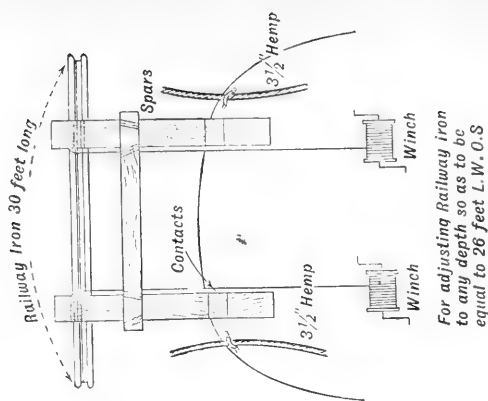


Fig. 106.



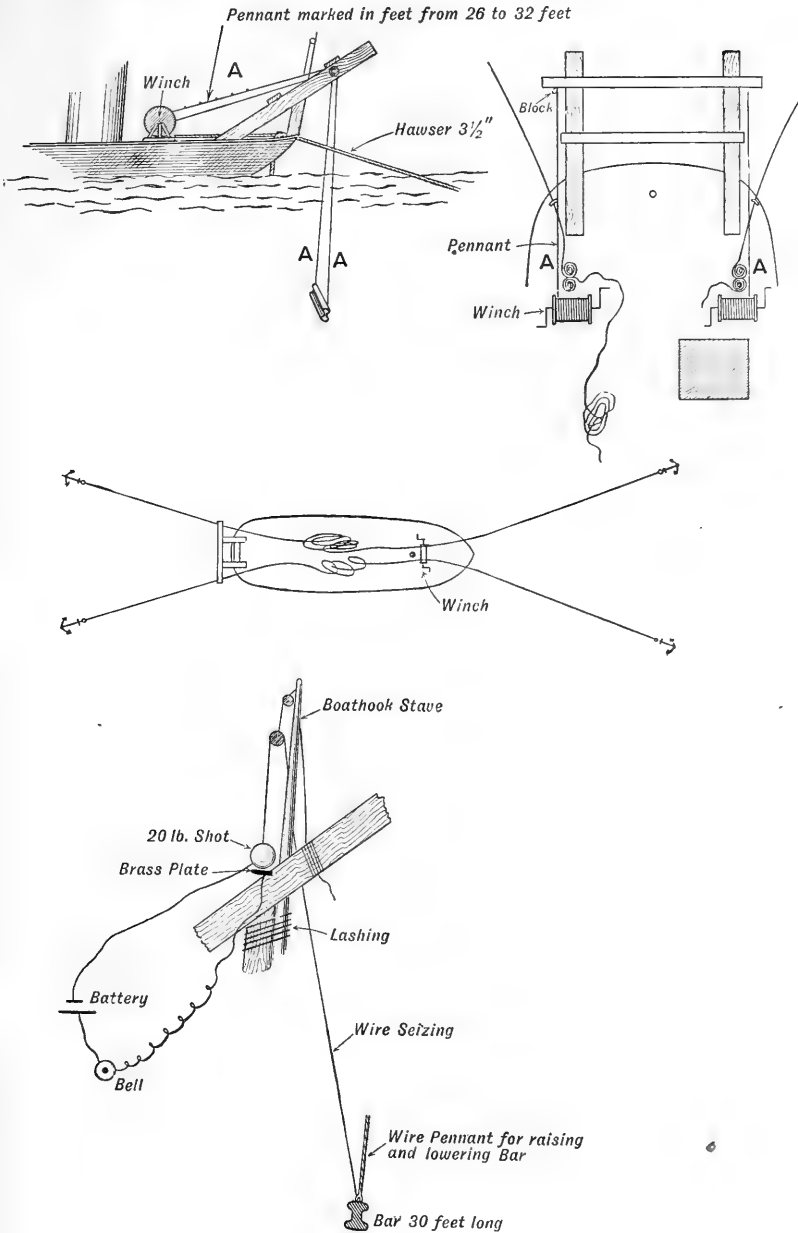


Fig. 107.

A winch was fitted in the bows of the lighter for hauling her forward or backward. The diameter of the drum on this winch was only 6 inches, so the rate of progress over the ground was extremely slow.

The time that was found suitable for this work was from 2 to 2½ hours before low-water to 1 hour after low-water, on account of the strong tide.

To determine the depth at which to set the bar a tide-pole was constantly read, and the bar set to 26 feet, plus the tide-pole reading, by means of the graduations on the wire pennants, the zero on the tide-pole being L.W.O.S., so that when the lighter was hauled over the ground, if there were any inequalities or lumps with a depth of less than 26 feet L.W.O.S., the bar would catch on the bottom and the spot could be immediately fixed.

A method by which a man did not have to handle the wire pennants to feel when the bar bumped was devised.

A piece of wire seizing was secured to each end of the bar and brought up to the lighter through a block (capable of being raised or lowered) made fast to a boat-hook stave lashed to one of the spars; to the inboard end of this wire a 20-pound shot was secured and an electric wire connected to this shot. Directly under the shot was a clean brass contact piece, connected by wire to a bell and battery (see Fig. 107), so that if the bar was lifted through coming in contact with an obstruction, the shot would drop, and, completing the circuit, the bell would ring. This was very successful. The shot could always be adjusted so as to be within a fraction of an inch of the brass contact plate so that the slightest lift could immediately be detected.

The rate of travelling over the ground was very slow indeed, about 100 yards being covered in half an hour—that is, about 900 square yards being thoroughly swept.

Excellent results were obtained when towed alongside a tug at a very slow speed.

This method of sweeping was very effective in the places where the bottom was hard, but for the muddy patches it was of practically little use.

The work was continuously checked by soundings being taken and recorded.

Fine weather was essential for this work, as, if not smooth,

it was found that the railway iron struck the bottom, owing to the motion of the lighter.

A somewhat similar method of sweeping was made use of at Gibraltar, using a 70-feet railway iron slung *fore and aft* under a lighter, the lighter being moved broadside on by ropes, thus obtaining the advantage of maximum length of sweep.

VACUUM TIDE GAUGE.

BY REAR-ADMIRAL H. E. PUREY-CUST, C.B., LATE
HYDROGRAPHER OF THE NAVY.

To one end of a length of rubber-tubing attach a glass barometer tube, place the latter vertically in a vessel of water above high-water mark, and lead the tubing down the beach to the sea. With very little difficulty the water may be made to syphon from the vessel to the sea, provided the highest portion of the tubing is not more than about 30 feet above the level of the sea.

Whilst the water is syphoning pour some mercury into the vessel (a jam-pot makes a convenient vessel), and then raise the latter until the end of the barometer tube dips under the mercury; the mercury will at once rise in the tube to a height AB (Fig. 108), equivalent to the height of a corresponding column of water CD. If CD varies owing to the surface of the water at D varying, then AB must likewise vary in the above proportion. Consequently, if D represent the surface of the tide, as the tide rises and falls so will the column of mercury AB fall and rise in the proportion of about one inch of mercury to one foot of water, and therefore, if a suitable scale be provided, AB may read off like a tide-pole.

The end of the tubing must always be below the surface of the water, but it is immaterial how much; it should, however, be kept clear of the bottom if of mud or sand, and may with advantage be attached to the under surface of a small buoy moored in a convenient spot.

As a very small amount of air leaking into the tubing will largely affect the vacuum, and, consequently, the height of the mercury column, two precautions are necessary, viz.—

- (a) Prevent leakage as much as possible by painting over all connections with several coats of "shellac varnish," made by breaking up sealing-wax into small pieces, placing in a bottle, and covering with methylated spirits, allowing to stand for two or three days before use.

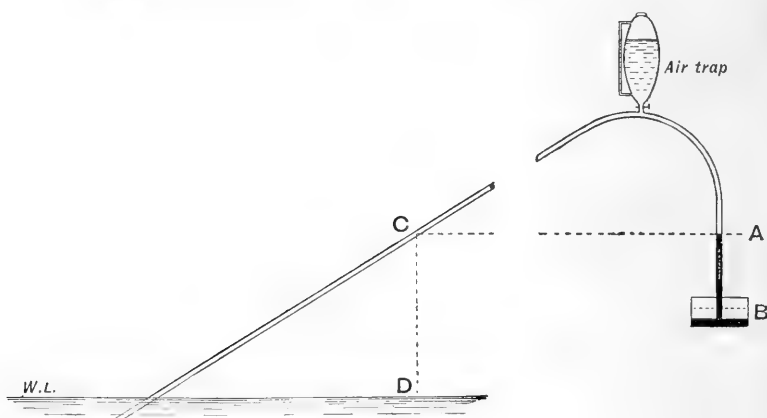


Fig. 108.

- (b) Connect a closed copper cylinder to the highest portion of the system to act as an "air trap," and fill it with water. Any air that leaks into the system rises into the cylinder, displacing a certain amount of water, but not affecting the reading of the gauge. A gauge-glass fixed outside the cylinder shows the amount of air and water inside. A screw plug is provided in the top of the cylinder for filling purposes, and a tap underneath the cylinder.

This gauge is not intended to supersede the ordinary tide-pole, but only to provide an alternative means of readily recording the tide when, for instance at night, it is inconvenient or impossible to obtain the direct readings of the pole. It is most useful, of course, on a shelving beach, as the mercury gauge, etc., can be fixed up inside the tide watcher's hut or tent, or as convenient, at some distance from the water, the limit being the amount of tubing available. The less height, however, the mercury gauge is above high-water, the less prac-

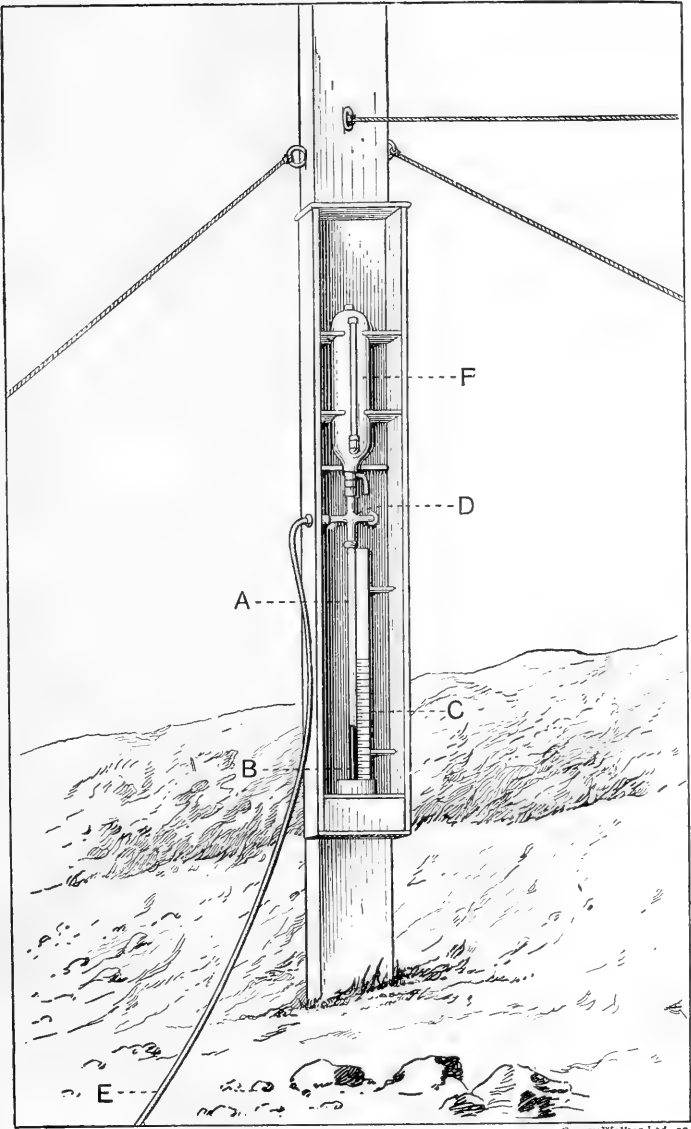


Fig. 109.

Emery Walker Ltd. sc.

tical difficulties there are likely to be with the gauge. Similar gauges have been used with perfect success by *Endeavour* and *Hearty*, using as much as 400 feet of tubing between sea and gauge.* No difficulties were experienced in keeping the vacuum, even when, as a severe test, it was left without attendance for a whole month. In practice, however, it is advisable to thoroughly wash through the whole apparatus once a week or oftener if there is much sediment in the water, as it is surprising what an amount of small particles of shell and other things find their way in. The washing through is easily performed by causing the water to syphon.

Fig. 109 shows the tide-gauge as set up at Uyca Sound, Shetland Islands.

On an upright placed above high-water was secured a casing to contain the gauge (door omitted in sketch).

A is the glass barometer tube, B the column of mercury, the height of which can be read off on the scale, C.

D is a four-way connecting piece, one end of which is connected to the glass barometer tube, one to the rubber-tubing, E, which latter leads down the beach to the water, one is blank flanged as not required, and the other is connected to F, the copper cylinder, which acts as an air-trap, on the front of which is seen a glass gauge for telling the height of the water inside, and underneath it is a tap.

TIDAL OBSERVATIONS FROM A SHIP AT ANCHOR.

The question of tidal observation in deep water having recently engaged attention at the Hydrographic Department of the Admiralty, an apparatus has been devised based on the principle of the pneumatic tide-gauge referred to at p. 223. This apparatus, for use from a ship at anchor, consists of indiarubber tubing having a bore of about $\frac{1}{8}$ inch, supplied in a sufficient number of lengths joined together to allow one end open to the sea to be attached to a weight lowered to the bottom near the anchor. The inboard end of the tubing is attached to the upper part of a closed vertical cylinder, 4 inches in diameter,

* On one occasion when in a gale of wind the tide-pole was knocked down, the gauge remained working satisfactorily.

and about 6 feet high, on the top of which is fitted a small Bourdon gauge of ordinary pattern. The lower part of the vertical cylinder is in connection with an air-reservoir, and is also connected, by a separate pipe of small diameter, with a large Bourdon gauge of special construction.

The air-reservoir, charged by a powerful air-pump, consists of four cylinders, each of which is similar in size and pattern to the vertical cylinder. The large Bourdon gauge is 12 inches in diameter, very delicately made, capable of indicating pressures up to 250 pounds on the square inch, and graduated on a reflecting surface to obviate the effect of parallax in reading off. It can be accurately read to within $\frac{1}{10}$ pound. The method of using the apparatus is as follows:

With the ship lying at anchor, and having sufficient cable veered, the indiarubber tubing should bear no strain. The 12-inch Bourdon gauge being shut off by a needle valve, controlling connection with the remainder of the apparatus, air is pumped into the air-reservoir flowing from thence to the sea through the vertical cylinder and indiarubber tubing. Pumping is continued until the small Bourdon gauge ceases to rise, thereby showing that all the water is expelled from the tubing, and that the air is escaping freely from the submerged end at each stroke of the pump.

The valve at the junction of the indiarubber tubing and vertical cylinder, controlling connection with the sea, is then closed, and the air reservoir and vertical cylinder charged to a pressure considerably exceeding that of the head of water due to the depth.

The compressed air being then admitted to the 12-inch Bourdon gauge by turning the needle valve, the whole apparatus is again placed in direct communication with the sea by means of the valve for that purpose.

The air pressure as shown by the 12-inch Bourdon gauge will then steadily fall as the air escapes into the sea and will continue to do so until the pressure in the apparatus exactly balances that due to the column of water represented by the depth over the submerged end of the indiarubber tubing. When the pointer of the 12-inch gauge ceases to fall and remains quite stationary the gauge is read off.

As a column of sea-water 1 foot high, with sectional area of

1 square inch, weighs 0.445 pound, it follows that the depth is obtained by the multiplication of that factor by the pressure in pounds per square inch as indicated by the gauge. The variation in pressure, provided the weight at the submerged end of the indiarubber tubing has not moved its position, is therefore a measure of the rise and fall of tide.

Observations with this apparatus have been made successfully in depths of 35 fathoms, and the results, when compared with observations of an ordinary tide-gauge on the beach in the immediate vicinity, were found to agree very closely; an occasional difference of 2 or 3 inches might be noted, but it seldom exceeded 1 inch, or even less.

PLOTTING.

Use of Arbitrary Plotting Points.

Reference may be made to the device of introducing one or more arbitrary plotting points to facilitate the accurate plotting of a series of secondary trigonometrical points. It sometimes happens that such points are connected with each other through primary trigonometrical stations which are too far distant to come within the limits of the plotting sheet.

An occasion for the use of this method may arise in the Home Surveys, which are usually based on the Ordnance Triangulation.

In Fig. 110, A, B, C, D, E, F are secondary stations not necessarily intervisible, lying in one general direction and in the relative position shown. The connection between any two of these stations can be found by the triangulation scheme provided.

Making a rough plot on a small scale, the most convenient position for the arbitrary plotting points X and Y are determined, and the angles A F Y, F A Y, A F X, F A X, are taken off to the nearest degree by station pointer.

Using the angles thus taken off, and the calculated distance A F, the triangles A F Y, A F X are solved. A and F being placed on the paper with regard to scale and bearing, X and Y are then plotted from A and F.

C, occupying a central position, may be plotted from A, X, and Y, which involves the solution of triangles A X C, X Y C,

and so on for the remaining points, confining the calculation to what is necessary to lay them down.

The series of points might, of course, be plotted simply on their relative angular measurements and distances, but the arbitrary plotting points afford means of cutting them in by long chords, which give a higher degree of accuracy, besides providing long lines in a transverse direction, which are convenient for carrying on the plotting of other points.

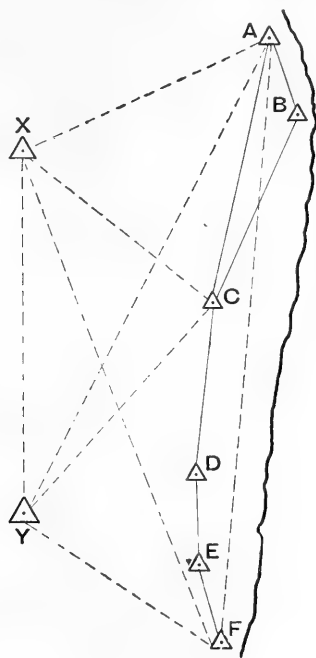


Fig. 110

Writing in the *Hydrographic Annual*, Rear-Admiral F. C. Learmonth, C.B., gives a further illustration of the use of an arbitrary plotting point, X in Fig. 111, which shows a triangulation suddenly contracting in width to a long narrow strait, in which sextant angles alone are possible. In such a case errors are bound to creep in, and the only check lies in carrying a true bearing observed at one end of the strait through the sextant triangulation and comparing it with a true bearing observed at the other end. If there is much discrepancy,

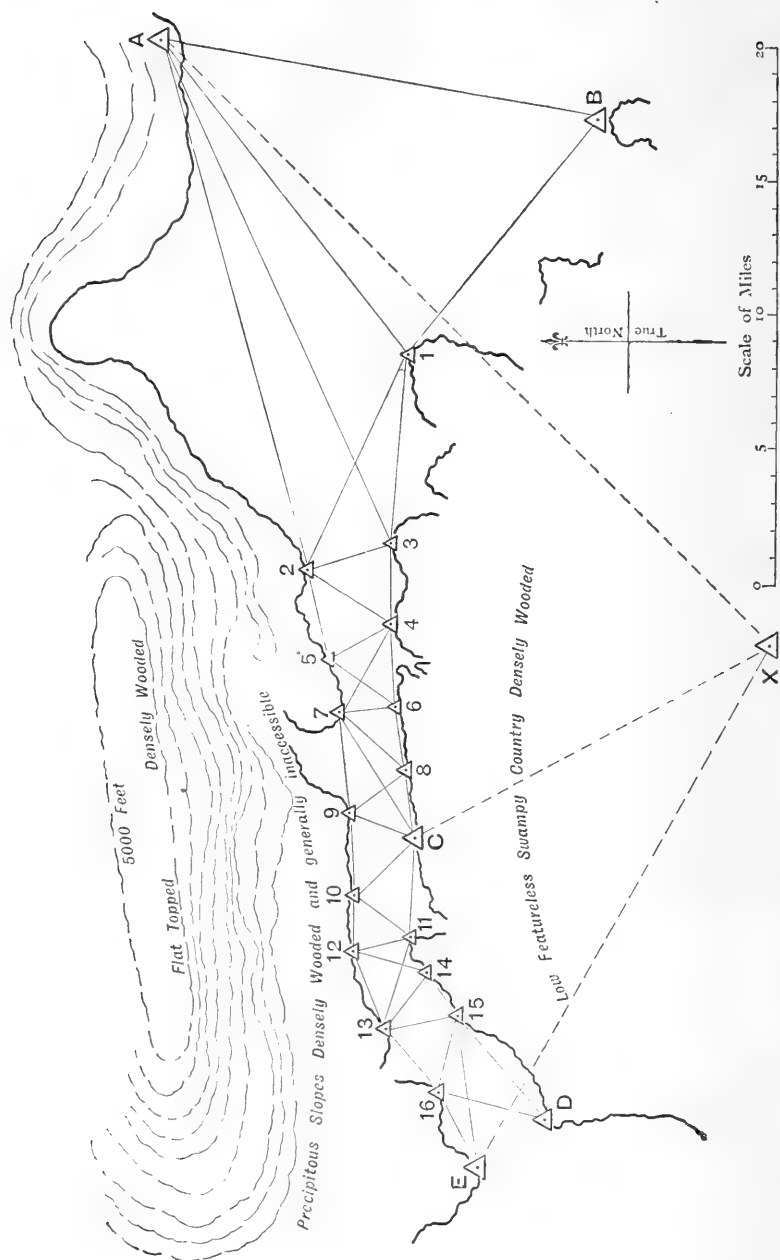


Fig. III.

some attempt at adjustment may be made, but in plotting the intermediate point C, whether by traverse from either end or by the introduction of an arbitrary point, the discrepancy is bound to show itself; it cannot be got rid of, and the only thing to be said is that, in plotting the points on either side of C, the error, whatever it may be, is a diminishing quantity.

PLOTTING WORK.

Captain T. H. Tizard, R.N., C.B., in the *Hydrographic Annual*, makes the following remarks, which may be useful on occasion:

One of the difficulties of plotting work is for the plotter to interpret the sketch-books of all the officers employed in the survey, more especially when their stations are of different heights and there are many islands visible between the stations.

Such a case occurred when the *Challenger* was surveying Kerguelen Island, and the manner in which the difficulty was surmounted was by the officer plotting the work taking a station on the summit of Hog Island, in Royal Sound, and carefully obtaining his own height first, from a base line measured by sound between his station and a low-water rock on which the gun was placed, and then by constructing a curve or curves of angles of depression, and by taking angles of depression to every low-water point seen (the rise and fall of tide being insignificant) he was able to plot a great number of points, roughly, from his own angles alone. His height was about 400 or 500 feet, and was got by the mean result of an angle of depression to the base rock and by an angle of elevation from the base rock.

In calculating for the curve it is, of course, necessary to add the dip for the distance of the point in the curve, and to subtract from the result the approximate refraction to obtain the correct angles of depression which would be obtained by the observer.

The following examples will show how it is done practically: Supposing the height of the observer to be 500 feet—

The dip for distances of 10, 9, 8, 7, 6, 5, 4, 3, 2, and 1 miles is respectively 88.25 feet; 72.48 feet; 56.48 feet; 43.24 feet; 31.77 feet; 22 feet; 14 feet; 8 feet; 3.5 feet; and 1 foot.

Angle of depression for 10 miles—take the mile
=6,080 feet:

Dist. for 10 miles,

60,800 feet - - log. 4.784332

Perp. 500 ÷ dip 88 =

588 feet - - „ 2.769377

sin 7.985045 = 0 33 13

- 50 Ref. = $\frac{1}{12}$ th
distance.

Angle of depression would be

from observer - - - 0 32 23

Worked out in a similar manner for distances of 9, 8, 7, 6, 5, 4, 3, 2, and 1 miles, the results will be found to be respectively 35' 10"; 38' 40"; 43' 18"; 49' 36"; 58' 38"; 1° 12' 20"; 1° 35' 30"; 2° 22' 13"; and 4° 43' 31".

The results are plotted on curves, so that the distance for any angle of depression up to 10 miles can be taken off. Two curves on different scales are used, as a larger scale is required for distances from 5 to 10 miles than for distances below 5 miles.

GRADUATION OF A GNOMONIC CHART.

By J. W. ATHERTON, ESQ., CARTOGRAPHER, HYDROGRAPHIC
DEPARTMENT.

The following method of graduating a gnomonic chart involves, perhaps, a little more labour than that given on pp. 383, *et seq.*, but it has certain features which make it worthy of consideration—viz., there are no very small distances to measure; each point of the projection, or each meridian, can be obtained independently of the others; the longitude scales are obtained indirectly, and thus are easily checked; the main points are plotted and measured directly from the original starting-point.

The necessary data required are one geodetic position, the meridian passing through that position and the scale of the chart.

In Fig. 112 A represents the station whose position is known, and A M the meridian through it. Along A M can be marked the points where the parallels cut it according to the scale, and if L be a point on the parallel which is nearest to the bottom border of the chart, the parallel through L can be constructed thus: Decide upon the number of points considered necessary to enable the curve to be drawn. It will depend upon the scale of the chart whether these points are one, five, ten, or more minutes of longitude apart, and although the position

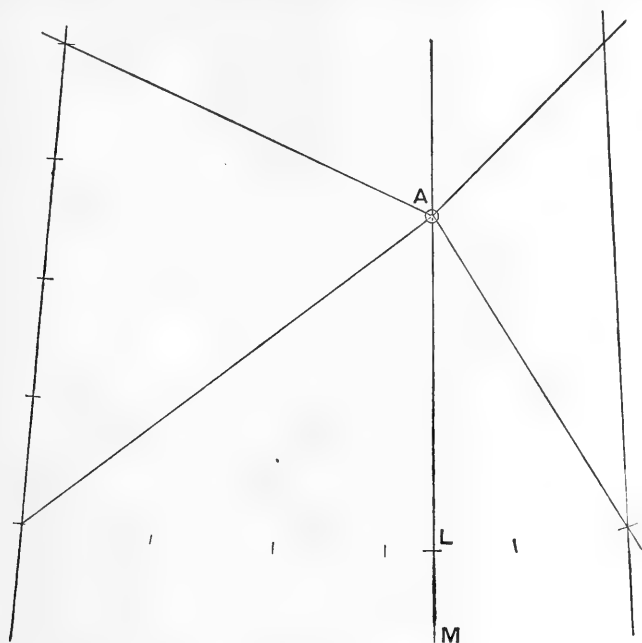


Fig. 112.

of A will be generally intermediate between two whole minutes of longitude, the positions along the parallel through L will be taken on certain meridians.

There are now, therefore, a certain number of known positions to plot in order to draw the curved parallel through L. The positions are known in so far that the latitude and longitude of each point are known, and, taking each position individually, the distance and bearing of it from A can be obtained readily.

When these positions are plotted they give, not only the points through which the curve representing the parallel has to be drawn, but also the actual position through which the meridians pass. These meridians can thus be drawn directly, because the bearing which each meridian makes with the line joining its particular position to A can be obtained whilst calculating the distance and bearing from A.

In practice it will be necessary to draw, perhaps, only three meridians in this manner, and the remainder inserted as follows: Along each of the extreme meridians the distances of the parallels of latitude are ticked off, measuring from the parallel already completed. When the uppermost and corresponding divisions on these extreme meridians are joined by a straight line, this line will suffice just as well as a curved parallel for marking along it the minutes of longitude into which the upper border has to be divided.

Meridians can then be drawn from the lower parallel to the corresponding longitudes marked on this upper line. If these meridians be divided off into the correct minutes of latitude, commencing at the lower parallel, as was done in the case of the extreme meridians, points will be obtained enabling the curve for any or all the other parallels to be drawn.

As a check, any particular intersection of a meridian and a parallel can be constructed by means of its bearing and distance from A.

This method could be limited to plotting the four corners of the chart from the initial position of A, and the remainder completed by the other methods, although what is considered its chief advantage would be ignored—viz., plotting a curved parallel without using very small dimensions. In any case it would provide a severe check in testing a gnomonic chart produced in any other way.

In connection with the subject of graduation of a gnomonic chart, reference should be made to a very useful "Table for the Graduation of Surveys and Charts on the Gnomonic Projection" published by the Hydrographic Department, Admiralty, and giving various alternative methods, with special tables for their use.

Mr. J. W. Atherton, of the Hydrographic Department, has also compiled "Tables for determining Geodetic Positions,

Latitudes 0° to 65° , together with Methods of using Co-ordinates" (sold by J. D. Potter, agent for sale of Admiralty Charts). These tables, used in connection with the specimen forms suggested for showing compactly the work necessary for determining geodetic positions, greatly facilitates the labour of computation.

FIXING BY STATION POINTER WHEN ALL OR ANY OF THE OBJECTS USED TO FIX WITH ARE BEYOND THE LIMITS OF THE POINTER.

By COMMANDER H. P. DOUGLAS, R.N., SUPERINTENDENT
OF CHARTS.

During surveying operations it very often happens that, when plotting positions, one or more of the objects observed fall beyond the limits of the legs of the station pointer.

The methods usually employed to overcome this difficulty are either to make use of a ruler to align the leg or legs to the distant point or points, or else to draw radiating lines from the distant object, and then align the leg of the pointer to one of these radial lines.

Neither of these methods is, at the best, satisfactory, but the following method, which simply means reducing the scale, appears more satisfactory, and is not, it is believed, generally known.

It is theoretically correct, and the only errors that can creep in are caused by (1) the prolongation of a line, (2) careless plotting, and (3) a bad fix.

(2) and (3) are not recognised by the surveyor, and the error due to (1) should be so small that the fix thus obtained is far more satisfactory than the above-mentioned methods.

It is generally known, when using certain objects to fix with, whether any of them will fall beyond the limits of the legs of the station pointer, and also approximately how much beyond.

Therefore, joining the three objects that are being used for fixing, divide the distance between them into $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc., as may be necessary. Then, setting the observed angles on the pointer, plot a position, using the centre object and the two objects obtained by division.

The true position will be on the prolongation of the line joining the centre object and the position found, using the assumed points at a distance from the centre object proportionate to the degree of division used.

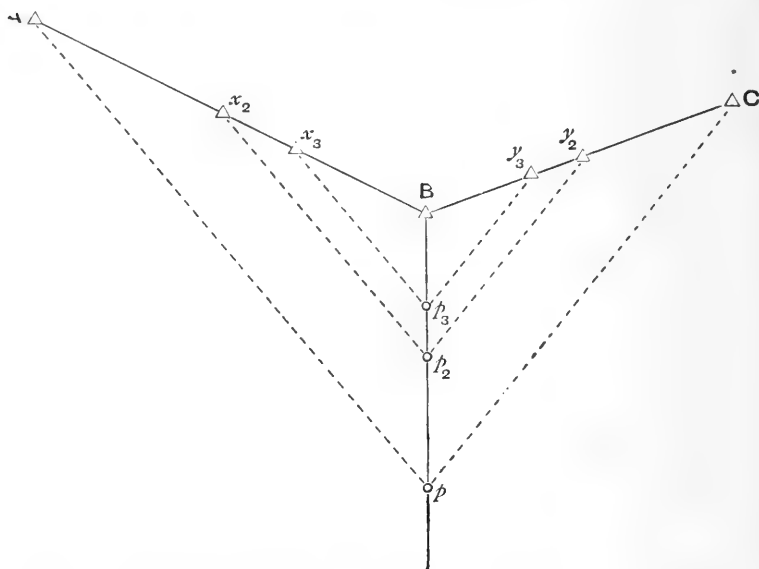


Fig. 113.

The following example (see Fig. 113) will explain this graphically:

| | | |
|-----------|----|---|
| A B C | .. | True positions of the objects. |
| $x_2 x_3$ | .. | Assumed positions of A, A B being divided into $\frac{1}{2}, \frac{1}{3}$. |
| $y_2 y_3$ | .. | Assumed positions of C, A C being divided into $\frac{1}{2}, \frac{1}{3}$. |
| P | .. | True position, using A, B, and C. |
| P_2 | .. | Position using $x_2 B y_2$ (1). |
| P_3 | .. | Position using $x_3 B y_3$ (2). |

Therefore, by (1) true position P is on the prolongation of the line B P_2 at the distance where $P_2 P = B P_2$.

And similarly by (2) true position P is on the prolongation of the line B P_3 at the distance where $P_3 P = 2B P_3$.

The problem stated in Example VIII., p. 141, may be utilised Fixing
Positions. for fixing the position of a rock when only two objects are in sight, the exact distance between the objects being known.

For instance, in Fig. 114, if the rock be at B, and a boat be stationed over the rock with an observer in it, another boat or the ship may be anchored at A, and simultaneous angles

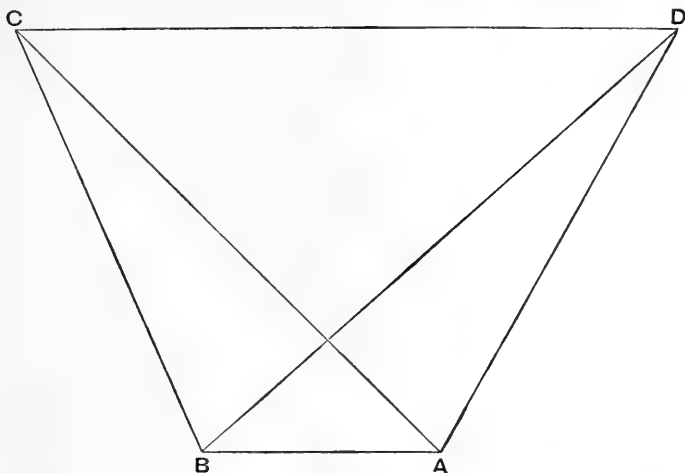


Fig. 114.

being taken at B between C and D and C and A, and also simultaneously angles at A between D and C and D and B, by assuming a distance between A and D or B and C, the other angles can be found as before, and the distance between C and D then utilised to find the distance of the rock B from both C and D.

RECTANGULAR CO-ORDINATES.

Naval surveyors are not infrequently called upon to deal with trigonometrical data furnished by the Ordnance, India, or Colonial Survey offices, which may be expressed in Rectangular Co-ordinates. It is necessary, therefore, to understand the meaning of such co-ordinates, and how they are calculated, in order to plot from them.

The following description is taken by permission from the "Textbook of Topographical and Geographical Surveying," by Colonel Sir C. F. Close, K.B.E., C.B., C.M.G., R.E.

The method of rectangular co-ordinates may be used if the distance east or west of the starting-point is not likely to exceed 150 miles, and if there is no probability of accurate geographical positions being required, nor of there being any further extension of the triangulation, and if the azimuths of the sides are not wanted.

The method is as follows:

(a) The lengths of all the sides of the triangles are calculated by plane trigonometry.

(b) One station is selected at the initial station and the meridian through that station is the initial meridian; then,

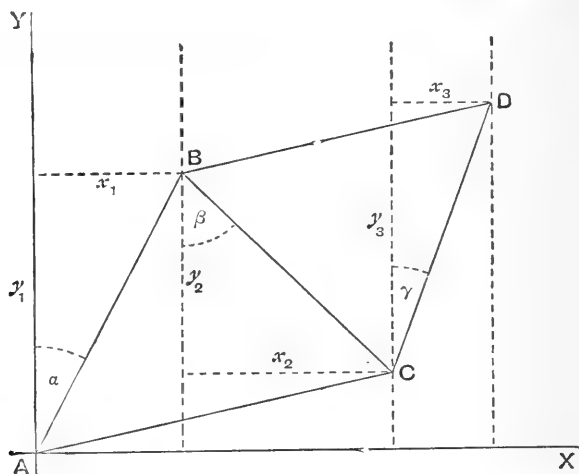


Fig. 115.

the earth's surface being treated as a plane, the *co-ordinates* of all the stations are calculated, the co-ordinates being measured along, and at right angles to, the initial meridian, thus:

If, as in Fig. 115, A be the initial point, or origin, A Y the initial meridian, and α the azimuth or true bearing, of the first side A B, then

$$x_1 \text{ (the co-ordinate at right angles to the meridian)} = AB \sin \alpha, \\ \text{and } y_1 = AB \cos \alpha.$$

A Y is called the axis of y_1 and A X, drawn at right angles to A Y, the axis of x .

(NOTE.—This represents a *great circle* of the earth, and is therefore not the same as the parallel of latitude of A, which is a *small circle*.)

If lines are drawn through B, C, D, parallel to A Y, it is clear that

$$\left. \begin{array}{l} x_2 = B C, \sin \beta \\ \text{and } y_2 = B C, \cos \beta \end{array} \right\} \text{where } \beta = A B C - a$$

$$\left. \begin{array}{l} x_3 = C D, \sin \gamma \\ \text{and } y_3 = C D, \cos \gamma \end{array} \right\} \text{where } \gamma = B C D - \beta$$

| | | | | | |
|---------------------------|---|-----|---|-------------------|-------------------|
| The co-ordinates of A are | | | | | o |
| „ | „ | „ B | „ | x_1 | y_1 |
| „ | „ | „ C | „ | $x_1 + x_2$ | $y_1 - y_2$ |
| „ | „ | „ D | „ | $x_1 + x_2 + x_3$ | $y_1 - y_2 + y_3$ |

All these points can now be plotted with reference to the origin, A, and its meridian.

It is obvious from Fig. 115 that the co-ordinates of C can be found direct from A, being—

$$C A \sin (a + B A C)$$

$$\text{and } C A \cos (a + B A C)$$

Similarly, the co-ordinates of D are—

$$D B \sin (180^\circ - \beta - D B C)$$

$$\text{and } D B \cos (180^\circ - \beta - D B C)$$

In this way an arithmetical check should be provided for every point.

The signs of rectangular co-ordinates are determined by two conventions:

(i.) x co-ordinates are considered as $\frac{+}{-}$ according as the point is $\frac{\text{east}}{\text{west}}$ of the origin.

(ii.) y co-ordinates are considered as $\frac{+}{-}$ according as the point is $\frac{\text{north}}{\text{south}}$ of the origin.

The co-ordinates are usually measured in feet. It is im-

portant to note that, once the initial meridian has been left, the angles β , γ , etc., formed by the sides with lines parallel to the initial meridian, are no longer the true bearings of these lines, and must never be used as such. They are usually called "false bearings," and differ from the true bearings by an amount equal to the "convergence of the meridians" (see p. 97 *et seq.*). It is clear that, if a chain of triangles run in an oblique direction, it may be convenient to plot along and at right angles to a line of known bearing, running in the same general direction.

RECENT DEVELOPMENTS.

The efforts of scientific men to meet needs arising out of the Great War have resulted in discoveries and inventions that may be expected in the near future to exercise a profound influence on nautical surveying.

Amongst these may be mentioned directional wireless, hydrophones, and the wire used as a ground log for measuring distances of considerable length. It is also to be anticipated that photography from aeroplanes may prove to be practicable for various purposes. Modern surveying vessels will no doubt be fitted with these appliances, but until further experience has been obtained it is yet too early to give an account of their practical use in the field. Their value, however, when placed at the disposal of the surveyor is obvious.

For work out of sight of land they will be of the utmost utility, but the main principles on which hydrographical surveying is conducted remain unaltered, and it is in the applications of the new methods to well-established principles that development may be expected.

The description that follows of the methods for surveying banks far from land, as used by Rear-Admiral F. C. Learmonth, C.B., need but little modification to conform with the procedure that must be adopted when hydrophones and the wire ground log are at hand.

The supreme value of directional wireless and hydrophones lies in the ability they give to the surveyor of obtaining a bearing between objects invisible from each other, within limits of accuracy which are negligible at long distances. As a means of

correction and adjustment of astronomical observations these inventions should be particularly useful, besides being applicable in all sorts of ways that will be apparent to the experienced surveyor.

Difference of longitude by W. T. places the surveyor in a position to be independent both of land lines and submarine cables.

CHAPTER XXI

REMARKS ON TRIANGULATION BY MEANS OF FLOATING MOORED BEACONS

AND

THE GENERAL PROCEDURE FOR VESSELS WHEN
CO-OPERATING OUT OF SIGHT OF LAND

BY REAR-ADMIRAL FREDERICK C. LEARMONTH, C.B., C.B.E.

PREFATORY REMARKS.

HYDROGRAPHIC surveys which depend almost entirely upon the use of floating beacons call for extra attention and vigilance on the part of those conducting them.

With the increasing size of vessels, whose load draught has already attained to beyond 36 feet, much of the future work of marine surveys will be necessarily occupied in a close re-examination of frequented areas, when any aids to accomplishing this end are gladly welcomed by the surveyor in the endeavour to keep in line with modern requirements.

There is nothing that can be described as new in the remarks that are here set forth, and no attempt has been made to lay down any hard-and-fast rules.

It is recognised that the circumstances of any two surveys are never likely to be the same, and that much must be left to the judgment of the officer in charge of a survey to meet the particular circumstances that may arise.

These remarks have been drawn up and are largely based upon the experiences of past years under varying conditions, including that gained in the North Sea Survey, which has been in progress since 1911, and carried out entirely by means of floating beacons by one directing vessel, together with one or more vessels co-operating.

As far as it is desirable, some attempt is here made to standardise methods and co-ordinate results, in order that the

ships engaged may be employed to the best advantage, when favourable opportunities of weather occur, a factor upon which all beacon work is more especially dependent.

When the vessels are so employed it may be taken as a safe indication of a proper organisation.

SECTION I.

SURVEYING FLOATING BEACONS, MOORINGS, AND FLAGS.

BEACONS.—The beacons now in use are the well-known cask type of surveying beacons, of the pattern of 1912, which embody various improvements (principally consisting of increased strength in the design as a whole), and include six galvanised iron tie-rods connecting the top and bottom pieces, also stouter sized poles and lengthened iron collars on the poles.

MOORINGS.—These are now universally of galvanised chain, size $\frac{1}{8}$ inch to $\frac{7}{8}$ inch—dependent upon the circumstances of depth, etc.—and are usually prepared beforehand in lengths of 35 fathoms.

Wire rope, size 2 inches to $1\frac{1}{4}$ inches special one-strand rope unit, can be used with advantage in deep water.

Owing to the liability to chafe, hemp moorings should be avoided, unless the depth renders it necessary.

In depths not exceeding 100 fathoms, about one and a half times the depth should be used for the length of the moorings.

ANCHORS.—The total weight of the anchors, backed with sinkers, in depths under 100 fathoms, should never be less than about one quarter of a ton, or about 500 pounds, and in a tideway this weight should be increased.

Various forms of anchors and sinkers have been used.

- (a) Sinkers: 5 cwt. Service pattern, flat bottomed, and mushroom shaped.
- (b) Anchors: 3 to 4 cwt., Admiralty pattern, with stock, backed with additional sinkers where necessary.
- (c) Boats' Anchors: Service pattern, 100 to 120 pounds' weight, backed up with eight $\frac{1}{2}$ -cwt. sinkers, either passed through or secured to the anchor stock.

Of these, the boats' anchors, (c), have proved the most effective, and held the best.

The 5-cwt. pattern sinkers (*a*), even when backed up by a small boat's anchor, have been found to drag unexpectedly; whilst the larger sized anchors (*b*) have, with the lapse of time, generally proved unsuitable owing to the mooring chain inevitably becoming entwined round the upturned fluke or arm of the stock, and consequently tripping the anchor.

ANCHORS—STOCKLESS.—It is proposed in future to employ stockless anchors of 3 to 4 cwt. (Byers' Patent Stockless Anchors, 4 cwt., are the best type), now generally adopted in the Service, as likely to give the best results. An extended trial with the Byers stockless anchors in the season of 1914 proved them to be very satisfactory.

A weight approximating to 250 pounds should be attached to the heel of the beacon spar in such a manner as will offer the least chance of fouling the moorings, and thus prevent the moorings canting the beacon spar.

The hydrographic pattern of 40 pounds galvanised ballast, with shackles, may be conveniently used as heel-weights. They can be supported by the one large shackle that is supplied, the pin of which is passed through the heel of the pole and securely moused.

To facilitate ready recognition and identification of each beacon from a distance, it is very desirable that each should be marked with large-sized numeral figures, painted in black or white, and occupying the full width of the cross-piece, top and bottom. Each beacon is further distinguished by a number of large-sized broad arrows cut on the top and bottom pieces for a depth of $\frac{1}{4}$ inch, together with metal tally-plates, with the name of the ship stamped on each tally, which latter is nailed to the cross-piece. This affords an additional means of recovery should the beacon be salvaged by fishing-vessels or by a stranger.

Tally-plates are also attached to the moorings, whilst the galvanised chain is marked at every 5 fathoms with a broad arrow.

BEACON FLAGS.—Approximately 15 feet in breadth along the staff and about 12 feet in length. Of special strengthened calico material of superior quality, made of variegated colours, sewn in *horizontal* widths of black and red, with occasionally a white strip. The darker colours are generally the more easily distinguished.

These large-sized flags require to be specially strengthened with sacking or canvas material about 6 inches wide, sewn on both sides of the calico along the length of the leech nearest the beacon spar. In addition, goring pieces should extend along the top and bottom for 18 inches. The edges of the flag are hemstitched throughout, including the edges of the com-

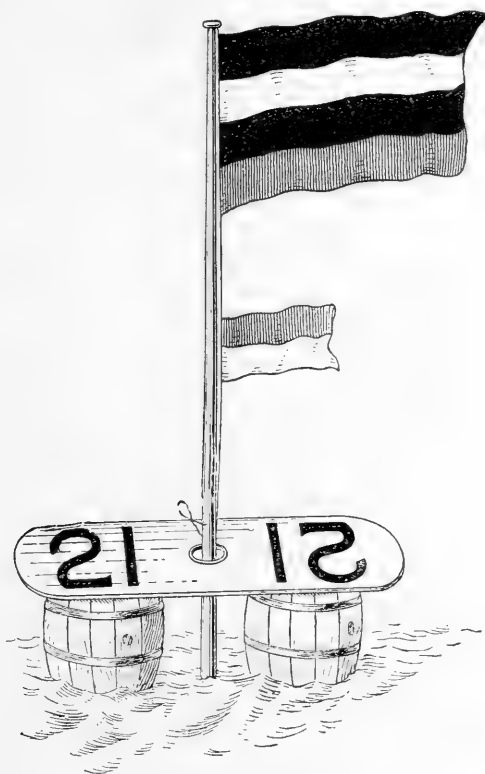


Fig. 116.

ponent parts. Such design best withstands the strain occasioned in a strong wind, and if so constructed will last a long time (first gradually fraying from the fly), and are well worth the additional time involved in making them.

When the loss of time that is involved should it become necessary to replace any beacon owing to the tattered condition of the flag is remembered, it will be seen that much

depends upon the maintenance of the flags. The risk of damaging the beacon is great should any attempt be made to replace the flag from a ship under way except under the most favourable conditions of weather.

A boat is not suitable for this, for we should not depend upon a block and halliards for securing a flag of this nature, which should be stopped along the staff at regular intervals from the metal grommets inserted along the strengthened canvas piece.

The bamboo being 35 feet in length, it will be found advantageous, and facilitate recognition, if one or more smaller sized flags are attached beneath the larger sized flag (see Fig. 116).

CAUTION.—The beacons should be distinguished as far as possible by their particular flags, bearing in mind that the colours should be arranged in such a manner as will not render them liable to be mistaken as National flags.

Under favourable conditions it is quite possible to reflect these beacon flags with a sextant when they are ten miles distant and the eye elevated not less than 30 feet.

A record should be kept in the data book giving full particulars of the dimensions of the bamboo.

This is often useful for obtaining short distances by subtense angle.

SECTION II.

SCHEME OF TRIANGULATION.

See Figs. I., II., III., IV. (at end of Section XXV.).

Dependent upon the nature of the work in hand and the area to be examined, some considered scheme should be adopted.

Generally speaking, if we are relying entirely upon astronomical observations we cannot do better than to lay out the beacons in parallel lines, either in a north and south or east and west direction, the lines of the beacons being approximately $3\frac{1}{2}$ miles apart, whilst the beacons take the form of equilateral triangles, the lengths of the sides being approximately 4 miles.

We then have the readiest means of checking the triangulation near its two extremes, either by difference of latitude or longitude, or other forms of position lines combined with observed true bearings.

The distance between the beacons should not exceed,

approximately, 4 miles. This will be found to give the best radius of action, station pointer fixes being the desiderata to be aimed at, unless the scale is less than one inch to the mile, when the compass and sextant may in certain cases be resorted to for fixing.

Should the area involved be in sight of shore objects, the positions of which are already known, and which are visible under favourable conditions either by day or by night—such as lighthouses, lightvessels, or distant mountain peaks—it would be well to arrange the floating triangulation so that it may be possible to obtain a station pointer fix or reliable sextant angles to these objects, combined with observed true bearings from positions near the two extremes of the triangulation.

It will thus appear very essential to prepare a rough diagram beforehand, especially when two or more ships are co-operating. The diagram should show an outline of the proposed scheme of the triangulation, giving the approximate positions and names for the beacons, and all other explanatory information required.

Fig. I. illustrates a beacon triangulation independent of shore or other objects, showing the disposition of thirteen beacons moored at distances of 4 miles apart in two parallel lines in a north and south direction $3\frac{1}{2}$ miles apart.

The irregular-shaped figure embraces an area of 395 square miles (approximately), within which a “station pointer” fix without resort to the compass is everywhere obtainable, with the beacons visible up to 8 miles.

Fig. II. illustrates the position of thirteen beacons arranged in three parallel lines at the same distances as in Fig. I., the area in this case being reduced to approximately 370 square miles.

The disposition here shown is generally more suitable when examining a bank or shoal area with the aid of boats, when a necessarily increased number of fixed objects are available in the central portion of the area.

It may be stated here, as a general guide, that about twelve beacons will be found to be a suitable number to have moored at any one time if conducting a general or extensive survey; but it is recognised that this number is largely dependent upon the varying circumstances and the number of ships engaged.

In the general survey of the North Sea, conducted generally on the 1-inch scale, with three ships engaged simultaneously, this number of beacons, which has not often been exceeded, has been found to give the best practical results, bearing in mind the special conditions—*i.e.*, prevailing haze, continued presence of fishing fleets and passing vessels, desirability of detailing an independent area for each vessel which is readily under the supervision of the controlling ship, and within which area the beacons may to a certain extent be safeguarded by the presence of that vessel; also, that information relating to those beacons can thereby be kept up to date and circulated.

Fig. III. illustrates a triangulation by beacons connected to shore objects by angles taken simultaneously from two ships at Nos. 1 and 13 to prominent lighthouses visible after dusk near their extreme range, the searchlights of the two ships being intervisible and true bearings obtained. The area to be surveyed is shown embraced by the pecked line.

Fig. IV. illustrates the floating triangulation connected to the shore objects, where it is desired to fix the position of the lightvessels in addition to examining the special area covered by the beacons.

The triangulation will be completed by three ships, X, Y, and Z, during one day and two successive evenings, X being the controlling ship.

Lightvessels and lighthouses visible at night in clear weather.

Assistants on board the four lightvessels.

Shore objects not visited.

OUTLINE OF GENERAL PROCEDURE.

First Day.

X drops Beacon 1 (also Mark Buoy, being controlled by Z at Beacon 1), and finally Beacon 2 (being controlled by Y at Mark Buoy).

(a) X, Y, and Z determine provisional scale for distance between Beacons 1 and 2 (for which see Section III.).

X drops Beacons 5, 7, 9, revisiting them in reverse order; also 1 and 2.

Y drops Beacons 3, 4, 6, 8, revisiting them in reverse order.

Z drops Beacons 10, 11, 12, 13, revisiting them in reverse order.

Vessels then unite at Beacon 1, and on data for provisional plot being made known by X, all vessels sound as detailed within the area to be surveyed.

First Evening.

X anchors near Beacon 9, after putting assistant on board Cromer Knoll and Outer Dowsing lightvessels.

Y anchors between Outer Dowsing and Dudgeon lightvessels, after putting an assistant on board Dudgeon lightvessel.

Z anchors north of Haisborough lightvessel, after putting an assistant on board Haisborough lightvessel.

Triangulation is carried out that evening as per diagram.

Second Day.

X, Y, and Z continue sounding, and pick up the assistants at the lightvessels, Y and Z communicating with X, as convenient, their triangulation data.

Second Evening.

X anchors near Beacon 9 as before.

Y anchors in a central position between Beacons 1 and 9.

Z anchors near Beacon 1.

Triangulation completed, and bearing and scale of beacon triangulation completed that evening.

The following day all three ships continue sounding after communicating their data to X. X picks up assistant at Cromer Knoll lightvessel, and completes the final calculations, transmitting the revised plot to Y and Z that evening, for which purpose all three vessels occupy some prearranged central positions for the night.

SECTION III.

PRELIMINARY PROCEDURE—OBTAINING THE SCALE.

The general procedure when laying out a beacon triangulation with three ships present would approximately be as follows:

Referring to Fig. III., which for our purpose will serve as a general illustration, the details can be adapted to meet varying circumstances.

FIGS. III. AND V., CASE 1.

Three ships present, X, Y, and Z, X being the conducting and preferably the largest vessel, and the scheme made known. No shore or other objects then in sight.

2. X drops Beacon 1 in a position where it is known, or may be reasonably anticipated, that the shore objects will be visible in clear weather, especially at night.

X then remains under way, keeping a close but safe distance from Beacon 1.

3. Ships Y and Z drop Beacons 2 and 3 respectively in their approximate prearranged positions, 2 miles in opposite directions from Beacon 1, and for which purpose the masthead angle and compass bearing of X will suffice.

OBTAINING THE SCALE.

Before proceeding to lay out further beacons we now require to obtain a scale of as accurate a nature as immediate circumstances permit, without which we cannot ensure the extension of the triangulation in the required direction and distance.

Upon this scale and the compass it will very likely be necessary to depend for some days, before clear skies and favourable weather enable us to obtain the necessary connection with the shore objects, and true bearing or observations by twilight stars for astronomical position, or whatever the nature of the circumstances may be upon which we ultimately depend.

4. On Beacons 2 and 3 being dropped, Y and Z take up positions at anchor equidistant from X, on opposite sides, and on a line of bearing from X approximating to that at right angles to the general direction of these beacons. The distance of Y and Z from X is governed by the sensitiveness of the angle subtended by the extreme length of X, situated near Beacon 1 when presenting her broadside approximately to Y and Z. Near the bow and stern of X well-defined marks, such as cones or balls, are exhibited about the same height; the "ensign" and "jackstaff" or "bowsprit end" generally serve for this, the desiderata being to obtain, if possible, a receiving angle at the extreme beacons subtended between X and each ship when in position of not less than about 40 degrees each.

This distance can be readily found by inspection of tables, or from the plot prepared beforehand, and approximates to $1\frac{3}{4}$ miles for a length of, say, 200 feet. Y and Z readily pick up their respective positions by cross bearings of the beacons from the rough plot now in progress on board each ship or the masthead angle of X.

It is not essential that Y and Z should anchor, but from what follows it will be seen that more reliable results are likely to ensue, and the difficulty of controlling simultaneous observations will be much diminished, if the ship from whose length it is intended to obtain the scale is the only one under way.

5. Y and Z, when ready to observe, display a large flag at the dip where best seen. X answers by one or more flags at the dip at each mast, which are mastheaded when X is stopped and steady, and presenting her broadside very nearly to both Y and Z, who answer the signal in a similar manner.

6. *Executive Signal* for observing is made by X smartly dipping flags, answered by Y and Z, when it will be seen that the following data and angles are required to be simultaneously observed on board the several ships.

Here it may be remarked that in all work of this kind nothing **Caution.** should be left to chance, for confusion, with inevitable mistakes, serious omissions and delays, would ensue, possibly jeopardising the whole plan, if not necessitating a repetition of some kind. It will therefore be one of the earliest duties of the commanding officer to detail to each assistant the angles to be observed by him and any compass bearings required, all of which should be clearly written down beforehand in the deckbook, with the approximate values and initials of the responsible assistant against them, and any information of value for identification or future discussion.

All entries should be made corrected for instrumental errors.

In this selection each commanding officer will be guided by the skill-experience of the assistant; his powers of vision; the importance attaching to the particular angle required; instruments available; visibility of objects. As far as possible some organised simple and generally applicable method should be adopted throughout—a fact which each assistant will soon appreciate.

All angles throughout should be taken from one selected spot

in the ships, preferably round the standard or bridge compass, above which should be displayed some suitably distinguishable object to which to observe from the other vessels, all of whom observe the upper part of the beacon flag.

FIGURE V., CASE 1.

The following essential data should be obtained simultaneously as far as possible:

On board X (the controlling ship, under way near Beacon 1).

1. The four angles between Y and Z and the Beacons 2 and 3, whose sum equals 360° .
2. The magnetic bearing of an object in the scheme easiest to observe, and at a suitable distance.
3. The direction of the ship's head.
4. The connection of Beacon 1 with the scheme.
5. G. M. T. or local mean time of the observation. (For identification with Y and Z.)

On board Y and Z.

1. Angles between X and the two extreme beacons measured through both X and the ship visible beyond X.
2. Horizontal subtense angle of the length presented by X between the two distinguishing marks near the extremes of the vessel (a varying quantity). Measured with observing sextant as accurately as possible.
3. Magnetic bearing of an object in the scheme easiest to observe, and at a suitable distance.
4. The connection of Beacon 1 with the scheme.
5. G. M. T. or local mean time of the observation. X should endeavour to take up a position so as not to mask the view between Y and Z.

The connection of Beacon 1 with the scheme can also now be made on board each ship, or subsequently, when revisiting the beacons.

The signals should be repeated after sufficient time for recording has elapsed to enable three independent sets of observations to be obtained, X remaining as steady as possible throughout.

FIGURE VI., CASE 2.

With two ships, X and Y, available only as in Fig. VI., the procedure would be something as follows:

Y drops Beacon 1 and X drops Beacon 2 on the prearranged bearing, approximately 2 miles from 1, X being controlled for distance by Y, who remains under way near 1, whilst observing masthead angle of X.

Y then takes up a position at anchor by the aid of Beacons 1 and 2, approximately $\frac{3}{4}$ mile from 1, and in a direction at right angles to that of the beacons. X next drops Beacon 3 on the opposite line of bearing, the distance being controlled from Y at anchor, who will communicate her position to X, if required, by cruiser arc lamp, or other suitable means, whilst X is proceeding from Beacon 2.

On Beacon 3 being dropped, approximately 2 miles from Beacon 1, X takes up a position under way about $1\frac{1}{2}$ miles from Y, and on the opposite side of Beacon 1, as in Fig. V., when a similar procedure will be adopted for obtaining the scale as previously described.

It will be seen that X and Y now occupy the best positions for determining the distance between Beacons 2 and 3, 4 miles apart—the receiving angle at these beacons between the two ships being approximately 45° .

CASE III.

Should one ship only be engaged, the procedure would be generally the same as in Fig. VI., a boat containing at least two assistants being substituted for the ship Y.

NOTE.—That side of the beacon where the light or background is best for observing X should be selected by the ship or boat when taking up her position.

SECTION IV.

EXTENDING THE TRIANGULATION.

In continuation of Section III., and referring again to Fig. III., which will serve as a general illustration of our purpose, the general procedure would be something as follows:

On completion of the last set of simultaneous observations

for scale determination, should no repetition be required, Y and Z at once weigh and close on X, for all ships now require the approximate bearing and distance of the two extreme beacons (Nos. 2 and 3) before proceeding farther.

From this Y and Z signal to X by some concise and simple prearranged method (for this see also Sections VIII. and XI.) their observations, which are repeated back to ensure the correct figures having been made and received.

Two assistants on board X, detailed as computers, at once deal with such data as will suffice for the calculation of, say, two independent values of the bearings and distance between Beacons 2 and 3.

This is quickly effected by the solution of the four plane triangles involved, such as : X Y Z (cosine rule), Y Z 2, Y Z 3, Y 2 3, Fig. V., the scale for which is found by the customary formula :

$$\left. \begin{array}{l} \text{Distance} \\ \text{in miles} \end{array} \right\} = \frac{\text{No. of feet subtended by ship} \times 34}{\text{Angle subtended in seconds by ship's length'}}$$

Each simultaneous set of observations admits of separate solutions for the distance between the extreme beacons dependent upon whether the observations of Y and Z are taken separately with those of X, or combined. The accepted value for this distance can be subsequently derived in the final consideration of all the observations when time permits.

Having thus determined the provisional bearing and distance, X communicates the same to Y and Z, when the rough plot on board each ship is revised accordingly.

SECTION V.

LAYING OUT BEACONS FOR TRIANGULATION.

All three ships at once proceed, on a prearranged plan, to drop the necessary beacons in their approximate assigned positions, bearing in mind that if we are to be dependent subsequently upon the connection of the triangulation with the shore objects, then the selected position for the terminal beacon cannot, in all probability, very largely be departed from with-

out incurring the risk of being unable to obtain a satisfactory fix at this terminal position (see Fig. III.).

A suitable arrangement would be for—

| | |
|---|--|
| X | to drop Beacons Nos. 4, 7, 10, and 13, |
| Y | „ „ „ 5, 8, and 11, |
| Z | „ „ „ 6, 9, 12, and 14, |

following in approximately numerical sequence.

The rough plot is kept up to date as we proceed by compass bearing and angle, and all available data recorded as each beacon is dropped.

SECTION VI.

OBTAINING ANGLES AT THE BEACONS.

On the necessary beacons being laid out, the work should be so arranged that it is now possible to revisit every beacon in quick succession whilst the tidal stream is running in one direction only, leaving a sufficient subsequent margin of time to enable the several ships to take up their assigned positions with certainty whilst daylight remains, should it be contemplated and the weather favourable for connecting the triangulation with the shore objects by night.

Slack water occurring about midday would be very suitable for the present work, which should not have up to now occupied more than four hours since commencing.

| | |
|---|---|
| X | would visit Beacons 13, 8, 5, 3, and 1 in that order. |
| Y | „ „ „ 11, 10, 7, 4, 2, in that order. |
| Z | „ „ „ 14, 12, 9, and 6 in that order. |

In obtaining these angles some care and forethought is necessary, involving a systematic procedure if we desire to obtain accurate results.

Some such method as the following should be adopted.

Before coming up to the Beacon.—1. Ship to be stemming the tidal stream at slow speed with the beacon about one cable distant, and the ship heading in the same direction as she will be when subsequently stopped at the beacon.

2. The assistants in readiness to observe the angles for which they are detailed, the approximate data being taken from

the plot and entered beforehand in the deck-book in light pencil. If more than one angle to be observed by any assistant, the order in which they are to be taken should be known.

3. All entries to be kept up to date in the deck-book. It will be found advisable to keep separate pages of the deck-book, which should be indexed, reserved for the entries at each beacon, for doubtless we shall require to revisit the beacons from time to time subsequently for various causes and much confusion would then arise with entries relating to the same beacon scattered throughout the book (see also Section XVIII.).

4. Some prearranged warning should be always given to the engine-room, such as will ensure prompt obedience to the telegraph.

5. It may be necessary to wait before going up to a beacon until any of the ships have quitted any beacon that they may be then at, or will obviously reach before us, so that nothing may be masked, a mutual arrangement that should be well understood.

With the Ship at the Beacon.—1. It will be found generally best for the commanding officer to handle the ship entirely. His attention being thus engaged, he will not therefore be available to observe any but the less important angles, should we be short-handed.

2. With proper preparation, and given clear weather, the time occupied at the beacon should not exceed two or three minutes, during which it will generally be found best (dependent upon the circumstances of the tidal stream, wind, and sea, etc.) to maintain the ship stationary, with the beacons on the port side nearly abreast of the bridge, say from 15 to 5 yards distant (for reason, see later), the bearing of the beacon and the subtended angle of the bamboo staff being observed from time to time by the commanding officer, or a range-finder used.

With the bridge forward as is now general the risk of fouling the beacon is very small, a slight movement of the engines and helm being all that is required, whilst the beacon should not be allowed to draw aft.

It will be found advantageous to make it a practice to pass the beacons on that side opposite to which the sounding gear

is rigged. This side should always be used for handling the beacons, the boat's davits being turned inboard or fore and aft, to avoid the possibility of fouling the flag or bamboo. Hands should be stationed in readiness with "bearing-out" spars should the ship get dangerously close, for, above all things, the ship must not fall on top of the beacon in an endeavour to get close to it.

3. As an orderly method, the same assistant should be detailed to make all the entries in the deck-book, being responsible to the commanding officer that this is done, and invariably reporting the fact whether all the angles arranged for have been obtained before the ship is out of position from the beacon.

SECTION VII.

ANGLES REQUIRED AT THE BEACONS.

It will be seen that the essential angles required at any one beacon are those between the beacons that are nearest to it, and which constitute the angles of each of the adjacent, approximately, equilateral triangles. If the weather is clear endeavours should also be made to observe as many of the more distant beacons as is possible in the readiest manner; but, generally speaking, any undue time striving for this is misspent, unless the ship is at some important point, when it may be necessary to make some extra effort from aloft, etc., in order to observe distant lightvessels or shore objects that may prove of subsequent value.

The officer in charge when arranging the sextant angles should make sure that their all-through connection is complete and obtained in the best possible way. Should there be many angles to obtain all round the horizon (such as in the centre line of beacons in Fig. III.), care in their preparation beforehand is necessary, and a diagram should be referred to so as to ensure that there are no omissions.

Visiting all the Beacons successively in the Manner arranged.—The three ships would unite about Beacon 1, when Y and Z would signal their angles to X, or, if the weather is suitable, time will be saved if typed copies already prepared be transmitted by boat.

SECTION VIII.

SIGNALLING ANGLES BETWEEN SHIPS ENGAGED.

When signalling a series of angles between ships, some simple method must be adopted that is not liable to confuse signalmen, and that will reduce the risk of mistakes.

It should be an invariable rule that all figures, bearings, and the essential data, should be repeated from the receiving ship until they are known to have been correctly received.

It is necessary to distinguish the beacons by prearranged one-syllable names.

A signal would read in order thus (the bearings being always magnetic unless otherwise stated, and the time quoted being that in use aboard all the ships, such as G.M.T. or local mean time):

Specimen Signal (see Fig. III.).

Endeavour at TRY Beacon. July 27th. 1.30 p.m.

| | | | | | |
|-------------|----|----|----|----|---|
| Beacon | .. | .. | .. | .. | N. 75° E. 12 yards. |
| Ship's head | .. | .. | .. | .. | S. 30° E. |
| Current | .. | .. | .. | .. | N. 30° W. Strong. |
| Wind | .. | .. | .. | .. | E. S. E. 3. |
| SEA | .. | .. | .. | .. | 58° 46' QUICK. |
| SEA | .. | .. | .. | .. | 121° 03' CLEAR. |
| CLEAR | .. | .. | .. | .. | 59° 11' LEG. |
| CLEAR | .. | .. | .. | .. | 116° 32' CORN. |
| CORN | .. | .. | .. | .. | 61° 02' MID. |
| CORN | .. | .. | .. | .. | 122° 25' SEA. |
| HASTE | .. | .. | .. | .. | 31° 11' QUICK. |
| VIEW | .. | .. | .. | .. | 40° 53' CORN. |
| DROP | .. | .. | .. | .. | 29° 27' CORN. |
| SHORE | .. | .. | .. | .. | 35° 43' SEA. |
| TURN | .. | .. | .. | .. | 4° 17' SEA. |
| TURN | .. | .. | .. | .. | Beacon flag foul. |
| DROP | .. | .. | .. | .. | Beacon very difficult to see, flag not blowing out. |

FAR and DUSK not visible.

Angles in all other cases easy to obtain.

SECTION IX.

PREPARATION OF THE ANGLES FOR CALCULATING
AND PLOTTING.

On board the directing ship X the angles as obtained at the beacons will have now been dealt with by the two or more computers, working in pairs, and the angles corrected for the various false stations involved before being tabulated for future use.

This at first sight might appear a laborious process, and of doubtful utility, but in actual practice it is not so, for it is necessary in any case to close the various triangles involved in the plotting, and this cannot be done without first treating the various false stations, some of which may be of large amount. We shall not require corrections nearer than $1'$ of arc, and these quantities can be readily obtained by inspection from a subtended angle table, and applied by the well-known method given at p. 90. To enable this to be done, time will be saved if the sextant angles at each position are expressed as theodolite angles from some convenient zero, the approximate bearing of which can be taken from the plot.

SECTION X.

RAPID PLOTTING.

The angles received from the co-operating vessels should be treated in a similar way on board the directing ship, and it is perhaps better that they should be transmitted in their original form, as any discrepancy can then be more readily discovered, and at once cleared up.

Dependent upon the care that has been taken, the various triangles, which must be made up correctly for calculation or plotting, should close with a total error that should not exceed $10'$, and will be generally less in practice.

If we require a rapid provisional plot, which is often the case, in order that the circumstances of the day may be taken full advantage of, it will be generally quite sufficient to plot the beacons in their order, adopting the provisional scale which has now been fully settled upon (see Section III.).

The use of large-size station pointers, combined with chords where necessary, with checks for the distances, computed as the plotting proceeds, will afford the readiest method.

For a provisional bearing we shall be mainly dependent upon carrying on that first obtained when determining the scale (see Sections III. and IV.), until opportunity occurs for a more reliable one to be obtained.

SECTION XI.

SIGNALLING THE PLOT IF UNABLE TO COMMUNICATE BY BOAT WITH THE VESSELS ENGAGED.

When desirous of communicating to other ships present the result of any plot, time will be saved, should the weather be unsuitable for communication by boat (when a tracing should be sent), if this is done in the form of a concise signal, the angles to the various objects being expressed as theodolite readings, and distances expressed in inches. This data may be taken in most cases with sufficient accuracy from the plot by means of reliable station pointers, whose errors are known, together with beam compasses, selecting for this purpose any two extremes of the triangulation as a common zero line, the bearing of which should also be given.

On this signal being repeated back correctly, it can be quickly plotted on board the receiving ships, the data given being sufficient to check any doubt or evident mistake should it arise.

Care, of course, is necessary in the first case, and the data to be signalled should be checked independently before transmission, a tracing of the plot being sent on the first opportunity in confirmation of the signal.

Specimen Signal.

Signalling the Plot (see Fig. IV.).

Signal.

At "HAD" Beacon.

| | | | | |
|---------|----|----|----------|-------------------------|
| HEIGHT | .. | .. | 360° 00' | 17.14 ins. N., 27.51 W. |
| WING | .. | .. | 2° 57' | 4.21 " |
| PLANE | .. | .. | 15° 13' | 15.37 " |
| VOL .. | .. | .. | 22° 17' | 11.78 " |
| SAIL .. | .. | .. | 30° 31' | 7.27 " |

At "HAD" Beacon—Continued.

| | | |
|-------------------|----------|------------------------|
| FLY | 62° 09' | 4.49 ins. N., 27.51 W. |
| PERCH | 116° 41' | 4.78 .. |
| CLAW | 188° 03' | 4.07 .. |
| TIP | 246° 07' | 4.19 .. |
| BEAK | 300° 43' | 4.15 .. |
| CROMER LT. VL. .. | 313° 01' | 12.38 .. |
| OUTER DOWSING LT. | | |
| VL. | 350° 33' | 28.77 .. |
| SEAT | 358° 29' | 8.41 .. |
| PASS | 539° 04' | 12.92 .. |

FINIS.

At "HEIGHT."

| | | |
|-----------|----------|-------------------------|
| HAD | 360° 00' | 17.14 ins. S., 27.51 E. |
|-----------|----------|-------------------------|

(Signal continues, etc., etc.)

SECTION XII.

ACCURACY OF THE PROVISIONAL PLOT.

It may be convenient to state here that many instances could be given in the writer's experience of the accuracy that it is possible to obtain by the methods we have so far employed, as manifested by the subsequent close agreement of the beacon triangulation when connected with the shore objects. A replot is not necessary, though some alteration in the provisional bearing may be expected. The soundings dependent upon the sextant angles to the beacons that may have already been in progress are not affected thereby.

SECTION XIII.

CONNECTION OF THE FLOATING TRIANGULATION WITH
THE SHORE OBJECTS AND DISPOSITION OF THE
SHIPS ENGAGED.

See Figs. III. and IV.

With conspicuous elevated shore objects this may be done during daytime dependent upon the weather. Any disposition of the ships that may be contemplated near the extremes of the triangulation should be so arranged that the ships are intervisible at the greatest distance possible.

With the three ships engaged, and dependent upon the visibility of the distant shore lights as triangulated points, a suitable method would be to place two ships at anchor near by beacons marking the two extremes of the triangulation, and with which they can be connected by compass bearing, the distance from each beacon being obtained by a sensitive mast-head angle or range-finder, whilst the ships occupy such positions as will enable them to connect the beacon triangulation by sextant angle with at least one shore object before darkness sets in, when the shore or other objects upon which we rely for our fix, such as lighthouses or lightvessels, will then become visible.

It would be well to so arrange matters that the searchlights or cruiser are lamps of these two ships may be intervisible, whilst the third ship occupies, before dusk, some suitable central position that will ensure that the two extreme ships and as many beacons as possible are visible, whilst as a secondary condition she is able to keep within visible range of as many shore objects as possible.

The selected positions for each vessel should be understood beforehand by all concerned.

Vessels so disposed afford the best means of connecting the triangulation with the shore objects and checking all previous work.

CONNECTING THE SHIP WITH A BEACON NEAR BY FOR THE PURPOSES OF TRIANGULATION.

With no range-finder available, it may be necessary to send a boat to the beacon to obtain the masthead angle before dusk, or that subtended by two lights, whose vertical or subtended horizontal distance apart is known.

If the weather is unfavourable for this, a reliable distance can be obtained by observing simultaneous angles to a particular part of the beacon by two observers intervisible, situated at the ends of the ship at some known distance apart, whilst the ship remains under way or is anchored, so as to have the beacon approximately abeam, and at a safe swinging distance when the ship has taken her cable.

SECTION XIV.

OBSERVATIONS AT NIGHT—GENERAL CONSIDERATIONS.

Endeavours should be made, if possible, to so arrange matters that whilst the ships are observing they may be all swung to the same general set of the tidal stream, and for this purpose slack water occurring near about midnight would be suitable, and before which all ships should have completed their observations, say, in the summer months.

It may be frequently necessary to obtain some of the angles from aloft, for which some preparation should be made beforehand. With no top or lookout platform available, it would be well to extemporise this, together with electric control switch for reading and writing, and thereby add much to the comfort and expedition when observing, and minimise the chance of injury to the instruments. An assistant should also be aloft to record, and a signalman to work the cruiser arc lamp.

No hard-and-fast rule should be laid down for working the searchlights or arc lamps when directing them for other vessels to observe, or when requiring their attention.

It will generally suffice when the ships are at such a distance as to render Morse signals impracticable, or their use involves much delay, if the following procedure be adopted:

1. A succession of short flashes directed from A to a vessel B indicates that A wishes to observe B.

2. B answers by a succession of short flashes, upon which A shuts off her light to B (unless B also requires to observe A).

3. B then maintains a steady light on A.

4. When A no longer requires B, A makes a succession of long flashes, which is answered by B shutting off her light.

It is as well to remark here that in order to obtain precision some practice is required in observing angles at night between the lights of lighthouses and lightvessels of varying periods.

It may frequently happen that their periods are such that they may only synchronise momentarily after long intervals of perhaps fully one hour, and then only for a comparatively short duration. This feature, together with uncertain and changing conditions of weather, motion of the ship, etc., increases the difficulties at night, and affords additional reasons

for not laying down too many rules between the ships; and it can be safely said that we cannot do wrong if we direct our signals steadily divided between the ships engaged, who may be experiencing like difficulties, and whose efforts may possibly be concentrated at one time in other directions than the other ships engaged.

It will greatly help the observer, and save fatigue, if he is prepared beforehand with the approximate value of the angles required, whilst the assistant, with a watch, times the periods of the lights, giving the observer sufficient warning beforehand of their reappearance.

The less difficult angles can be obtained from the bridge, also the true bearing, by the other assistants.

Should the searchlight be screened by ships' fixtures, etc., obstructing the view in the direction it is required to use it, the situation may be often saved by adapting the searchlight as a heliostat, using the simple form of portable mirror upon which to direct the beam, from which mirror rays can be directed in the required direction with the aid of an extemporised movable directing arm.

It is often possible to observe reliable angles to the beam of a searchlight when it is dipped beneath the horizon, if care is taken, in the first place, to train the projector in the direction it is required to be observed from.

It should be the duty of an officer to control the searchlight and ensure that it is being properly trained. This should not be left to the crew, who soon become inattentive, especially if the ship has motion and is yawing about and it is not possible to see the horizon or the object upon which it is required to train; or, again, on a dark night with no guide, such as a star, etc.

A boat's compass near at hand is often a help, and occasional comparison with a ship's compass should ensure the ray being observed if a sufficient margin of training is allowed for.

All data inserted in the deck-book should include the time being entered against each observation, for should the angles differ to any extent from various causes, it may be necessary on a subsequent review to select those obtained simultaneously at a particular period in order to best combine them with the observations obtained by the other vessels engaged.

When treating the angles, it is necessary to consider the various corrections occasioned by change of position in each ship, searchlights situated at remote distances, angles observed from different positions on board, all of which should be referred to a definite point, say the standard or bridge compass of each vessel, from which positions they can be then easily referred to the beacon near by if necessary. Such corrections are quickly dealt with in the manner previously described if the necessary data are recorded at the time and not left to the memory. A rough diagram illustrating the positions of the various ships, their searchlights, masthead flashing lamps, etc., and the beacons near by, will much facilitate this operation should the corrections appear somewhat involved in their application.

It is well to remember that the succeeding morning twilight may often help to fill up any important gaps that there may have been overnight, and thereby save the situation. We should also be ready to take advantage of the clear atmosphere that often occurs and precedes the haze that increases as the heat of the day progresses during a period of fine weather.

The vessels engaged should therefore, as a general rule, not quit their overnight positions before a stated time in the morning, and display a large flag or other prominent mark from the masthead whilst in position during daylight.

The vessels will in the morning then be doubtless swung to the opposite tidal stream, and the necessary corrections must be applied before treating the angles then obtained with any former that have been taken when swung in a different direction.

SECTION XV.

ADJUSTMENT OF THE TRIANGULATION.

When the data thus obtained are received by the directing vessel, the necessary calculations can be made for determining the several ships' positions. When station pointer fixes have been obtained this is as described on p. 170.

The bearings calculated from the fixes which are dependent upon the original known data relating to the triangulated shore objects will here afford a valuable check with those that we have been able to observe. Should there be a disagreement in the

results, a comparison of all the bearings derived from the different sources when referred to two common points, with the aid of the sextant angles connecting them, will enable us to determine the best value to adopt for the bearing and scale, after due consideration has been given to the conditions appertaining to each position. If the conditions are favourable, and proper care has been taken, the difference should never amount to much.

It may be here remarked that at any ship's position it is possible we may have:

(a) A bearing actually observed.

(b) A bearing calculated from the station pointer fix.

And thus, (c) the comparison of (a) and (b) with similar data obtained independently from the other vessels engaged, all of which are referred in the most direct way to a common line of bearing joining two suitable points.

SECTION XVI.

ASTRONOMICAL OBSERVATIONS AND THE CONSIDERATIONS ATTACHING TO THEM.

If entirely dependent upon astronomical observations, it is well to arrange the daily programme of sounding so as to obtain solar position lines, a.m. and p.m., which should include ex. meridian altitudes, direct and supplemental when possible, on opposite lines of bearing by each ship on successive days, taken near the extreme beacons embracing the triangulation which are the best situated for obtaining the scale.

To reduce the possibilities of error, it is best when observing to maintain the ship so that the position line then derived will also pass approximately close to the beacon.

Twilight stars, a.m. or p.m., taken on two or more successive nights at suitable positions near the extremes of the triangulation, will give the best results, and from this the scale can be derived.

A reliable bearing is best obtained by means of the ships disposed at night as in Section XIII., or in a manner that the third ship is situated so as best to connect the two extreme ships should they not be intervisible.

In order to obtain any degree of precision it is necessary to observe every precaution, amongst which may be mentioned:

Each observer to use the same instrument throughout, whose errors are satisfactorily ascertained from time to time.

The observer's eye from a protected position to be always at the same height above the sea, and this as low as possible as will enable a clear definition of the horizon with due regard to the then state of the sea and motion of the ship.

Before commencing observations it is often very necessary to scan that part of the horizon upon which we are dependent carefully with a pair of binoculars.

This is more especially the case when passing clouds or showers give way to bursts of sunshine, or in calm weather when haze or the smoke of passing vessels lie in a horizontal strata. An inspection will often reveal a false horizon due to these causes, which otherwise would not have been detected, and thus necessitate waiting until better conditions prevail, although the likelihood of the heavenly body remaining unobserved appearing uncertain would form an inducement not to do so.

SECTION XVII.

CHECKING THE POSITION OF THE BEACONS AND THE SCALE SUBSEQUENT TO THE ORIGINAL TRIANGULATION.

It should be an invariable rule, throughout the progress of a survey dependent upon beacons, for all the vessels engaged to check the positions of the beacons at an early opportunity each day, on commencing work, or if remaining at anchor, when an all-round fix taken to all the beacons that are then in sight should be tested. Whilst sounding is in progress, check angles when fixing, the occurrence of transits, compass bearings; simultaneous fixes, using different objects, will serve for this without causing any undue delay.

It is not always an easy matter to determine at once how many, and which, of the beacons at any time no longer occupy their original positions when a discrepancy has been revealed. As soon as the "erring beacons" can be identified, and their new positions ascertained, they should be treated with caution as "doubtful quantities," likely to move again, probably owing

to the moorings having tripped, and it may be generally best to replace them at once, dropping the new beacons beforehand.

The scale can be readily checked at any convenient time and place should there be reason to suppose that inaccuracy has crept in. This should always be done after heavy weather, or whenever it may be expected that the beacons have dragged or may be out of position from any cause.

SECTION XVIII.

ENTRIES IN THE DECK-BOOK RELATING TO BEACONS.

(See also Sections III. and VI., para. 3, and also Section XIV.)

In beacon work it is a good plan to reserve a space in the deck-book where all information relating to every beacon is there entered in a convenient tabular form, lists being prepared for each ship.

A specimen would include something as follows:

MACK BEACON.

| | | |
|----------------------------------|---|--|
| Name and No. | .. | MACK, No. 21. |
| Date dropped, and by whom; depth | 7 a.m., Monday, July 19th, 1911. | Depth, 23 fathoms. By <i>Hearty</i> . |
| Approximate position | Near north end of Swarte Bank. Lat., 53° 30' 12" N.; Long., 1° 58' 46" E. | |
| Distinguishing flag | .. | See Fig. 116. |
| Bamboo | | Length, 35' 4". |
| Casks | | Black colour. |
| Visited | | 1 p.m., July 19th, at S.E. stream (see page 14 in deck-book). |
| „ | | 8 a.m., July 26th, at N.W. stream (see page 15 in deck-book). |
| „ | | 6.30 p.m., July 31st, at S.E. stream (see page 16 in deck-book). |
| Moorings | | 35 fathoms. Galvanised chain, $\frac{5}{16}$. One 120-lb. boat's anchor, eight 56-lb. Baillee sinkers. Heel-weights, six 40-lb. galvanised iron ballast with shackles. |

| | |
|--------------------------------|--|
| Date recovered, and by whom | 4 p.m., August 5th, complete, by <i>Daisy</i> , and in correct position. Flag much tattered, one cask quarter full, which has been drained, repaired, and marked for future guidance. Moor- ings clear soft mud bottom. |
| General remarks .. | Which should include, if not recovered, by whom, when, and where last seen or known to have disappeared or be out of position. |
| Salvage | Particulars relating to subsequent sal- vage, if applicable. |

Without this information in a concise form, much uncertainty and confusion may subsequently arise, for these data will at once enable us to readily identify any beacon at any time from whatever cause, whether broken adrift, damaged, subsequently salvaged, or fouled by passing vessels, and also afford the means of tracing the losses or an interchange of stores, as it by no means follows that the vessel that dropped a particular beacon will necessarily be the one detailed to recover it as the triangulation progresses or when the survey is completed.

The necessity for all entries being at once made in the deck-book, where successive pages are reserved for each beacon, has been referred to in Sections VI. and XIV. whenever a beacon is dropped, visited, or recovered, whole or in part.

The responsible heads of departments affected, executive officer, and boatswain, extracting the necessary information for their guidance that they may require.

SECTION XIX.

RIGGING, HANDLING, AND GENERAL MANAGEMENT OF BEACONS.

This is referred to on p. 55.

The following points may be profitably touched upon. As it is most desirable to achieve success and celerity some organised method is essential when ships are mutually engaged.

Some such method as the following is found to work well in practice:

Rigging and unrigging, dropping, and picking up beacons should be considered to take the form of a recognised evolution.

The particulars as to length of moorings, ground moorings, distinguishing flags, etc., etc., having been settled beforehand, so many are then prepared as there is deck space available for, with one complete beacon triced up in position by the derrick or fore yard, with the heel of the spar secured over the ship's side in readiness for the beacon to be dropped. Particular attention should be given to the seizings securing the bamboo to the upper part of the staff, also those for the flag at top and bottom, and at each division of the different coloured breadths of calico.

All pins of shackles, whether fitted with screw or forelock, rings of slips, etc., etc., upon which we are dependent for the security of the beacon, should be well moused with seizing wire, for they will otherwise inevitably work loose with the constant wave motion and the straining of the beacon in the tidal stream.

The ship's berthing and rails, and all head gear included, with bowsprit or jib-boom if so fitted, should be kept unrigged and snugged down as far as is possible whilst engaged upon beacon work, with the boat's forward davits turned fore and aft on that side of the ship that is being used.

It will be found a good plan to always work the beacons on one side only, preferably the port side, leaving the starboard side with the sounding gear rigged.

Arrived at a spot for dropping a beacon, the commanding officer, whilst manœuvring the ship with due regard to the set of the tidal stream, wind, etc., in such a manner that, on the beacon being dropped, the ship will draw clear astern, gives the necessary instructions to the forecastle and to the assistants on the bridge.

If in doubt as to the set of the tidal stream, a boat's wire sounding machine will enable us to readily ascertain this, for we shall always require to know the depth beforehand.

The executive officer during beacon work, whenever he can be spared from the bridge, should himself direct the handling operations of the beacon from the forecastle, whilst the boat-swain with his party is more especially responsible for the moorings, which are better treated as distinct from the beacon itself.

The fact of the moorings reaching the bottom should always

be reported to the bridge as a warning to all; the beacon is then lowered to the water when all is ready and clear, the last operation being the cutting of the strand or seizing attached to the mooring chain, as this procedure is more likely to ensure the beacon getting clear of the ship on becoming finally released.

The flags should be rolled up as a sail, ready to be let fall, otherwise they will inevitably become torn round the derrick head, or catch in the fittings on the ship's side, especially if the wind be blowing directly on to the engaged side.

A good plan is to secure the rolled-up flags with a few cotton stops, which are set free by a light "ripping line" secured to the head of the bamboo, round which the stops are passed. On this line being pulled the flags can be freed at the proper moment.

The angles, however, taken when dropping the beacon, if any accuracy is being aimed for throughout any triangulation, cannot be considered very satisfactory, owing to the possible uncertainty as to the actual position that the moorings then occupy. It is better, therefore, to subsequently revisit the beacon for this purpose, as described in Section VI., after the moorings have taken up their final positions.

When picking up a Beacon.—The procedure is generally in the reverse order to that previously described. It should be the duty of a selected man to hook on (spring hook) the single 4-inch hemp pendant used as the lifting purchase to the wire strop, which is in place on the beacon. This requires some activity and skill, dependent upon the weather conditions. Jacob's ladders, grappels, and heaving lines should be in readiness where required, and also light bearing-out spars, terminating with wooden jaws, together with a long bamboo, on the outer end of which is placed a detachable wooden plug carrying a large iron hook with hemp line attached inboard in the form of a lance. This latter will frequently enable the staff of the beacon to be first grappled in rough weather, when the beacon can then be drawn to the ship's side and the man safely lowered.

The moorings are freed from the beacon as soon as the slip can be reached, being weighed independently by the most suitable mechanical means, whilst the beacon, being treated independently and dealt with as required, will ensure as little delay

as possible. Care is necessary to prevent the flags becoming fouled aloft as the beacon is lifted.

It should be made a practice for the ship's artisans to examine and report the condition of the beacons on being recovered, draining the casks, and tightening the hoops where necessary.

An examination of the moorings, and whether the mooring chain has become entwined round the anchor, will frequently give a ready clue as to the reason of a beacon being out of position, and afford useful knowledge for safeguarding this in future.

Should it be necessary to replace at any time a beacon that has become dismasted, dragged, etc., but which may still occupy about the position we require, it is generally a better plan to drop the new beacon first close to and with reference to the original beacon, whose position may be known but is not determined at the time owing to the weather conditions, such as mist, etc.; sufficient room must be left to manœuvre the ship clear of both beacons when subsequently engaged in picking up the damaged beacon.

As a useful guide, it may be stated that the time occupied for one vessel with a speed of 8 knots when engaged in picking up successively ten beacons moored in about 20 fathoms at distances of $4\frac{1}{2}$ miles apart (which necessitates steaming not less than 40 miles), should not under normal circumstances exceed seven hours, and has in the writer's experience often been less.

As a rough guide, not more than forty-five minutes should be allowed for the interval between recovering successive beacons when moored at this distance.

SECTION XX.

THE PROTECTION AND DISTINGUISHING OF BEACONS BY NIGHT.

LIGHTING OF BEACONS.

No suitable means have yet been found in practice for lighting the beacons in such a lasting manner as will indicate their presence at night to passing vessels.

It is out of the question to attempt to constantly renew any lamp of whatever character that it may be possible to attach in the first place to the staff, either from the ship or from a boat in suitable weather; the delays and risk of damage in-

volved entirely militate against any such attempt being made. All that has been attempted at present is to circulate as widely as possible in general terms by a notice to mariners, etc., information relating to the presence of the ships and beacons, and this especially to the fishery fleets, through H.M. Customs, when working in home waters.

When anchoring at night, and desirous of protecting the more exposed beacons situated near the general lines of traffic, each vessel, as far as is prudent, can assist in this by selecting a berth near them during fine weather, when no danger is likely to ensue—occupying such a position that passing vessels will, in keeping clear, naturally pass on the side of the ship remote from the beacon, whilst the ship herself displays the lights prescribed for a vessel engaged with telegraph cables.

Dependent upon the circumstances, it may frequently happen that a beacon may be moored for a full month or more in one position.

Trawling vessels in the North Sea, when working independent of fishery fleets, and desirous of maintaining a particular fishing ground by night, generally attach a hand acetylene lamp to the staff of a Dan buoy; this light, which burns continuously for upwards of thirty hours, is readily distinguishable at a distance of 4 miles in clear weather.

SECTION XXI.

LIGHTVESSELS AND THEIR MOORINGS AND THEIR USE IN TRIANGULATION.

The mooring-chains of lightvessels, unless moored in restricted positions, generally consist of a single chain of considerable length—150 to 200 fathoms' scope of cable is not unusual in a depth of 25 fathoms with exposed positions, the nip in the hawse-pipe being by regulation constantly altered. This should be borne in mind when conducting a triangulation by the aid of lightvessels. Some arrangement and understanding can, however, by the courtesy of those responsible on board, be made during the fine weather that we shall depend upon whilst engaged upon triangulation. Sextant angles are best obtained, especially at night, from a position

above the lantern, and where the observer is screened from the glare and reflection of the lantern.

Most lightvessels have a watch-buoy reliably moored a short distance from them. This fact can often advantageously be made use of when recovering a position, etc.

In the United Kingdom there is generally to be found at least one trained signalman on board each lightvessel, and a megaphone is included in their equipment.

By day, when anxious to observe a lightvessel at the farthest possible distance, the situation can often be saved by making an extra amount of smoke from their galley funnel at a pre-concerted time, where it would be undesirable, owing to passing vessels, to hoist any flags on board the lightvessel.

SECTION XXII.

EMPLOYMENT OF BOATS FOR SOUNDING AND AS MARK BOATS.

Boats can be usefully employed in fine weather to examine particular areas where desirable, for which purpose it should not be necessary to render them dependent for "fixing" upon the position of the ship when at anchor, for the ship will generally be required to be engaged more advantageously in other directions.

In order to facilitate fixing by station pointer with the beacons within the area to be examined by the boats, it may be necessary to lay out one or more additional temporary beacons, or mark boats, in previously selected spots, where a reliable fix can be obtained when dropping them or visiting one of the original beacons if necessary for this purpose, when the new beacon may be "shot up."

When using a mark boat, it is often a good plan to both drop and weigh the moorings from the ship, passing the upper end of the moorings to and from the boat whilst the ship obtains the fixing angles.

One mechanically propelled boat or two, if available, can often, under favourable conditions of weather, be employed sounding, keeping in company with the ship on parallel lines at some previously assigned distance, the boats being dependent upon the ship for fixing.

The following points should then be attended to: To obtain the best results there should be as few alterations of course as possible—*i.e.*, the lines of soundings to be run should be long and fairly straight.

An assistant, together with a signaller, to be specially detailed to concentrate his attention upon each boat from the ship, recording all times, bearings, and signal hoists in a book kept for the purpose, whilst the sounding in the ship proceeds uninterruptedly, the speed being regulated to suit the boats, and alterations of course to be signalled in sufficient time beforehand.

The boats, whilst remaining approximately on either beam and maintaining the course directed, in other respects control independently their own movements, in order to preserve approximately their correct distance and stop to sound as often as may be necessary.

The boats should not range beyond easy visual signal distance—say, half a mile—and display by hand successive numeral flags up to 99 (when the series is repeated) whenever they require to be observed from the ship, at the same time observing the masthead angle and the time, etc. The ship, when answering each boat by a particular signal—distinguished by the position from which it is displayed—then observes the boat's compass bearing and the number displayed by her, the fix being further identified by the time. The boat's traverse can be subsequently plotted from that of the ship, and the soundings filled in, the assistant on board being responsible that the entries appear in their correct positions, and can be identified with regard to the ship's traverse as the plotting proceeds.

This system has been found to work well, and can be used with advantage in open off-shore or bank work, when the boats would be otherwise unable to fix owing to the distance of the objects, the difficulty of distinguishing them, or when out of sight of land, etc. A range-finder can also be usefully employed from the ship when observing the boats.

CAUTION.—In addition to the usual precautions (signalling by day and night, extra provisions, etc.) it should be an invariable rule that all boats when detached from the ship, under conditions that render them liable to rapid exposure on a

sudden change of weather conditions imperilling their safety, should be provided with specific instructions with regard to acting upon such an emergency, including fog. This should specify a rendezvous at some prearranged beacon, the duty of the mechanically propelled boats being to first succour the mark boats, who should be always prepared to slip and buoy their moorings rather than court any danger there may be in the attempt to weigh them.

SECTION XXIII.

THE USE OF BEACONS WITH SHIP TRIANGULATION DURING IRREGULAR OR DETAILED SURVEYS.

The subject is sufficiently referred to in Chapters V. and VI., where various examples are given of the employment of beacons when carrying out an irregular triangulation of a coast by means of the ship. The previous sections have more especial reference to the use of beacons when conducting a survey in the open sea or out of sight of land or other objects, etc.

Floating beacons can often be resorted to in detailed surveys on the larger scales, when, owing to the nature or trend of the coast, the lack of drying banks, off-shore depths, etc., any other method would present much difficulty or involve great delay. This often occurs, where it may be desired to extend a triangulation, or commence from some starting-point on shore, on coasts that are generally straight, low, featureless, and densely wooded, but may afford desirable anchorages. In such cases the beacons should be moored with two or more bridles, and with as small swinging area as may be practicable, for, in order to obtain the necessary precision, we shall almost entirely depend upon the simultaneous intersection of such beacons by observers at shore stations.

When conducting an extensive coast survey by beacons where landing is difficult, such as on the West Coast of Africa, it will greatly assist matters, and largely reduce the chances of errors accumulating and remaining undetected as the triangulation proceeds, if advantage is taken of at least one accessible elevated station, commanding a view as occasion offers, where an observer can be stationed whilst the ship is placed in the best

position, and with a heliostat to assist the observer to distinguish and observe the more distant beacons.

In detailed surveys, beacons, in conjunction with shore objects, can also be made to serve as objects for use when fixing, which might be rendered otherwise difficult owing to the trend of the coast or the delay involved in extending a triangulation in order to obtain additional points.

SECTION XXIV.

THE EMPLOYMENT OF FLOATING BEACONS IN SHALLOW WATER.

Surveying beacons, pattern 1912, owing to the length of the submerged staff, cannot be used where the depths at low water approximate to about 24 feet or less.

If, however, it is not desired to surmount a bamboo and flag of the full sizes mentioned in Section I., the submerged spar can be reduced in length with due regard to the stability of the beacon when supporting those of the lesser dimensions that it is intended to employ.

Two 18-gallon casks, floating on their bilge, and lying parallel when firmly secured together by a stout wooden exterior framework and filling-pieces, can be used when it is desired to improvise without delay a beacon of modified form with the materials usually found on board ship. The weighted staff, of convenient length, passes through the centre filling-piece, being well secured by close interfitting chocks nailed or bolted to both staff and filling-pieces. Lengths of flat iron boltstaves, about 30 inches in length, being firmly secured to the staff, and used as fishes above and below the middle filling-piece, will ensure the staff being able to withstand the strain that is here brought upon it. Beacons of this description will support a flag that is easily visible at 8 miles distance.

In the delta of the River Niger, where it is necessary to moor beacons, in depths between 6 and 18 feet, and tidal stream of 3 to 4 knots, it was found that boats could conveniently handle beacons consisting of lengths of stout planking about 10 feet in length, bolted and dovetailed together in triangular form, with central crosspiece supporting the weighted staff. The

lengths of planking being separately distinguished, the beacons could be readily prepared and dismantled in the boats, to be used on future occasions as required.

It is not necessary here to make any special reference to any other form of beacons, which may take the form of targets or mark buoys, such as cross-planks or single cask, each surmounting a staff and flag, or Dan buoys, all of which can be used with advantage within their known limits of vision and depth of water.

SECTION XXV.

OBTAINING THE DISTANCE BETWEEN ANY SELECTED POSITIONS NEAR THE EXTREMES OF THE SHIP.

Where no reliable fitted drawing of the ship on a sufficiently large scale is available it may be sometimes necessary to determine this. This is best done by means of a distance measured on shore made at a convenient distance from the ship when secured to a neighbouring wharf, etc.

Two intervisible terminal points, commanding a view of both ends of the ship, should be selected on shore near abreast the ends of the ship, the intervening ground being adapted for a reliable measurement to be made between these points, which are carefully marked, and in such a manner that they can be used as reference-marks, the one from the other, with a theodolite when placed at each terminal point.

Theodolites are set up exactly over these spots, and horizontal angles measured from each as a reference point to the selected points near the extremes of the ship, where vertical battens are erected and their positions accurately marked on the deck.

Two observers should be employed on shore, and simultaneous intersections made from each end when the ship is motionless, for obviously any movement of the ship during the interval between the intersections would vitiate the result.

The necessary computations can then be made for determining the horizontal distance between the points intersected on board the ship, the receiving angles in each case approximating to a right angle.

We may require to determine the horizontal distance between, say, the centre of the trucks of the ensign and jackstaff or a

mark on the bowsprit end, etc., all of which are suitable for subsequent use when determining the distance of the observer from the ship by means of the horizontal angle subtended between them. These objects should therefore also be intersected with the theodolites; but as the former are liable to removal, alteration, or damage, we shall depend upon the deck marks, from which any desired extension can be made at any future time by actual measurement on board.

Reliable results are equally obtainable under favourable conditions, such as a calm and no tidal stream, when the ship is at anchor and the observers stationed in readiness at suitable positions on shore, not too far distant from the ship.

It is generally impossible to make any direct measurement on board between two positions situated near the ends of the ship, whether they are selected out of the central line or not, owing to the numerous objects that generally intervene.

If it is desired to determine this distance without resort to some measurement made on shore, a reliable result can be obtained by selecting successive intervisible theodolite positions at suitable intervals and at about the same relative height, between which reliable measurements can be made either along the intervening straight deck or by suspending wire, etc.

The computation of the horizontal distance between the selected extreme points, which need not necessarily be intervisible, is the solution of the connected traverse made between them.

Before finally removing the instruments it is as well to make sure that all the necessary data for this have been obtained, and that all the steps are complete.

When great accuracy is required, or should the measured distances depart much from the horizontal, the distances must be referred to the horizontal plan; readings taken on the vertical arc of the theodolite will suffice for this, and it is therefore necessary to select a time when the ship is practically motionless.

In all cases the draught of water of the vessel should be carefully recorded, and the vessel as nearly upright as possible.

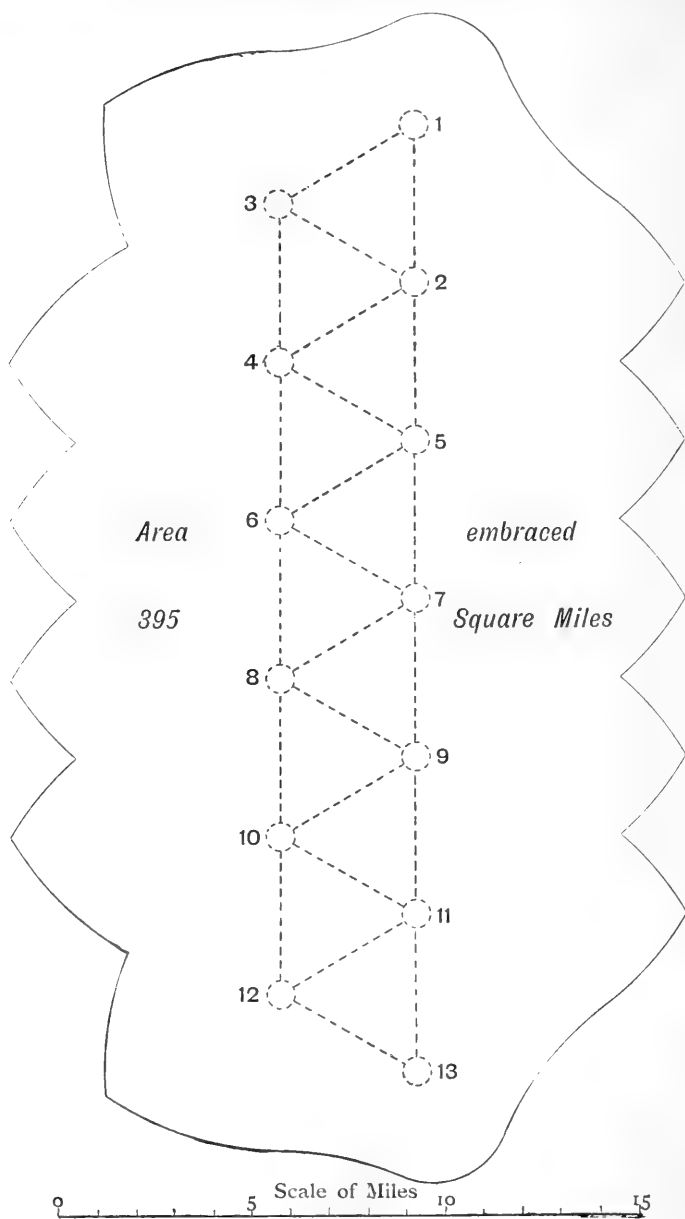


Fig. 1.

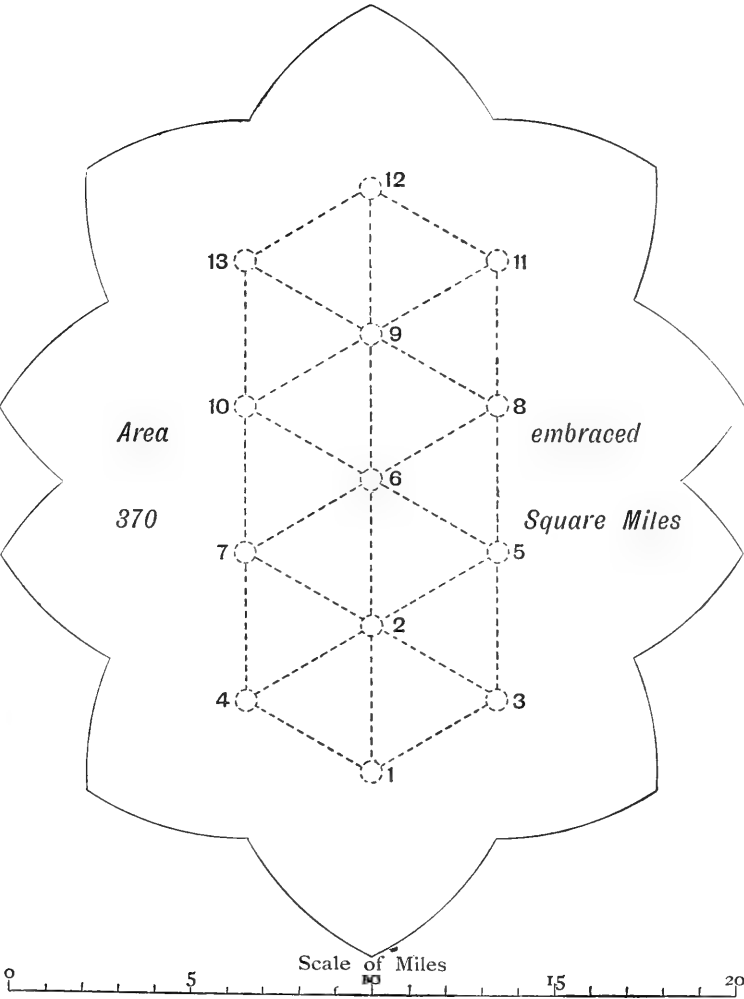


Fig. II.

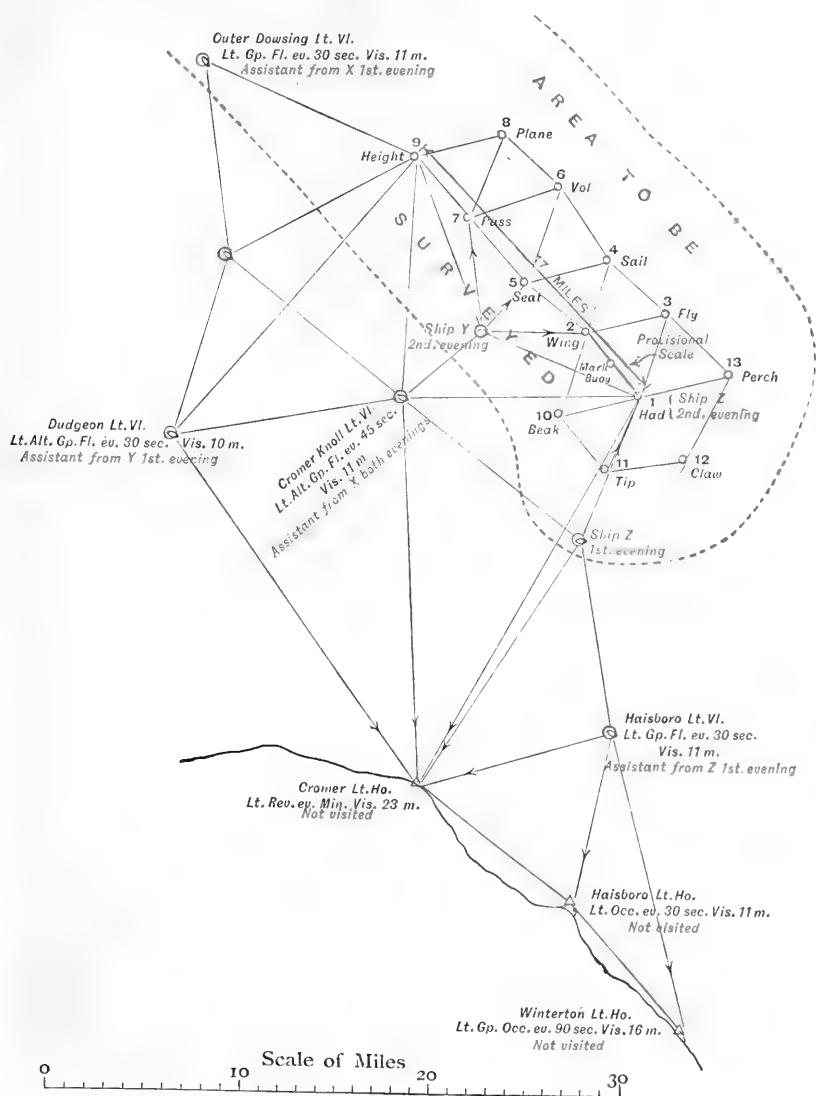


Fig. IV. — Illustrating a Floating Triangulation connected to the Shore Objects, where it is desired to fix the Positions of the Lightvessels in addition to examining the Area covered by the Beacons. The Co-ordinates of the Shore Objects are known.

Three vessels, X, Y, Z, engaged during two successive evenings and one day.
X the controlling vessel.

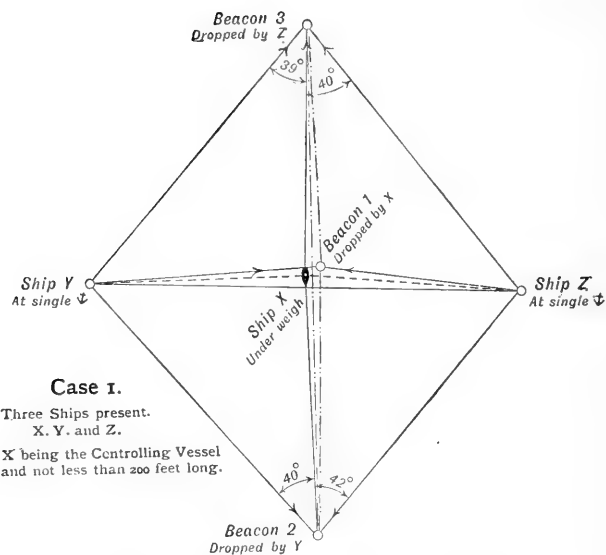
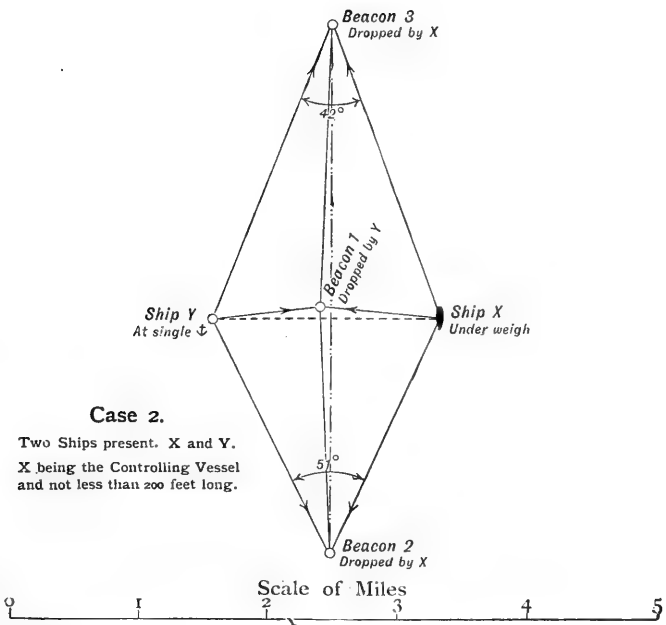


Fig. V.—Obtaining the Scale provisionally.



Case 2.

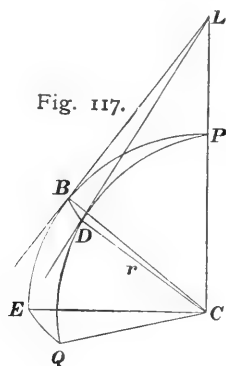
Two Ships present. X and Y.
X being the Controlling Vessel
and not less than 200 feet long.

Fig. VI —Obtaining the Scale provisionally.

APPENDIX.



A.—To prove that Tan Convergency = Tan Dep. Tan Mid Lat.



Here C is the centre of the earth, P is the pole, EP, QP, two meridians a known distance apart. BL, EL, are two tangents to the meridians, at the middle latitude known, in the same plane as the meridian, and meeting one another and the axis of the earth CP, produced, in L.

Then BLD is the Convergency required, and DLC is the middle latitude, and BCD the departure. DC is a radius of the earth = r .

Now as BD is small, it can be taken as a straight line without sensible error.

We can also assume BLD and BCD to be right-angled triangles.

Then $BD = DL \times \tan BLD$.

Similarly $BD = r \times \tan BCD$.

Equating, we have $DL \times \tan BLD = r \times \tan BCD$.

But $DL = r \times \cot DLC$;

$\therefore r \times \cot DLC \times \tan BLD = r \times \tan BCD$,

or $\tan BLD = \tan BCD \times \tan DLC$,

or $\tan \text{Convergency} = \tan \text{dep} \times \tan \text{Mid Lat.}$,

and when Convergency is very small, we can say

$\text{Convergency} = \text{Dep} \times \tan \text{Mid Lat.}$

Bisect C K G in K N, making G K N or X K Z = $\frac{1}{2}$ convergency,

Then E where K N intersects P F is on the parallel K C, or P E = P K.

Bisect K P E in P X.

Now M K Z = K Z P + K P Z,

but M K Z = $90^\circ + \frac{1}{2}$ Convergency (by construction) and

K P Z = $\frac{1}{2}$ Convergency;

$\therefore 90^\circ + \frac{1}{2} \text{Conv} = \text{K Z P} + \frac{1}{2} \text{Conv};$

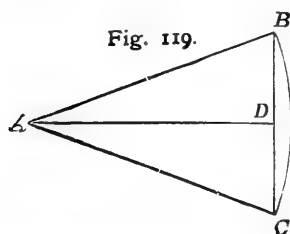
$\therefore \text{K Z P} = 90^\circ = \text{P Z E},$

and as K P Z = Z P E and P Z is common, the Δ s K Z P & P Z E are equal and similar;

$\therefore \text{P E} = \text{P K}.$

Q. E. D.

C.—To prove $\text{Chord} = 2 \text{ rad} \left\{ \text{Vers} \left(90 + \frac{\theta}{2} \right) - 1 \right\}.$



Let $\angle CAB = \theta$, the angle whose chord is required.

At any radius $AC = r$, describe arc CB .

Join CB , then CB is Chord required.

Bisect BC in D and join AD .

Then $\angle DAB = \frac{\theta}{2}.$

Now $DB = AB \sin DAB$

$$= r \sin \frac{\theta}{2};$$

but $BC = 2 DB$;

$$\therefore BC = 2r \sin \frac{\theta}{2} \dots \dots \dots (a).$$

But $\text{Versine} \frac{\theta}{2} = 1 - \cos \frac{\theta}{2};$

$$\therefore \text{Versine} \left(90 + \frac{\theta}{2} \right) = 1 - \cos \left(90 + \frac{\theta}{2} \right)$$

$$= 1 - \left(-\sin \frac{\theta}{2} \right)$$

$$= 1 + \sin \frac{\theta}{2}$$

$$\therefore \sin \frac{\theta}{2} = \text{Vers} \left(90 + \frac{\theta}{2} \right)$$

Substituting this in (a) we get

$$BC = 2r \left\{ \text{Vers} \left(90 + \frac{\theta}{2} \right) - 1 \right\}$$

D.—To prove *Reduction to the Meridian* = $\frac{\cos l \cdot \cos d}{\sin z} \cdot \frac{\text{Vers } h}{\sin 1''}$

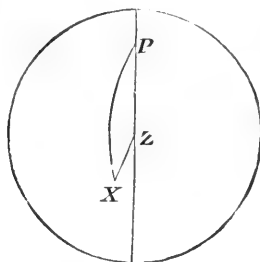


Fig. 120.

Let X be a heavenly body near the Meridian, P the pole, Z the Zenith.

Let Hour Angle Z P X = h , Latitude = $90 - PZ = l$, Zenith distance X Z = z , Declination = $90 - PX = d$.

$$\text{Then } \cos Z P X = \frac{\cos XZ - \cos PX \cdot \cos PZ}{\sin PX \cdot \sin PZ},$$

$$\text{or } \cos h = \frac{\cos z - \sin l \cdot \sin d}{\cos l \cdot \cos d};$$

$$\begin{aligned} \therefore \cos z - \sin l \cdot \sin d &= \cos l \cdot \cos d \cdot \cos h \\ &= \cos l \cdot \cos d \cdot (1 - \text{Vers } h) \\ &= \cos l \cdot \cos d - \cos l \cdot \cos d \cdot \text{Vers } h; \end{aligned}$$

$$\begin{aligned} \therefore \cos z + \cos l \cdot \cos d \cdot \text{Vers } h &= \cos l \cdot \cos d + \sin l \cdot \sin d \\ &= \cos (l \infty d) \\ &= 1 - \text{Vers } (l \infty d); \end{aligned}$$

$$\therefore \text{Vers } (l \infty d) = 1 - \cos z - \cos l \cdot \cos d \cdot \text{Vers } h$$

$$= \text{Vers } z - \cos l \cdot \cos d \cdot \text{Vers } h.$$

$$\begin{aligned} \text{Working with Declination} &= 90 + PX, \text{ we shall get} \\ &= \text{Vers } z - \cos l \cdot \cos d \cdot \text{Vers } h. \end{aligned}$$

But $l \infty d$ or $l + d$ is the Meridian Zenith Distance = Z .

Then $\text{Vers } Z = \text{Vers } z - \cos l \cdot \cos d \cdot \text{Vers } h$.

$$\begin{aligned} -\cos l \cdot \cos d \cdot \text{Vers } h &= \text{Vers } Z - \text{Vers } z \\ &= 1 - \cos Z - 1 + \cos z \\ &= \cos z - \cos Z \\ &= -2 \sin \frac{z+Z}{2} \cdot \sin \frac{z-Z}{2}; \end{aligned}$$

but z and Z are nearly alike, so $\frac{z+Z}{2}$ may be taken = z

and $z - Z$ is very small $\therefore 2 \sin \frac{z-Z}{2}$ may be taken = $(z - Z) \sin 1''$;

$$\therefore \cos l \cdot \cos d \cdot \text{Vers } h = \sin z \cdot (z - Z) \sin 1'',$$

$$\text{or } z - Z = \frac{\cos l \cdot \cos d}{\sin z} \cdot \frac{\text{Vers } h}{\sin 1''}$$

but $z - Z$ is the Reduction to the Meridian;

$$\therefore \text{Reduction to Mer.} = \frac{\cos l \cdot \cos d}{\sin z} \cdot \frac{\text{Vers } h}{\sin 1''}$$

E.—To show that the Distance of Horizon in English Miles

$$= \sqrt{\frac{3}{2} \text{ height in feet.}}$$

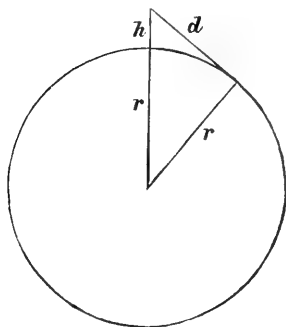


Fig. 121.

Let r be radius of earth.

h height of observer in feet.

d distance of horizon.

$$d^2 + r^2 = (h + r)^2$$

$$= h^2 + r^2 + 2hr,$$

h^2 being small may be omitted.

$$d^2 = 2hr;$$

but h in Eng. miles is $\frac{h}{5280}$

and $2r$ „ „ is 7910;

$$\therefore d^2 = \frac{7910}{5280} h$$

$$= \frac{3}{2} h \text{ very nearly;}$$

$$\therefore d = \sqrt{\frac{3}{2} h}.$$

This is the distance disregarding refraction, which has the effect of increasing the distance of the visible horizon. If having found d as above, we subtract $\frac{1}{18}$ of itself from it, the remainder will be the true distance in sea miles very nearly, with the effects of refraction taken into consideration.

F.—Base by Sound.

To prove that $T = \frac{2 t t^1}{t + t^1}$

Let d be distance in feet between stations,
 v the velocity of sound, } in feet per
 x „ of the wind, } second,
 t the observed interval in seconds with the wind,
 t_1 „ against the wind,
 T the mean interval required.

Then $d = v T$.

By observation $d = v t + x t$
 and $d = v t_1 - x t_1$.

Dividing by t and t_1 we get

$$\frac{d}{t} = v + x,$$

$$\frac{d}{t_1} = v - x.$$

Adding, we have

$$\frac{d}{t} + \frac{d}{t_1} = 2 v,$$

$$d \left(\frac{1}{t} + \frac{1}{t_1} \right) = 2 v,$$

$$d \frac{t^1 + t}{t t^1} = 2 v,$$

$$d = v \frac{2 t t^1}{t + t^1};$$

but we have also

$$d = v T,$$

and by equating

$$T = \frac{2 t t^1}{t + t_1}.$$

FORM H.

Chronometer Comparison Book.

Max.

Ther.

Min.

| Chron. | Time. | Check. | Slow on A. | 2nd Diff. | |
|--------|-------|--------|------------|-----------|--|
| A | | | | | |
| B | | | | | |
| A | | | | | |
| C | | | | | |
| A | | | | | |
| D | | | | | |
| A | | | | | |
| E | | | | | |
| A | | | | | |
| F | | | | | |
| A | | | | | |
| G | | | | | |
| A | | | | | |
| H | | | | | |
| A | | | | | |
| I | | | | | |
| A | | | | | |
| K | | | | | |
| A | | | | | |
| L | | | | | |

TABLE J.—*Table of Chords of Arcs from 0° to 60°, to facilitate the Projection of Angles. Radius = 10. By T. H. Tizard, Navigating Lieut. R.N.*

| Min. | 0° | Parts for " | 1° | Parts for " | 2° | Parts for " | 3° | Parts for " | 4° | Parts for " |
|------|---------|-------------------|---------|-------------------|---------|-------------------|---------|-------------------|---------|-------------------|
| 0 | 0°00000 | 0 | 0°17453 | 0 | 0°34905 | 0 | 0°52354 | 0 | 0°69799 | 0 |
| 1 | °00291 | 5 | °17744 | 5 | °35195 | 5 | °52644 | 5 | °70080 | 5 |
| 2 | °00582 | 10 | °18035 | 10 | °35486 | 10 | °52935 | 10 | °70380 | 10 |
| 3 | °00873 | 15 | °18326 | 15 | °35777 | 15 | °53226 | 15 | °70671 | 15 |
| 4 | °01164 | 20 | °18617 | 20 | °36068 | 20 | °53517 | 20 | °70962 | 20 |
| 5 | °01455 | 25 | °18907 | 25 | °36359 | 25 | °53808 | 25 | °71252 | 25 |
| 6 | °01746 | 30 | °19198 | 30 | °36650 | 30 | °54098 | 30 | °71543 | 30 |
| 7 | °02037 | 35 | °19489 | 35 | °36941 | 35 | °54389 | 35 | °71834 | 35 |
| 8 | °02328 | 39 | °19780 | 39 | °37231 | 39 | °54680 | 39 | °72129 | 39 |
| 9 | °02618 | 44 | °20070 | 44 | °37522 | 44 | °54970 | 44 | °72415 | 44 |
| 10 | °02908 | 49 | °20361 | 49 | °37813 | 49 | °55261 | 49 | °72706 | 49 |
| 11 | °03199 | 53 | °20652 | 53 | °38104 | 53 | °55552 | 53 | °72996 | 53 |
| 12 | °03490 | 58 | °20943 | 58 | °38395 | 58 | °55843 | 58 | °73287 | 58 |
| 13 | °03781 | 63 | °21234 | 63 | °38685 | 63 | °56134 | 63 | °73578 | 63 |
| 14 | °04072 | 68 | °21525 | 68 | °38976 | 68 | °56425 | 68 | °73869 | 68 |
| 15 | °04363 | 73 | °21816 | 73 | °39267 | 73 | °56715 | 73 | °74159 | 73 |
| 16 | °04654 | 78 | °22107 | 78 | °39558 | 78 | °57006 | 78 | °74450 | 78 |
| 17 | °04945 | 82 | °22398 | 82 | °39849 | 82 | °57297 | 82 | °74741 | 82 |
| 18 | °05236 | 87 | °22689 | 87 | °40140 | 87 | °57588 | 87 | °75032 | 87 |
| 19 | °05527 | 92 | °22979 | 92 | °40430 | 92 | °57878 | 92 | °75322 | 92 |
| 20 | °05818 | 97 | °23270 | 97 | °40721 | 97 | °58169 | 97 | °75613 | 97 |
| 21 | °06109 | 102 | °23561 | 102 | °41012 | 102 | °58460 | 102 | °75903 | 102 |
| 22 | °06400 | 107 | °23852 | 107 | °41303 | 107 | °58751 | 107 | °76194 | 107 |
| 23 | °06691 | 112 | °24143 | 112 | °41594 | 112 | °59042 | 112 | °76485 | 112 |
| 24 | °06982 | 117 | °24434 | 117 | °41884 | 117 | °59333 | 117 | °76775 | 117 |
| 25 | °07272 | 122 | °24725 | 122 | °42175 | 122 | °59623 | 122 | °77066 | 122 |

ERRATA

APPENDIX, TABLE J

| Page | ° | Min. | Chord Shown. | Correct Chord. |
|------|-----|------|--------------|------------------|
| 525 | 0° | 34 | 0°09808 | 0°09890 |
| 525 | 0° | 49 | 0°14153 | 0°14253 |
| 525 | 2° | 43 | 0°41410 | 0°47410 |
| 526 | 5° | 29 | 0°05665 | 0°95665 |
| 526 | 6° | 45 | 1°17441 | 1°17741 |
| 526 | 9° | 26 | 1°64556 | 1°64456 |
| 530 | 28° | 35 | 4°93615 | 4°93715 |
| 530 | 28° | 56 | 4°90633 | 4°99633 |
| 531 | 30° | 58 | 5°35916 | 5°33916 |
| 531 | 32° | 1 | 5°51654 | 5°51554 |
| 531 | 33° | 35 | 5°77885 | 5°77785 |
| 531 | 34° | 40 | 5°98560 | 5°95860 |
| 525 | 4° | 59 | Parts for " | for 226 read 286 |

| | | | | | | | | | | |
|----|--------|-----|--------|-----|--------|-----|--------|-----|--------|-----|
| 55 | °15999 | 266 | °33450 | 266 | °50900 | 266 | °68345 | 266 | °85785 | 266 |
| 56 | °16290 | 271 | °33741 | 271 | °51191 | 271 | °68636 | 271 | °86076 | 271 |
| 57 | °16580 | 276 | °34032 | 276 | °51481 | 276 | °68927 | 276 | °86367 | 276 |
| 58 | °16871 | 281 | °34323 | 281 | °51772 | 281 | °69217 | 281 | °86657 | 281 |
| 59 | °17162 | 286 | °34614 | 286 | °52063 | 286 | °69508 | 286 | °86948 | 286 |
| 60 | °17453 | 291 | °34905 | 291 | °52354 | 291 | °69799 | 291 | °87239 | 291 |

FORM H.

*Chronometer Comparison Book.**Max.**Ther.**Min.*

| Chrons. | Time. | Check. | Slow on A. | 2nd Diff. | |
|---------|-------|--------|------------|-----------|--|
| A | | | | | |
| B | | | | | |
| A | | | | | |
| C | | | | | |
| A | | | | | |
| D | | | | | |
| A | | | | | |

TABLE J.—*Table of Chords of Arcs from 0° to 60°, to facilitate the Projection of Angles. Radius = 10. By T. H. TIZARD, Navigating Lieut. R.N.*

| Min. | 0° | Parts for " | 1° | Parts for " | 2° | Parts for " | 3° | Parts for " | 4° | Parts for " |
|------|---------|-------------------|---------|-------------------|---------|-------------------|---------|-------------------|---------|-------------------|
| 0 | 0°00000 | 0 | 0°17453 | 0 | 0°34905 | 0 | 0°52354 | 0 | 0°69799 | 0 |
| 1 | 0°00291 | 5 | 0°17744 | 5 | 0°35195 | 5 | 0°52644 | 5 | 0°70080 | 5 |
| 2 | 0°00582 | 10 | 0°18035 | 10 | 0°35486 | 10 | 0°52935 | 10 | 0°70380 | 10 |
| 3 | 0°00873 | 15 | 0°18326 | 15 | 0°35777 | 15 | 0°53226 | 15 | 0°70671 | 15 |
| 4 | 0°01164 | 20 | 0°18617 | 20 | 0°36068 | 20 | 0°53517 | 20 | 0°70962 | 20 |
| 5 | 0°01455 | 25 | 0°18907 | 25 | 0°36359 | 25 | 0°53808 | 25 | 0°71252 | 25 |
| 6 | 0°01746 | 30 | 0°19198 | 30 | 0°36650 | 30 | 0°54098 | 30 | 0°71543 | 30 |
| 7 | 0°02037 | 35 | 0°19489 | 35 | 0°36941 | 35 | 0°54389 | 35 | 0°71834 | 35 |
| 8 | 0°02328 | 39 | 0°19780 | 39 | 0°37231 | 39 | 0°54680 | 39 | 0°72129 | 39 |
| 9 | 0°02618 | 44 | 0°20070 | 44 | 0°37522 | 44 | 0°54970 | 44 | 0°72415 | 44 |
| 10 | 0°02908 | 49 | 0°20361 | 49 | 0°37813 | 49 | 0°55261 | 49 | 0°72706 | 49 |
| 11 | 0°03199 | 53 | 0°20652 | 53 | 0°38104 | 53 | 0°55552 | 53 | 0°72996 | 53 |
| 12 | 0°03490 | 58 | 0°20943 | 58 | 0°38395 | 58 | 0°55843 | 58 | 0°73287 | 58 |
| 13 | 0°03781 | 63 | 0°21234 | 63 | 0°38686 | 63 | 0°56134 | 63 | 0°73578 | 63 |
| 14 | 0°04072 | 68 | 0°21525 | 68 | 0°38976 | 68 | 0°56425 | 68 | 0°73869 | 68 |
| 15 | 0°04363 | 73 | 0°21816 | 73 | 0°39267 | 73 | 0°56715 | 73 | 0°74159 | 73 |
| 16 | 0°04654 | 78 | 0°22107 | 78 | 0°39558 | 78 | 0°57006 | 78 | 0°74450 | 78 |
| 17 | 0°04945 | 82 | 0°22398 | 82 | 0°39849 | 82 | 0°57297 | 82 | 0°74741 | 82 |
| 18 | 0°05236 | 87 | 0°22689 | 87 | 0°40140 | 87 | 0°57588 | 87 | 0°75032 | 87 |
| 19 | 0°05527 | 92 | 0°22979 | 92 | 0°40430 | 92 | 0°57878 | 92 | 0°75322 | 92 |
| 20 | 0°05818 | 97 | 0°23270 | 97 | 0°40721 | 97 | 0°58169 | 97 | 0°75613 | 97 |
| 21 | 0°06109 | 102 | 0°23561 | 102 | 0°41012 | 102 | 0°58460 | 102 | 0°75903 | 102 |
| 22 | 0°06400 | 107 | 0°23852 | 107 | 0°41303 | 107 | 0°58751 | 107 | 0°76194 | 107 |
| 23 | 0°06691 | 112 | 0°24143 | 112 | 0°41594 | 112 | 0°59042 | 112 | 0°76485 | 112 |
| 24 | 0°06982 | 117 | 0°24434 | 117 | 0°41884 | 117 | 0°59333 | 117 | 0°76775 | 117 |
| 25 | 0°07272 | 122 | 0°24725 | 122 | 0°42175 | 122 | 0°59623 | 122 | 0°77066 | 122 |
| 26 | 0°07563 | 127 | 0°25016 | 127 | 0°42466 | 127 | 0°59914 | 127 | 0°77356 | 127 |
| 27 | 0°07854 | 132 | 0°25306 | 132 | 0°42757 | 132 | 0°60205 | 132 | 0°77647 | 132 |
| 28 | 0°08145 | 137 | 0°25597 | 137 | 0°43048 | 137 | 0°60495 | 137 | 0°77938 | 137 |
| 29 | 0°08436 | 141 | 0°25888 | 141 | 0°43339 | 141 | 0°60786 | 141 | 0°78229 | 141 |
| 30 | 0°08727 | 145 | 0°26179 | 145 | 0°43629 | 145 | 0°61077 | 145 | 0°78519 | 145 |
| 31 | 0°09017 | 150 | 0°26470 | 150 | 0°43920 | 150 | 0°61368 | 150 | 0°78810 | 150 |
| 32 | 0°09308 | 155 | 0°26761 | 155 | 0°44211 | 155 | 0°61658 | 155 | 0°79101 | 155 |
| 33 | 0°09599 | 160 | 0°27052 | 160 | 0°44502 | 160 | 0°61949 | 160 | 0°79392 | 160 |
| 34 | 0°09880 | 165 | 0°27342 | 165 | 0°44793 | 165 | 0°62240 | 165 | 0°79682 | 165 |
| 35 | 0°10181 | 170 | 0°27633 | 170 | 0°45084 | 170 | 0°62531 | 170 | 0°79973 | 170 |
| 36 | 0°10472 | 175 | 0°27924 | 175 | 0°45374 | 175 | 0°62821 | 175 | 0°80264 | 175 |
| 37 | 0°10763 | 180 | 0°28215 | 180 | 0°45665 | 180 | 0°63112 | 180 | 0°80554 | 180 |
| 38 | 0°11054 | 185 | 0°28506 | 185 | 0°45956 | 185 | 0°63403 | 185 | 0°80845 | 185 |
| 39 | 0°11344 | 190 | 0°28797 | 190 | 0°46247 | 190 | 0°63694 | 190 | 0°81135 | 190 |
| 40 | 0°11635 | 194 | 0°29088 | 194 | 0°46538 | 194 | 0°63984 | 194 | 0°81426 | 194 |
| 41 | 0°11926 | 199 | 0°29378 | 199 | 0°46828 | 199 | 0°64275 | 199 | 0°81717 | 199 |
| 42 | 0°12217 | 204 | 0°29669 | 204 | 0°47119 | 204 | 0°64566 | 204 | 0°82007 | 204 |
| 43 | 0°12508 | 209 | 0°29960 | 209 | 0°47410 | 209 | 0°64857 | 209 | 0°82297 | 209 |
| 44 | 0°12799 | 214 | 0°30251 | 214 | 0°47701 | 214 | 0°65147 | 214 | 0°82588 | 214 |
| 45 | 0°13090 | 219 | 0°30542 | 219 | 0°47992 | 219 | 0°65438 | 219 | 0°82879 | 219 |
| 46 | 0°13381 | 223 | 0°30833 | 223 | 0°48283 | 223 | 0°65728 | 223 | 0°83170 | 223 |
| 47 | 0°13672 | 228 | 0°31123 | 228 | 0°48574 | 228 | 0°66019 | 228 | 0°83461 | 228 |
| 48 | 0°13963 | 232 | 0°31414 | 232 | 0°48864 | 232 | 0°66310 | 232 | 0°83751 | 232 |
| 49 | 0°14253 | 237 | 0°31705 | 237 | 0°49155 | 237 | 0°66601 | 237 | 0°84042 | 237 |
| 50 | 0°14544 | 242 | 0°31996 | 242 | 0°49446 | 242 | 0°66892 | 242 | 0°84332 | 242 |
| 51 | 0°14835 | 247 | 0°32287 | 247 | 0°49737 | 247 | 0°67182 | 247 | 0°84623 | 247 |
| 52 | 0°15126 | 252 | 0°32578 | 252 | 0°50027 | 252 | 0°67473 | 252 | 0°84913 | 252 |
| 53 | 0°15417 | 257 | 0°32869 | 257 | 0°50318 | 257 | 0°67764 | 257 | 0°85204 | 257 |
| 54 | 0°15708 | 262 | 0°33160 | 262 | 0°50609 | 262 | 0°68055 | 262 | 0°85495 | 262 |
| 55 | 0°15999 | 266 | 0°33450 | 266 | 0°50900 | 266 | 0°68345 | 266 | 0°85785 | 266 |
| 56 | 0°16290 | 271 | 0°33741 | 271 | 0°51191 | 271 | 0°68636 | 271 | 0°86076 | 271 |
| 57 | 0°16580 | 276 | 0°34032 | 276 | 0°51481 | 276 | 0°68927 | 276 | 0°86367 | 276 |
| 58 | 0°16871 | 281 | 0°34323 | 281 | 0°51772 | 281 | 0°69217 | 281 | 0°86657 | 281 |
| 59 | 0°17162 | 286 | 0°34614 | 286 | 0°52063 | 286 | 0°69508 | 286 | 0°86948 | 286 |
| 60 | 0°17453 | 291 | 0°34905 | 291 | 0°52354 | 291 | 0°69799 | 291 | 0°87239 | 291 |

| Min. | 5° | Parts for " | 6° | Parts for " | 7° | Parts for " | 8° | Parts for " | 9° | Parts for " |
|------|---------|-------------------|---------|-------------------|---------|-------------------|---------|-------------------|---------|-------------------|
| 0 | 0°87239 | 0 | 1°04672 | 0 | 1°22097 | 0 | 1°39513 | 0 | 1°56918 | 0 |
| 1 | °87529 | 5 | °04962 | 5 | °22387 | 5 | °39803 | 5 | °57208 | 5 |
| 2 | °87820 | 10 | °05252 | 10 | °22677 | 10 | °40093 | 10 | °57498 | 10 |
| 3 | °88110 | 15 | °05543 | 15 | °22968 | 15 | °40383 | 15 | °57788 | 15 |
| 4 | °88401 | 20 | °05833 | 20 | °23258 | 20 | °40673 | 20 | °58078 | 20 |
| 5 | °88691 | 25 | °06124 | 25 | °23548 | 25 | °40964 | 25 | °58368 | 25 |
| 6 | °88982 | 30 | °06414 | 30 | °23839 | 30 | °41254 | 30 | °58658 | 30 |
| 7 | °89273 | 35 | °06705 | 35 | °24129 | 35 | °41544 | 35 | °58948 | 35 |
| 8 | °89563 | 39 | °06995 | 39 | °24419 | 39 | °41834 | 39 | °59238 | 39 |
| 9 | °89854 | 44 | °07286 | 44 | °24710 | 44 | °42124 | 44 | °59528 | 44 |
| 10 | °90144 | 49 | °07576 | 48 | °25000 | 48 | °42415 | 48 | °59818 | 48 |
| 11 | 0°90435 | 53 | 1°07867 | 53 | 1°25292 | 53 | 1°42705 | 53 | 1°60108 | 53 |
| 12 | °90726 | 58 | °08157 | 58 | °25581 | 58 | °42995 | 58 | °60398 | 58 |
| 13 | °91016 | 63 | °08448 | 63 | °25871 | 63 | °43285 | 63 | °60688 | 63 |
| 14 | °91307 | 68 | °08738 | 68 | °26161 | 68 | °43575 | 68 | °60978 | 68 |
| 15 | °91597 | 73 | °09029 | 73 | °26452 | 73 | °43866 | 73 | °61267 | 73 |
| 16 | °91888 | 78 | °09319 | 78 | °26742 | 78 | °44156 | 78 | °61557 | 78 |
| 17 | °92178 | 82 | °09610 | 82 | °27032 | 82 | °44446 | 82 | °61847 | 82 |
| 18 | °92469 | 87 | °09900 | 87 | °27323 | 87 | °44736 | 87 | °62137 | 87 |
| 19 | °92759 | 92 | °10190 | 92 | °27613 | 92 | °45026 | 92 | °62427 | 92 |
| 20 | °93050 | 97 | °10481 | 97 | °27903 | 97 | °45316 | 97 | °62717 | 97 |
| 21 | 0°93341 | 102 | 1°10771 | 102 | 1°28194 | 102 | 1°45607 | 102 | 1°63007 | 102 |
| 22 | °93631 | 107 | °11062 | 107 | °28484 | 107 | °45897 | 107 | °63297 | 107 |
| 23 | °93922 | 112 | °11352 | 112 | °28774 | 112 | °46187 | 112 | °63587 | 112 |
| 24 | °94212 | 117 | °11643 | 117 | °29064 | 117 | °46477 | 117 | °63876 | 117 |
| 25 | °94503 | 122 | °11933 | 122 | °29355 | 122 | °46767 | 122 | °64166 | 122 |
| 26 | °94794 | 127 | °12223 | 127 | °29645 | 127 | °47057 | 127 | °64456 | 127 |
| 27 | °95084 | 132 | °12514 | 132 | °29935 | 132 | °47347 | 132 | °64746 | 132 |
| 28 | °95375 | 137 | °12804 | 137 | °30225 | 137 | °47637 | 137 | °65036 | 137 |
| 29 | °95665 | 141 | °13095 | 141 | °30516 | 141 | °47927 | 141 | °65326 | 141 |
| 30 | °95956 | 145 | °13385 | 145 | °30806 | 145 | °48217 | 145 | °65616 | 145 |
| 31 | 0°96246 | 150 | 1°13676 | 149 | 1°31096 | 149 | 1°48507 | 149 | 1°65906 | 149 |
| 32 | °96537 | 155 | °13966 | 154 | °31387 | 154 | °48797 | 154 | °66196 | 154 |
| 33 | °96827 | 160 | °14257 | 159 | °31677 | 159 | °49088 | 159 | °66486 | 159 |
| 34 | °97118 | 165 | °14547 | 164 | °31967 | 164 | °49378 | 164 | °66776 | 164 |
| 35 | °97409 | 170 | °14837 | 169 | °32257 | 169 | °49668 | 169 | °67066 | 169 |
| 36 | °97699 | 175 | °15128 | 174 | °32547 | 174 | °49958 | 174 | °67355 | 174 |
| 37 | °97990 | 180 | °15418 | 179 | °32838 | 179 | °50248 | 179 | °67645 | 179 |
| 38 | °98280 | 185 | °15709 | 184 | °33128 | 184 | °50538 | 184 | °67935 | 184 |
| 39 | °98571 | 190 | °15999 | 189 | °33418 | 189 | °50828 | 189 | °68225 | 189 |
| 40 | °98861 | 194 | °16289 | 193 | °33709 | 193 | °51118 | 193 | °68515 | 193 |
| 41 | 0°99152 | 199 | 1°16580 | 198 | 1°33999 | 198 | 1°51408 | 198 | 1°68805 | 198 |
| 42 | °99442 | 204 | °16870 | 203 | °34289 | 203 | °51698 | 203 | °69095 | 203 |
| 43 | °99733 | 209 | °17160 | 208 | °34579 | 208 | °51988 | 208 | °69384 | 208 |
| 44 | 1°00023 | 214 | °17451 | 213 | °34869 | 213 | °52278 | 213 | °69674 | 213 |
| 45 | °00314 | 219 | °17441 | 218 | °35160 | 218 | °52568 | 218 | °69964 | 218 |
| 46 | °00605 | 223 | °18031 | 222 | °35450 | 222 | °52858 | 222 | °70254 | 222 |
| 47 | °00895 | 228 | °18322 | 227 | °35740 | 227 | °53148 | 227 | °70544 | 227 |
| 48 | °01185 | 232 | °18612 | 231 | °36030 | 231 | °53438 | 231 | °70833 | 231 |
| 49 | °01476 | 237 | °18903 | 236 | °36321 | 236 | °53728 | 236 | °71123 | 236 |
| 50 | °01766 | 242 | °19193 | 241 | °36611 | 241 | °54018 | 241 | °71413 | 241 |
| 51 | 1°02057 | 247 | 1°19483 | 246 | 1°36901 | 246 | 1°54308 | 246 | 1°71703 | 246 |
| 52 | °02348 | 252 | °19774 | 251 | °37191 | 251 | °54598 | 251 | °71993 | 251 |
| 53 | °02638 | 257 | °20064 | 256 | °37481 | 256 | °54888 | 256 | °72283 | 256 |
| 54 | °02929 | 262 | °20354 | 261 | °37771 | 261 | °55178 | 261 | °72572 | 261 |
| 55 | °03219 | 266 | °20645 | 265 | °38062 | 265 | °55468 | 265 | °72862 | 265 |
| 56 | °03510 | 271 | °20935 | 270 | °38352 | 270 | °55758 | 270 | °73152 | 270 |
| 57 | °03800 | 276 | °21226 | 275 | °38642 | 275 | °56048 | 275 | °73442 | 275 |
| 58 | °04090 | 281 | °21516 | 280 | °38932 | 280 | °56338 | 280 | °73732 | 280 |
| 59 | °04381 | 286 | °21806 | 285 | °39222 | 285 | °56628 | 285 | °74021 | 285 |
| 60 | °04672 | 291 | °22097 | 290 | °39513 | 290 | °56918 | 290 | °74311 | 290 |

| Min | 10° | Parts for " | 11° | Parts for " | 12° | Parts for " | 13° | Parts for " | 14° | Parts for " |
|-----|----------|-------------------|----------|-------------------|----------|-------------------|----------|-------------------|----------|-------------------|
| 0 | 1° 74311 | 0 | 1° 91691 | 0 | 2° 09057 | 0 | 2° 26407 | 0 | 2° 43738 | 0 |
| 1 | 74601 | 5 | 91980 | 5 | 09346 | 5 | 26696 | 5 | 44026 | 5 |
| 2 | 74891 | 10 | 92270 | 10 | 09635 | 10 | 26985 | 10 | 44315 | 10 |
| 3 | 75181 | 15 | 92560 | 15 | 09924 | 15 | 27274 | 15 | 44604 | 15 |
| 4 | 75470 | 20 | 92849 | 20 | 10214 | 20 | 27563 | 20 | 44892 | 20 |
| 5 | 75760 | 25 | 93139 | 25 | 10503 | 25 | 27852 | 25 | 45181 | 25 |
| 6 | 76050 | 30 | 93428 | 30 | 10792 | 30 | 28141 | 30 | 45470 | 30 |
| 7 | 76340 | 35 | 93718 | 35 | 11082 | 35 | 28430 | 35 | 45758 | 35 |
| 8 | 76630 | 39 | 94008 | 39 | 11371 | 39 | 28719 | 39 | 46047 | 39 |
| 9 | 76919 | 44 | 94297 | 44 | 11660 | 44 | 29008 | 44 | 46336 | 44 |
| 10 | 77209 | 48 | 94587 | 48 | 11950 | 48 | 29297 | 48 | 46625 | 48 |
| 11 | 1° 77499 | 53 | 1° 94876 | 53 | 2° 12239 | 53 | 2° 29586 | 53 | 2° 46913 | 53 |
| 12 | 77789 | 58 | 95166 | 58 | 12528 | 58 | 29875 | 58 | 47202 | 58 |
| 13 | 78078 | 62 | 95455 | 62 | 12817 | 62 | 30164 | 62 | 47491 | 62 |
| 14 | 78368 | 67 | 95745 | 67 | 13106 | 67 | 30453 | 67 | 47779 | 67 |
| 15 | 78658 | 72 | 96034 | 72 | 13396 | 72 | 30742 | 72 | 48068 | 72 |
| 16 | 78947 | 77 | 96324 | 77 | 13685 | 77 | 31031 | 77 | 48357 | 77 |
| 17 | 79237 | 81 | 96613 | 81 | 13974 | 81 | 31320 | 81 | 48645 | 81 |
| 18 | 79527 | 86 | 96903 | 86 | 14263 | 86 | 31609 | 86 | 48934 | 86 |
| 19 | 79816 | 91 | 97192 | 91 | 14552 | 91 | 31898 | 91 | 49223 | 91 |
| 20 | 80106 | 96 | 97482 | 96 | 14842 | 96 | 32187 | 96 | 49512 | 96 |
| 21 | 1° 80396 | 101 | 1° 97771 | 101 | 2° 15131 | 101 | 2° 32475 | 101 | 2° 49800 | 101 |
| 22 | 80686 | 106 | 98060 | 106 | 15420 | 106 | 32764 | 106 | 50089 | 106 |
| 23 | 80975 | 111 | 98350 | 111 | 15709 | 111 | 33053 | 111 | 50377 | 111 |
| 24 | 81265 | 116 | 98669 | 116 | 15998 | 116 | 33342 | 116 | 50666 | 116 |
| 25 | 81555 | 121 | 98929 | 121 | 16288 | 121 | 33631 | 121 | 50955 | 121 |
| 26 | 81844 | 126 | 99218 | 126 | 16577 | 126 | 33919 | 126 | 51243 | 126 |
| 27 | 82134 | 131 | 99507 | 131 | 16866 | 131 | 34208 | 131 | 51532 | 131 |
| 28 | 82424 | 136 | 99797 | 136 | 17155 | 136 | 34497 | 136 | 51821 | 136 |
| 29 | 82713 | 140 | 2° 00086 | 140 | 17444 | 140 | 34786 | 140 | 52110 | 140 |
| 30 | 83003 | 144 | 00376 | 144 | 17734 | 144 | 35075 | 144 | 52399 | 144 |
| 31 | 1° 83292 | 148 | 2° 00665 | 148 | 2° 18023 | 148 | 2° 35363 | 148 | 2° 52687 | 148 |
| 32 | 83582 | 153 | 00954 | 153 | 18312 | 153 | 35652 | 153 | 52976 | 153 |
| 33 | 83872 | 158 | 01244 | 158 | 18601 | 158 | 35941 | 158 | 53264 | 158 |
| 34 | 84161 | 163 | 01533 | 163 | 18890 | 163 | 36230 | 163 | 53553 | 163 |
| 35 | 84451 | 168 | 01823 | 168 | 19179 | 168 | 36519 | 168 | 53841 | 168 |
| 36 | 84740 | 173 | 02112 | 173 | 19468 | 173 | 36807 | 173 | 54130 | 173 |
| 37 | 85030 | 178 | 02401 | 178 | 19757 | 178 | 37096 | 178 | 54418 | 178 |
| 38 | 85320 | 183 | 02691 | 183 | 20046 | 183 | 37385 | 183 | 54707 | 183 |
| 39 | 85609 | 188 | 02980 | 188 | 20335 | 188 | 37674 | 188 | 54995 | 188 |
| 40 | 85899 | 192 | 03270 | 192 | 20625 | 192 | 37963 | 192 | 55284 | 192 |
| 41 | 1° 86188 | 197 | 2° 03559 | 197 | 2° 20914 | 197 | 2° 38251 | 197 | 2° 55572 | 197 |
| 42 | 86478 | 202 | 03848 | 202 | 21203 | 202 | 38540 | 202 | 55861 | 202 |
| 43 | 86768 | 207 | 04138 | 207 | 21492 | 207 | 38829 | 207 | 56149 | 207 |
| 44 | 87056 | 212 | 04427 | 212 | 21781 | 212 | 39118 | 212 | 56438 | 212 |
| 45 | 87347 | 217 | 04717 | 217 | 22070 | 217 | 39407 | 217 | 56726 | 217 |
| 46 | 87637 | 221 | 05006 | 221 | 22359 | 221 | 39695 | 221 | 57015 | 221 |
| 47 | 87926 | 226 | 05295 | 226 | 22648 | 226 | 39984 | 226 | 57303 | 226 |
| 48 | 88216 | 230 | 05585 | 230 | 22937 | 230 | 40273 | 230 | 57592 | 230 |
| 49 | 88506 | 235 | 05874 | 235 | 23226 | 235 | 40562 | 235 | 57880 | 235 |
| 50 | 88796 | 240 | 06164 | 240 | 23516 | 240 | 40851 | 240 | 58169 | 240 |
| 51 | 1° 89085 | 245 | 2° 06453 | 245 | 2° 23805 | 245 | 2° 41139 | 245 | 2° 58457 | 245 |
| 52 | 89375 | 250 | 06742 | 250 | 24094 | 250 | 41428 | 250 | 58745 | 250 |
| 53 | 89664 | 255 | 07031 | 255 | 24383 | 255 | 41717 | 255 | 59034 | 255 |
| 54 | 89954 | 260 | 07321 | 260 | 24672 | 260 | 42005 | 260 | 59322 | 260 |
| 55 | 90243 | 264 | 07610 | 264 | 24961 | 264 | 42294 | 264 | 59611 | 264 |
| 56 | 90533 | 269 | 07899 | 269 | 25250 | 269 | 42583 | 269 | 59899 | 269 |
| 57 | 90822 | 274 | 08189 | 274 | 25539 | 274 | 42871 | 274 | 60187 | 274 |
| 58 | 91112 | 279 | 08478 | 279 | 25828 | 279 | 43160 | 279 | 60476 | 279 |
| 59 | 91401 | 284 | 08767 | 284 | 26117 | 284 | 43449 | 284 | 60764 | 284 |
| 60 | 91691 | 289 | 09057 | 289 | 26407 | 289 | 43738 | 289 | 61053 | 288 |

| Min. | 15° | Parts for " | 16° | Parts for " | 17° | Parts for " | 18° | Parts for " | 19° | Parts for " |
|------|---------|-------------------|---------|-------------------|---------|-------------------|---------|-------------------|---------|-------------------|
| 0 | 2°61053 | 0 | 2°78346 | 0 | 2°95619 | 0 | 3°12868 | 0 | 3°30095 | 0 |
| 1 | *61341 | 5 | *78634 | 5 | *95906 | 5 | *13155 | 5 | *30382 | 5 |
| 2 | *61629 | 10 | *78922 | 10 | *96194 | 10 | *13442 | 10 | *30669 | 10 |
| 3 | *61917 | 15 | *79210 | 15 | *96482 | 15 | *13729 | 15 | *30955 | 15 |
| 4 | *62206 | 20 | *79498 | 20 | *96769 | 20 | *14017 | 20 | *31242 | 20 |
| 5 | *62494 | 25 | *79786 | 25 | *97057 | 25 | *14304 | 25 | *31529 | 25 |
| 6 | *62782 | 30 | *80074 | 30 | *97345 | 30 | *14591 | 30 | *31816 | 30 |
| 7 | *63071 | 35 | *80362 | 35 | *97632 | 35 | *14879 | 35 | *32103 | 35 |
| 8 | *63359 | 39 | *80651 | 39 | *97920 | 39 | *15166 | 39 | *32390 | 39 |
| 9 | *63647 | 44 | *80939 | 44 | *98208 | 44 | *15453 | 44 | *32677 | 44 |
| 10 | *63936 | 48 | *81227 | 48 | *98496 | 48 | *15741 | 48 | *32964 | 48 |
| 11 | 2°64224 | 53 | 2°81515 | 53 | 2°98783 | 53 | 3°16028 | 53 | 3°33251 | 53 |
| 12 | *64512 | 58 | *81803 | 58 | *99071 | 58 | *16315 | 58 | *33537 | 58 |
| 13 | *64800 | 62 | *82091 | 62 | *99358 | 62 | *16602 | 62 | *33824 | 62 |
| 14 | *65089 | 67 | *82379 | 67 | *99646 | 67 | *16889 | 67 | *34111 | 67 |
| 15 | *65377 | 72 | *82667 | 72 | *99934 | 72 | *17177 | 72 | *34398 | 72 |
| 16 | *65665 | 77 | *82955 | 77 | 3°00221 | 77 | *17464 | 77 | *34685 | 77 |
| 17 | *65954 | 81 | *83243 | 81 | *00509 | 81 | *17751 | 81 | *34971 | 81 |
| 18 | *66242 | 86 | *83531 | 86 | *00796 | 86 | *18038 | 86 | *35258 | 86 |
| 19 | *66530 | 91 | *83819 | 91 | *01084 | 91 | *18325 | 91 | *35545 | 91 |
| 20 | *66819 | 96 | *84107 | 96 | *01372 | 96 | *18613 | 96 | *35832 | 95 |
| 21 | 2°67107 | 101 | 2°84394 | 101 | 3°01659 | 101 | 3°18900 | 101 | 3°36118 | 100 |
| 22 | *67395 | 106 | *84682 | 106 | *01947 | 106 | *19187 | 106 | *36405 | 105 |
| 23 | *67683 | 111 | *84970 | 111 | *02234 | 111 | *19474 | 111 | *36692 | 110 |
| 24 | *67971 | 116 | *85258 | 116 | *02522 | 116 | *19762 | 116 | *36979 | 115 |
| 25 | *68260 | 121 | *85546 | 121 | *02809 | 121 | *20049 | 121 | *37265 | 120 |
| 26 | *68548 | 126 | *85834 | 126 | *03097 | 126 | *20336 | 126 | *37552 | 125 |
| 27 | *68836 | 131 | *86122 | 131 | *03384 | 131 | *20623 | 131 | *37839 | 130 |
| 28 | *69124 | 136 | *86410 | 136 | *03672 | 136 | *20910 | 136 | *38125 | 135 |
| 29 | *69412 | 140 | *86698 | 140 | *03959 | 140 | *21198 | 140 | *38412 | 139 |
| 30 | *69701 | 144 | *86986 | 144 | *04247 | 144 | *21485 | 144 | *38699 | 143 |
| 31 | 2°69989 | 148 | 2°87273 | 148 | 3°04534 | 148 | 3°21772 | 148 | 3°38985 | 147 |
| 32 | *70277 | 153 | *87561 | 153 | *04821 | 153 | *22059 | 153 | *39272 | 152 |
| 33 | *70565 | 158 | *87849 | 158 | *05109 | 158 | *22346 | 158 | *39559 | 157 |
| 34 | *70853 | 163 | *88137 | 163 | *05396 | 163 | *22633 | 163 | *39845 | 162 |
| 35 | *71142 | 168 | *88425 | 168 | *05684 | 168 | *22920 | 168 | *40132 | 167 |
| 36 | *71430 | 173 | *88712 | 173 | *05971 | 173 | *23207 | 173 | *40418 | 172 |
| 37 | *71718 | 178 | *89000 | 178 | *06258 | 178 | *23494 | 178 | *40705 | 177 |
| 38 | *72006 | 183 | *89288 | 183 | *06546 | 183 | *23782 | 183 | *40992 | 182 |
| 39 | *72294 | 188 | *89576 | 188 | *06833 | 188 | *24069 | 188 | *41278 | 187 |
| 40 | *72583 | 192 | *89864 | 192 | *07121 | 192 | *24356 | 192 | *41565 | 191 |
| 41 | 2°72871 | 197 | 2°90151 | 197 | 3°07408 | 197 | 3°24643 | 197 | 3°41851 | 196 |
| 42 | *73159 | 202 | *90439 | 202 | *07695 | 202 | *24930 | 202 | *42138 | 200 |
| 43 | *73447 | 207 | *90727 | 207 | *07983 | 207 | *25217 | 207 | *42425 | 205 |
| 44 | *73735 | 212 | *91015 | 212 | *08270 | 212 | *25504 | 212 | *42711 | 210 |
| 45 | *74024 | 217 | *91303 | 217 | *08558 | 217 | *25791 | 217 | *42998 | 215 |
| 46 | *74312 | 221 | *91590 | 221 | *08845 | 221 | *26078 | 221 | *43284 | 219 |
| 47 | *74600 | 226 | *91878 | 226 | *09132 | 226 | *26365 | 226 | *43571 | 224 |
| 48 | *74888 | 230 | *92166 | 230 | *09420 | 230 | *26652 | 230 | *43858 | 228 |
| 49 | *75176 | 235 | *92454 | 235 | *09707 | 235 | *26939 | 235 | *44144 | 233 |
| 50 | *75465 | 240 | *92742 | 240 | *09995 | 240 | *27226 | 239 | *44431 | 238 |
| 51 | 2°75753 | 245 | 2°93029 | 245 | 3°10282 | 244 | 3°27513 | 244 | 3°44717 | 243 |
| 52 | *76041 | 250 | *93317 | 250 | *10569 | 249 | *27800 | 249 | *45004 | 248 |
| 53 | *76329 | 255 | *93605 | 255 | *10856 | 254 | *28086 | 254 | *45290 | 253 |
| 54 | *76617 | 260 | *93892 | 260 | *11144 | 259 | *28373 | 259 | *45577 | 258 |
| 55 | *76905 | 264 | *94180 | 264 | *11431 | 263 | *28660 | 263 | *45863 | 262 |
| 56 | *77193 | 269 | *94468 | 269 | *11718 | 268 | *28947 | 268 | *46150 | 267 |
| 57 | *77481 | 274 | *94755 | 274 | *12006 | 273 | *29234 | 273 | *46436 | 272 |
| 58 | *77769 | 279 | *95043 | 279 | *12293 | 278 | *29521 | 278 | *46723 | 277 |
| 59 | *78057 | 284 | *95331 | 284 | *12580 | 283 | *29808 | 283 | *47009 | 282 |
| 60 | *78346 | 288 | *95619 | 288 | *12868 | 287 | *30095 | 287 | *47296 | 286 |

| Min. | 20° | Parts for " | 21° | Parts for " | 22° | Parts for " | 23° | Parts for " | 24° | Parts for " |
|------|---------|-------------------|---------|-------------------|---------|-------------------|---------|-------------------|---------|-------------------|
| 0 | 3°47296 | 0 | 3°64471 | 0 | 3°81618 | 0 | 3°98735 | 0 | 4°15824 | 0 |
| 1 | °47582 | 5 | °64757 | 5 | °81904 | 5 | °99020 | 5 | °16109 | 5 |
| 2 | °47869 | 10 | °65043 | 10 | °82189 | 10 | °99305 | 10 | °16393 | 9 |
| 3 | °48155 | 15 | °65329 | 15 | °82475 | 15 | °99590 | 15 | °16678 | 14 |
| 4 | °48441 | 20 | °65615 | 20 | °82760 | 20 | °99875 | 20 | °16962 | 19 |
| 5 | °48728 | 25 | °65901 | 25 | °83046 | 25 | 4°00160 | 25 | °17247 | 24 |
| 6 | °49014 | 30 | °66187 | 30 | °83331 | 30 | °00445 | 30 | °17531 | 29 |
| 7 | °49301 | 35 | °66473 | 35 | °83617 | 35 | °00730 | 35 | °17816 | 34 |
| 8 | °49587 | 39 | °66759 | 39 | °83902 | 39 | °01015 | 39 | °18100 | 38 |
| 9 | °49873 | 44 | °67045 | 44 | °84188 | 44 | °01300 | 44 | °18385 | 43 |
| 10 | °50160 | 48 | °67331 | 48 | °84473 | 48 | °01585 | 48 | °18669 | 47 |
| 11 | 3°50446 | 53 | 3°67617 | 53 | 3°84758 | 53 | 4°01870 | 53 | 4°18953 | 52 |
| 12 | °50732 | 58 | °67903 | 58 | °85044 | 58 | °02155 | 58 | °19238 | 57 |
| 13 | °51019 | 62 | °68189 | 62 | °85329 | 62 | °02440 | 62 | °19522 | 61 |
| 14 | °51305 | 67 | °68475 | 67 | °85615 | 67 | °02725 | 67 | °19807 | 66 |
| 15 | °51591 | 72 | °68761 | 72 | °85900 | 72 | °03010 | 72 | °20091 | 71 |
| 16 | °51878 | 77 | °69046 | 77 | °86185 | 77 | °03294 | 77 | °20375 | 76 |
| 17 | °52164 | 81 | °69332 | 81 | °86471 | 81 | °03579 | 81 | °20660 | 80 |
| 18 | °52450 | 86 | °69718 | 86 | °86756 | 86 | °03864 | 86 | °20944 | 85 |
| 19 | °52736 | 91 | °70004 | 91 | °87042 | 91 | °04149 | 91 | °21229 | 90 |
| 20 | °53023 | 95 | °70190 | 95 | °87327 | 95 | °04434 | 95 | °21513 | 94 |
| 21 | 3°53309 | 100 | 3°70476 | 100 | 3°87612 | 100 | 4°04719 | 100 | 4°21797 | 99 |
| 22 | °53595 | 105 | °70762 | 105 | °87898 | 105 | °05004 | 105 | °22082 | 104 |
| 23 | °53882 | 110 | °71047 | 110 | °88183 | 110 | °05289 | 110 | °22366 | 109 |
| 24 | °54168 | 115 | °71333 | 115 | °88468 | 115 | °05574 | 115 | °22650 | 114 |
| 25 | °54454 | 120 | °71619 | 120 | °88754 | 120 | °05859 | 120 | °22935 | 119 |
| 26 | °54741 | 125 | °71905 | 125 | °89039 | 125 | °06143 | 125 | °23219 | 124 |
| 27 | °55027 | 130 | °72191 | 130 | °89324 | 130 | °06428 | 130 | °23503 | 129 |
| 28 | °55313 | 135 | °72476 | 135 | °89609 | 135 | °06713 | 135 | °23787 | 134 |
| 29 | °55600 | 139 | °72762 | 139 | °89895 | 139 | °06998 | 139 | °24072 | 138 |
| 30 | °55886 | 143 | °73048 | 143 | °90180 | 143 | °07283 | 143 | °24356 | 142 |
| 31 | 3°56172 | 147 | 3°73334 | 147 | 3°90465 | 147 | 4°07568 | 147 | 4°24640 | 146 |
| 32 | °56458 | 152 | °73619 | 152 | °90750 | 152 | °07853 | 152 | °24924 | 151 |
| 33 | °56745 | 157 | °73905 | 157 | °91036 | 157 | °08137 | 157 | °25209 | 156 |
| 34 | °57031 | 162 | °74191 | 162 | °91321 | 161 | °08422 | 161 | °25493 | 160 |
| 35 | °57317 | 167 | °74477 | 167 | °91606 | 166 | °08707 | 166 | °25777 | 165 |
| 36 | °57603 | 172 | °74762 | 172 | °91891 | 171 | °08992 | 171 | °26061 | 170 |
| 37 | °57889 | 177 | °75048 | 177 | °92176 | 176 | °09277 | 176 | °26345 | 175 |
| 38 | °58176 | 182 | °75334 | 182 | °92462 | 181 | °09561 | 181 | °26630 | 180 |
| 39 | °58462 | 187 | °75619 | 187 | °92747 | 186 | °09846 | 186 | °26914 | 185 |
| 40 | °58748 | 191 | °75905 | 191 | °93032 | 190 | °10131 | 190 | °27198 | 189 |
| 41 | 3°59034 | 196 | 3°76191 | 196 | 3°93317 | 195 | 4°10416 | 195 | 4°27482 | 194 |
| 42 | °59320 | 200 | °76476 | 200 | °93602 | 199 | °10700 | 199 | °27766 | 198 |
| 43 | °59607 | 205 | °76762 | 205 | °93888 | 204 | °10985 | 204 | °28050 | 203 |
| 44 | °59893 | 210 | °77048 | 210 | °94173 | 209 | °11270 | 209 | °28334 | 208 |
| 45 | °60179 | 215 | °77334 | 215 | °94458 | 214 | °11555 | 214 | °28519 | 213 |
| 46 | °60465 | 219 | °77619 | 219 | °94743 | 218 | °11839 | 218 | °28903 | 217 |
| 47 | °60752 | 224 | °77905 | 224 | °95028 | 223 | °12124 | 223 | °29187 | 222 |
| 48 | °61038 | 228 | °78191 | 228 | °95314 | 227 | °12409 | 227 | °29471 | 226 |
| 49 | °61324 | 233 | °78476 | 233 | °95599 | 232 | °12693 | 232 | °29755 | 231 |
| 50 | °61610 | 238 | °78762 | 238 | °95884 | 237 | °12978 | 237 | °30039 | 236 |
| 51 | 3°61896 | 243 | 3°79048 | 243 | 3°96169 | 242 | 4°13263 | 242 | 4°30323 | 241 |
| 52 | °62182 | 248 | °79333 | 248 | °96454 | 247 | °13547 | 247 | °30607 | 246 |
| 53 | °62468 | 253 | °79619 | 253 | °96739 | 252 | °13832 | 252 | °30891 | 251 |
| 54 | °62754 | 258 | °79904 | 258 | °97024 | 257 | °14116 | 257 | °31175 | 256 |
| 55 | °63041 | 262 | °80190 | 262 | °97310 | 261 | °14401 | 261 | °31459 | 260 |
| 56 | °63327 | 267 | °80476 | 267 | °97595 | 266 | °14686 | 266 | °31743 | 265 |
| 57 | °63613 | 272 | °80761 | 272 | °97880 | 271 | °14970 | 271 | °32027 | 270 |
| 58 | °63899 | 277 | °81047 | 277 | °98165 | 276 | °15255 | 276 | °32311 | 275 |
| 59 | °64185 | 282 | °81332 | 282 | °98450 | 281 | °15539 | 281 | °32595 | 280 |
| 60 | °64471 | 286 | °81618 | 286 | °98735 | 285 | °15824 | 285 | °32879 | 284 |

| Min. | 25° | Parts for " | 26° | Parts for " | 27° | Parts for " | 28° | Parts for " | 29° | Parts for " |
|------|---------|-------------------|---------|-------------------|---------|-------------------|---------|-------------------|---------|-------------------|
| 0 | 4°32879 | 0 | 4°49901 | 0 | 4°66890 | 0 | 4°83843 | 0 | 5°00760 | 0 |
| 1 | °33163 | 5 | °50184 | 5 | °67173 | 5 | °84125 | 5 | °01042 | 5 |
| 2 | °33447 | 9 | °50468 | 9 | °67456 | 9 | °84407 | 9 | °01323 | 9 |
| 3 | °33730 | 14 | °50751 | 14 | °67738 | 14 | °84690 | 14 | °01605 | 14 |
| 4 | °34015 | 19 | °51035 | 19 | °68021 | 19 | °84972 | 19 | °01886 | 19 |
| 5 | °34298 | 24 | °51318 | 24 | °68304 | 24 | °85254 | 24 | °02168 | 24 |
| 6 | °34582 | 29 | °51601 | 29 | °68587 | 29 | °85536 | 29 | °02450 | 29 |
| 7 | °34866 | 34 | °51885 | 34 | °68870 | 34 | °85818 | 34 | °02731 | 34 |
| 8 | °35150 | 38 | °52168 | 38 | °69152 | 38 | °86101 | 38 | °03013 | 38 |
| 9 | °35434 | 43 | °52452 | 43 | °69435 | 43 | °86383 | 43 | °03294 | 43 |
| 10 | °35718 | 47 | °52735 | 47 | °69718 | 47 | °86665 | 47 | °03576 | 47 |
| 11 | 4°36002 | 52 | 4°53018 | 52 | 4°70001 | 52 | 4°86947 | 51 | 5°03857 | 51 |
| 12 | °36286 | 57 | °53302 | 57 | °70283 | 57 | °87229 | 56 | °04139 | 56 |
| 13 | °36569 | 61 | °53585 | 61 | °70566 | 61 | °87511 | 61 | °04420 | 61 |
| 14 | °36853 | 66 | °53868 | 66 | °70849 | 66 | °87793 | 66 | °04702 | 66 |
| 15 | °37137 | 71 | °54152 | 71 | °71133 | 71 | °88075 | 71 | °04983 | 71 |
| 16 | °37421 | 76 | °54435 | 76 | °71414 | 76 | °88358 | 76 | °05265 | 76 |
| 17 | °37705 | 80 | °54718 | 80 | °71697 | 80 | °88640 | 80 | °05546 | 80 |
| 18 | °37988 | 85 | °55001 | 85 | °71980 | 85 | °88922 | 85 | °05828 | 85 |
| 19 | °38272 | 90 | °55285 | 90 | °72262 | 90 | °89204 | 90 | °06109 | 90 |
| 20 | °38556 | 94 | °55568 | 94 | °72545 | 94 | °89486 | 94 | °06391 | 94 |
| 21 | 4°38840 | 99 | 4°55851 | 98 | 4°72828 | 98 | 4°89768 | 98 | 5°06672 | 98 |
| 22 | °39123 | 104 | °56134 | 103 | °73110 | 103 | °90050 | 103 | °06954 | 103 |
| 23 | °39407 | 109 | °56418 | 108 | °73393 | 108 | °90332 | 108 | °07235 | 108 |
| 24 | °39691 | 114 | °56701 | 113 | °73675 | 113 | °90614 | 113 | °07516 | 113 |
| 25 | °39975 | 119 | °56984 | 118 | °73958 | 118 | °90896 | 118 | °07797 | 118 |
| 26 | °40258 | 124 | °57267 | 123 | °74241 | 123 | °91178 | 123 | °08079 | 123 |
| 27 | °40542 | 129 | °57550 | 128 | °74523 | 128 | °91460 | 128 | °08360 | 128 |
| 28 | °40826 | 134 | °57834 | 133 | °74806 | 133 | °91742 | 133 | °08641 | 133 |
| 29 | °41109 | 138 | °58117 | 137 | °75088 | 137 | °92024 | 137 | °08923 | 137 |
| 30 | °41393 | 142 | °58400 | 141 | °75371 | 141 | °92306 | 141 | °09204 | 141 |
| 31 | 4°41677 | 146 | 4°58683 | 145 | 4°75653 | 145 | 4°92588 | 145 | 5°09485 | 145 |
| 32 | °41960 | 151 | °58966 | 150 | °75936 | 150 | °92870 | 150 | °09766 | 150 |
| 33 | °42244 | 156 | °59249 | 155 | °76218 | 155 | °93152 | 155 | °10048 | 155 |
| 34 | °42528 | 160 | °59532 | 159 | °76501 | 159 | °93434 | 159 | °10329 | 159 |
| 35 | °42812 | 165 | °59816 | 164 | °76783 | 164 | °93715 | 164 | °10610 | 164 |
| 36 | °43095 | 170 | °60099 | 169 | °77066 | 169 | °93997 | 169 | °10891 | 169 |
| 37 | °43379 | 175 | °60382 | 174 | °77348 | 174 | °94279 | 174 | °11172 | 174 |
| 38 | °43663 | 180 | °60665 | 179 | °77631 | 179 | °94561 | 179 | °11454 | 179 |
| 39 | °43946 | 185 | °60948 | 184 | °77913 | 184 | °94843 | 184 | °11735 | 184 |
| 40 | °44230 | 189 | °61231 | 188 | °78196 | 188 | °95125 | 188 | °12016 | 188 |
| 41 | 4°44514 | 194 | 4°61514 | 193 | 4°78478 | 193 | 4°95407 | 193 | 5°12297 | 193 |
| 42 | °44797 | 198 | °61797 | 197 | °78761 | 197 | °95689 | 197 | °12578 | 197 |
| 43 | °45080 | 203 | °62080 | 202 | °79043 | 202 | °95970 | 202 | °12859 | 202 |
| 44 | °45363 | 208 | °62363 | 207 | °79326 | 207 | °96252 | 207 | °13140 | 207 |
| 45 | °45647 | 213 | °62646 | 212 | °79608 | 212 | °96534 | 212 | °13421 | 212 |
| 46 | °45931 | 217 | °62929 | 216 | °79890 | 216 | °96816 | 216 | °13703 | 216 |
| 47 | °46214 | 222 | °63212 | 221 | °80173 | 221 | °97098 | 221 | °13984 | 221 |
| 48 | °46498 | 226 | °63495 | 225 | °80455 | 225 | °97379 | 225 | °14265 | 225 |
| 49 | °46781 | 231 | °63778 | 230 | °80738 | 230 | °97661 | 230 | °14546 | 230 |
| 50 | °47065 | 236 | °64061 | 235 | °81020 | 235 | °97943 | 235 | °14827 | 235 |
| 51 | 4°47349 | 241 | 4°64344 | 240 | 4°81302 | 240 | 4°98225 | 239 | 5°15108 | 239 |
| 52 | °47632 | 246 | °64627 | 245 | °81585 | 245 | °98506 | 244 | °15389 | 244 |
| 53 | °47916 | 251 | °64910 | 250 | °81867 | 250 | °98788 | 249 | °15670 | 249 |
| 54 | °48199 | 256 | °65193 | 255 | °82149 | 255 | °99070 | 254 | °15951 | 254 |
| 55 | °48483 | 260 | °65475 | 259 | °82431 | 259 | °99351 | 258 | °16232 | 258 |
| 56 | °48767 | 265 | °65758 | 264 | °82714 | 264 | °99633 | 263 | °16513 | 263 |
| 57 | °49050 | 270 | °66041 | 269 | °82996 | 269 | °99915 | 268 | °16794 | 268 |
| 58 | °49334 | 275 | °66324 | 274 | °83278 | 274 | 5°00197 | 273 | °17075 | 273 |
| 59 | °49617 | 280 | °66607 | 279 | °83561 | 279 | °00478 | 278 | °17356 | 278 |
| 60 | °49901 | 284 | °66890 | 283 | °83843 | 283 | °00760 | 282 | °17637 | 282 |

| Min. | 30° | Parts for " | 31° | Parts for " | 32° | Parts for " | 33° | Parts for " | 34° | Parts for " |
|------|---------|-------------------|---------|-------------------|---------|-------------------|---------|-------------------|---------|-------------------|
| 0 | 5°17638 | 0 | 5°34476 | -0 | 5°51274 | 0 | 5°68030 | 0 | 5°84744 | 0 |
| 1 | *17919 | 5 | *34757 | 5 | *51654 | 5 | *68309 | 5 | *85022 | 5 |
| 2 | *18200 | 9 | *35038 | 9 | *51834 | 9 | *68588 | 9 | *85300 | 9 |
| 3 | *18481 | 14 | *35318 | 14 | *52114 | 14 | *68867 | 14 | *85578 | 14 |
| 4 | *18762 | 19 | *35598 | 19 | *52394 | 19 | *69146 | 19 | *85856 | 19 |
| 5 | *19043 | 23 | *35878 | 23 | *52673 | 23 | *69425 | 23 | *86134 | 23 |
| 6 | *19324 | 28 | *36158 | 28 | *52952 | 28 | *69704 | 28 | *86412 | 28 |
| 7 | *19605 | 33 | *36438 | 33 | *53232 | 32 | *69983 | 32 | *86690 | 32 |
| 8 | *19886 | 37 | *36718 | 37 | *53512 | 36 | *70262 | 36 | *86968 | 36 |
| 9 | *20167 | 42 | *36999 | 42 | *53791 | 41 | *70541 | 41 | *87246 | 41 |
| 10 | *20448 | 47 | *37280 | 47 | *54070 | 46 | *70820 | 46 | *87524 | 46 |
| 11 | 5°20729 | 51 | 5°37560 | 51 | 5°54350 | 50 | 5°71098 | 50 | 5°87802 | 50 |
| 12 | *21010 | 56 | *37840 | 56 | *54630 | 55 | *71376 | 55 | *88080 | 55 |
| 13 | *21290 | 61 | *38120 | 61 | *54909 | 60 | *71655 | 60 | *88358 | 60 |
| 14 | *21570 | 65 | *38400 | 65 | *55188 | 65 | *71934 | 65 | *88636 | 65 |
| 15 | *21851 | 70 | *38680 | 70 | *55468 | 70 | *72213 | 70 | *88914 | 70 |
| 16 | *22132 | 75 | *38960 | 75 | *55748 | 75 | *72492 | 75 | *89192 | 75 |
| 17 | *22413 | 79 | *39240 | 79 | *56027 | 79 | *72771 | 79 | *89470 | 79 |
| 18 | *22694 | 84 | *39520 | 84 | *56306 | 84 | *73050 | 84 | *89748 | 84 |
| 19 | *22975 | 89 | *39800 | 89 | *56585 | 89 | *73328 | 89 | *90026 | 89 |
| 20 | *23256 | 93 | *40080 | 93 | *56864 | 93 | *73606 | 93 | *90304 | 93 |
| 21 | 5°23537 | 98 | 5°40360 | 98 | 5°57144 | 97 | 5°73885 | 97 | 5°90582 | 97 |
| 22 | *23818 | 103 | *40640 | 103 | *57424 | 102 | *74164 | 102 | *90860 | 102 |
| 23 | *24098 | 108 | *40920 | 108 | *57703 | 106 | *74443 | 106 | *91138 | 106 |
| 24 | *24378 | 112 | *41200 | 112 | *57982 | 111 | *74722 | 111 | *91416 | 111 |
| 25 | *24659 | 117 | *41481 | 117 | *58261 | 116 | *75000 | 116 | *91694 | 116 |
| 26 | *24940 | 122 | *41762 | 122 | *58540 | 121 | *75278 | 121 | *91972 | 121 |
| 27 | *25221 | 126 | *42042 | 126 | *58820 | 125 | *75557 | 125 | *92250 | 125 |
| 28 | *25502 | 131 | *42322 | 131 | *59100 | 130 | *75836 | 130 | *92528 | 130 |
| 29 | *25782 | 136 | *42601 | 136 | *59379 | 135 | *76114 | 135 | *92806 | 135 |
| 30 | *26062 | 140 | *42880 | 140 | *59658 | 139 | *76392 | 139 | *93084 | 139 |
| 31 | 5°26343 | 145 | 5°43160 | 145 | 5°59937 | 144 | 5°76671 | 144 | 5°93361 | 144 |
| 32 | *26624 | 150 | *43440 | 150 | *60216 | 149 | *76950 | 148 | *93638 | 148 |
| 33 | *26905 | 154 | *43720 | 154 | *60496 | 153 | *77228 | 152 | *93916 | 152 |
| 34 | *27186 | 159 | *44000 | 159 | *60776 | 158 | *77506 | 157 | *94194 | 157 |
| 35 | *27466 | 164 | *44280 | 164 | *61055 | 163 | *77885 | 162 | *94472 | 162 |
| 36 | *27746 | 168 | *44560 | 168 | *61334 | 167 | *78064 | 166 | *94750 | 166 |
| 37 | *28027 | 173 | *44840 | 173 | *61613 | 172 | *78342 | 171 | *95028 | 171 |
| 38 | *28308 | 178 | *45120 | 178 | *61892 | 177 | *78620 | 176 | *95306 | 176 |
| 39 | *28588 | 183 | *45400 | 183 | *62171 | 182 | *78899 | 181 | *95583 | 181 |
| 40 | *28868 | 187 | *45680 | 187 | *62450 | 186 | *79178 | 185 | *95860 | 185 |
| 41 | 5°29149 | 192 | 5°45960 | 192 | 5°62729 | 191 | 5°79456 | 190 | 5°96138 | 190 |
| 42 | *29430 | 196 | *46240 | 196 | *63008 | 195 | *79734 | 194 | *96416 | 194 |
| 43 | *29710 | 201 | *46520 | 201 | *63287 | 200 | *80013 | 199 | *96694 | 199 |
| 44 | *29990 | 206 | *46800 | 206 | *63566 | 205 | *80292 | 204 | *96972 | 204 |
| 45 | *30271 | 210 | *47079 | 210 | *63845 | 209 | *80570 | 208 | *97249 | 208 |
| 46 | *30552 | 215 | *47358 | 215 | *64124 | 214 | *80848 | 213 | *97526 | 213 |
| 47 | *30832 | 220 | *47638 | 219 | *64404 | 218 | *81126 | 217 | *97804 | 217 |
| 48 | *31112 | 224 | *47918 | 223 | *64684 | 222 | *81404 | 221 | *98082 | 221 |
| 49 | *31393 | 229 | *48198 | 228 | *64963 | 227 | *81683 | 226 | *98359 | 226 |
| 50 | *31674 | 234 | *48478 | 233 | *65242 | 232 | *81962 | 231 | *98636 | 231 |
| 51 | 5°31954 | 238 | 5°48758 | 237 | 5°65521 | 236 | 5°82240 | 235 | 5°98914 | 235 |
| 52 | *32234 | 243 | *49038 | 242 | *65800 | 241 | *82518 | 240 | *99192 | 240 |
| 53 | *32514 | 248 | *49317 | 247 | *66079 | 246 | *82796 | 245 | *99469 | 245 |
| 54 | *32794 | 252 | *49596 | 251 | *66358 | 250 | *83074 | 249 | *99746 | 249 |
| 55 | *33075 | 257 | *49876 | 256 | *66637 | 255 | *83352 | 254 | *00024 | 254 |
| 56 | *33356 | 262 | *50156 | 261 | *66916 | 260 | *83630 | 259 | *00302 | 259 |
| 57 | *33636 | 266 | *50436 | 265 | *67194 | 264 | *83909 | 263 | *00579 | 263 |
| 58 | *33916 | 271 | *50716 | 270 | *67472 | 269 | *84188 | 268 | *00856 | 268 |
| 59 | *34196 | 276 | *50995 | 275 | *67751 | 274 | *84466 | 273 | *01134 | 273 |
| 60 | *34476 | 281 | *51274 | 280 | *68030 | 279 | *84744 | 278 | *01412 | 278 |

| Min. | 35° | Parts for " | 36° | Parts for " | 37° | Parts for " | 38° | Parts for " | 39° | Parts for " |
|------|---------|-------------------|---------|-------------------|---------|-------------------|---------|-------------------|---------|-------------------|
| 0 | 6°01412 | 0 | 6°16034 | 0 | 6°34610 | 0 | 6°51136 | 0 | 6°67614 | 0 |
| 1 | *01689 | 5 | *18311 | 5 | *34886 | 5 | *51411 | 5 | *67888 | 5 |
| 2 | *01966 | 9 | *18588 | 9 | *35162 | 9 | *51686 | 9 | *68162 | 9 |
| 3 | *02244 | 14 | *18864 | 14 | *35437 | 14 | *51961 | 14 | *68436 | 14 |
| 4 | *02522 | 18 | *19140 | 18 | *35712 | 18 | *52236 | 18 | *68710 | 18 |
| 5 | *02799 | 23 | *19417 | 23 | *35988 | 23 | *52511 | 23 | *68984 | 23 |
| 6 | *03076 | 28 | *19694 | 28 | *36264 | 28 | *52786 | 28 | *69258 | 27 |
| 7 | *03353 | 32 | *19970 | 32 | *36540 | 32 | *53061 | 32 | *69532 | 31 |
| 8 | *03630 | 37 | *20246 | 37 | *36816 | 37 | *53336 | 37 | *69806 | 36 |
| 9 | *03908 | 41 | *20523 | 41 | *37092 | 41 | *53611 | 41 | *70081 | 40 |
| 10 | *04186 | 46 | *20800 | 46 | *37368 | 46 | *53886 | 46 | *70356 | 45 |
| 11 | 6°04463 | 50 | 6°21076 | 50 | 6°37643 | 50 | 6°54161 | 50 | 6°70630 | 50 |
| 12 | *04740 | 55 | *21352 | 55 | *37918 | 55 | *54436 | 55 | *70904 | 55 |
| 13 | *05017 | 59 | *21629 | 59 | *38194 | 59 | *54711 | 59 | *71178 | 59 |
| 14 | *05294 | 64 | *21906 | 64 | *38470 | 64 | *54986 | 64 | *71452 | 64 |
| 15 | *05571 | 69 | *22182 | 69 | *38746 | 68 | *55261 | 68 | *71726 | 68 |
| 16 | *05848 | 74 | *22458 | 74 | *39022 | 73 | *55536 | 73 | *72000 | 73 |
| 17 | *06126 | 78 | *22735 | 78 | *39297 | 77 | *55810 | 77 | *72274 | 77 |
| 18 | *06404 | 83 | *23012 | 83 | *39572 | 82 | *56084 | 82 | *72548 | 82 |
| 19 | *06681 | 88 | *23288 | 88 | *39848 | 87 | *56359 | 87 | *72822 | 87 |
| 20 | *06958 | 92 | *23564 | 92 | *40124 | 91 | *56634 | 91 | *73096 | 91 |
| 21 | 6°07235 | 96 | 6°23841 | 96 | 6°40399 | 95 | 6°56909 | 95 | 6°73369 | 95 |
| 22 | *07512 | 101 | *24118 | 101 | *40674 | 100 | *57184 | 100 | *73642 | 100 |
| 23 | *07789 | 105 | *24394 | 105 | *40950 | 104 | *57459 | 104 | *73916 | 104 |
| 24 | *08066 | 110 | *24670 | 110 | *41226 | 109 | *57734 | 109 | *74190 | 109 |
| 25 | *08343 | 115 | *24946 | 115 | *41502 | 114 | *58008 | 114 | *74464 | 114 |
| 26 | *08620 | 120 | *25222 | 120 | *41778 | 119 | *58282 | 119 | *74738 | 119 |
| 27 | *08897 | 124 | *25499 | 124 | *42053 | 123 | *58557 | 123 | *75012 | 123 |
| 28 | *09174 | 129 | *25776 | 129 | *42328 | 128 | *58832 | 128 | *75286 | 128 |
| 29 | *09451 | 134 | *26052 | 134 | *42604 | 133 | *59107 | 133 | *75560 | 133 |
| 30 | *09728 | 138 | *26328 | 138 | *42880 | 137 | *59382 | 137 | *75834 | 137 |
| 31 | 6°10005 | 143 | 6°26604 | 143 | 6°43155 | 142 | 6°59656 | 142 | 6°76108 | 142 |
| 32 | *10282 | 147 | *26880 | 147 | *43430 | 146 | *59930 | 146 | *76382 | 146 |
| 33 | *10559 | 151 | *27156 | 151 | *43705 | 150 | *60205 | 150 | *76655 | 150 |
| 34 | *10836 | 156 | *27432 | 156 | *43980 | 155 | *60480 | 155 | *76928 | 155 |
| 35 | *11113 | 161 | *27709 | 161 | *44256 | 160 | *60754 | 160 | *77202 | 160 |
| 36 | *11390 | 165 | *27986 | 165 | *44532 | 164 | *61028 | 164 | *77476 | 164 |
| 37 | *11667 | 170 | *28262 | 170 | *44807 | 169 | *61303 | 169 | *77750 | 169 |
| 38 | *11944 | 175 | *28538 | 175 | *45082 | 174 | *61578 | 174 | *78024 | 173 |
| 39 | *12221 | 180 | *28814 | 180 | *45357 | 179 | *61852 | 179 | *78297 | 178 |
| 40 | *12498 | 184 | *29090 | 184 | *45632 | 183 | *62126 | 183 | *78570 | 182 |
| 41 | 6°12775 | 189 | 6°29366 | 189 | 6°45908 | 188 | 6°62401 | 188 | 6°78844 | 187 |
| 42 | *13052 | 193 | *29642 | 193 | *46184 | 192 | *62676 | 192 | *79118 | 191 |
| 43 | *13329 | 198 | *29918 | 198 | *46459 | 197 | *62950 | 197 | *79392 | 196 |
| 44 | *13606 | 203 | *30194 | 203 | *46734 | 202 | *63224 | 202 | *79666 | 201 |
| 45 | *13883 | 207 | *30470 | 207 | *47009 | 206 | *63499 | 206 | *79939 | 205 |
| 46 | *14160 | 212 | *30746 | 212 | *47284 | 211 | *63774 | 211 | *80212 | 210 |
| 47 | *14437 | 216 | *31022 | 216 | *47559 | 215 | *64048 | 215 | *80486 | 214 |
| 48 | *14714 | 220 | *31298 | 220 | *47834 | 219 | *64322 | 219 | *80760 | 218 |
| 49 | *14990 | 225 | *31574 | 225 | *48110 | 224 | *64597 | 224 | *81033 | 223 |
| 50 | *15266 | 230 | *31850 | 230 | *48386 | 229 | *64872 | 229 | *81306 | 228 |
| 51 | 6°15543 | 234 | 6°32126 | 234 | 6°48661 | 233 | 6°65146 | 233 | 6°81580 | 232 |
| 52 | *15820 | 239 | *32402 | 239 | *48936 | 238 | *65420 | 238 | *81854 | 237 |
| 53 | *16097 | 244 | *32678 | 244 | *49211 | 243 | *65694 | 243 | *82127 | 242 |
| 54 | *16374 | 248 | *32954 | 248 | *49486 | 247 | *65968 | 247 | *82400 | 246 |
| 55 | *16651 | 253 | *33230 | 253 | *49761 | 252 | *66242 | 252 | *82673 | 251 |
| 56 | *16928 | 258 | *33506 | 257 | *50036 | 256 | *66516 | 256 | *82946 | 255 |
| 57 | *17204 | 262 | *33782 | 261 | *50311 | 260 | *66791 | 260 | *83220 | 259 |
| 58 | *17480 | 267 | *34058 | 266 | *50586 | 265 | *67066 | 265 | *83494 | 264 |
| 59 | *17757 | 272 | *34334 | 271 | *50861 | 270 | *67340 | 270 | *83767 | 269 |
| 60 | *18034 | 277 | *34610 | 276 | *51136 | 275 | *67614 | 275 | *84040 | 274 |

| Min. | 40° | Parts for " | 41° | Parts for " | 42° | Parts for " | 43° | Parts for " | 44° | Parts for " |
|------|---------|-------------------|---------|-------------------|---------|-------------------|---------|-------------------|---------|-------------------|
| 0 | 6°84040 | 0 | 7°00414 | 0 | 7°16736 | 0 | 7°33002 | 0 | 7°49214 | 0 |
| 1 | °84314 | 5 | °00687 | 5 | °17008 | 5 | °33273 | 5 | °49483 | 5 |
| 2 | °84588 | 9 | °00960 | 9 | °17280 | 9 | °33544 | 9 | °49752 | 9 |
| 3 | °84861 | 14 | °01232 | 14 | °17551 | 14 | °33815 | 14 | °50022 | 14 |
| 4 | °85134 | 18 | °01504 | 18 | °17822 | 18 | °34086 | 18 | °50292 | 18 |
| 5 | °85407 | 23 | °01777 | 23 | °18094 | 23 | °34356 | 23 | °50562 | 23 |
| 6 | °85680 | 27 | °02050 | 27 | °18366 | 27 | °34626 | 27 | °50832 | 27 |
| 7 | °85953 | 31 | °02322 | 31 | °18637 | 31 | °34897 | 31 | °51101 | 31 |
| 8 | °86226 | 36 | °02594 | 36 | °18908 | 36 | °35168 | 36 | °51370 | 36 |
| 9 | °86500 | 40 | °02866 | 40 | °19179 | 40 | °35438 | 40 | °51640 | 40 |
| 10 | °86774 | 45 | °03138 | 45 | °19450 | 45 | °35708 | 45 | °51910 | 45 |
| 11 | 6°87047 | 50 | 7°03411 | 50 | 7°19722 | 50 | 7°35979 | 50 | 7°52179 | 50 |
| 12 | °87320 | 55 | °03684 | 55 | °19994 | 55 | °36250 | 54 | °52448 | 54 |
| 13 | °87593 | 59 | °03956 | 59 | °20265 | 59 | °36520 | 59 | °52718 | 59 |
| 14 | °87866 | 64 | °04228 | 64 | °20536 | 64 | °36790 | 63 | °52988 | 63 |
| 15 | °88139 | 68 | °04500 | 68 | °20808 | 68 | °37060 | 68 | °53257 | 68 |
| 16 | °88412 | 73 | °04772 | 73 | °21080 | 73 | °37330 | 72 | °53526 | 72 |
| 17 | °88685 | 77 | °05044 | 77 | °21351 | 77 | °37601 | 77 | °53796 | 77 |
| 18 | °88958 | 82 | °05316 | 82 | °21622 | 81 | °37872 | 81 | °54066 | 81 |
| 19 | °89231 | 87 | °05589 | 87 | °21893 | 86 | °38142 | 86 | °54335 | 86 |
| 20 | °89504 | 91 | °05862 | 91 | °22164 | 90 | °38412 | 90 | °54604 | 90 |
| 21 | 6°89777 | 95 | 7°06134 | 95 | 7°22435 | 94 | 7°38683 | 95 | 7°54873 | 95 |
| 22 | °90050 | 100 | °06406 | 100 | °22706 | 99 | °38954 | 99 | °55142 | 99 |
| 23 | °90323 | 104 | °06678 | 104 | °22978 | 103 | °39224 | 104 | °55412 | 104 |
| 24 | °90596 | 109 | °06950 | 109 | °23250 | 108 | °39494 | 108 | °55682 | 108 |
| 25 | °90869 | 113 | °07222 | 113 | °23521 | 112 | °39764 | 113 | °55951 | 113 |
| 26 | °91142 | 118 | °07494 | 118 | °23792 | 117 | °40034 | 117 | °56220 | 117 |
| 27 | °91415 | 122 | °07766 | 122 | °24063 | 121 | °40304 | 122 | °56489 | 122 |
| 28 | °91688 | 127 | °08038 | 127 | °24334 | 126 | °40574 | 126 | °56758 | 126 |
| 29 | °91961 | 132 | °08310 | 132 | °24605 | 131 | °40844 | 131 | °57028 | 131 |
| 30 | °92234 | 136 | °08582 | 136 | °24876 | 135 | °41114 | 135 | °57298 | 135 |
| 31 | 6°92507 | 141 | 7°08854 | 141 | 7°25147 | 140 | 7°41385 | 140 | 7°57567 | 140 |
| 32 | °92780 | 146 | °09126 | 146 | °25418 | 145 | °41656 | 144 | °57836 | 144 |
| 33 | °93053 | 150 | °09398 | 150 | °25689 | 149 | °41926 | 149 | °58105 | 149 |
| 34 | °93326 | 155 | °09670 | 155 | °25960 | 154 | °42196 | 153 | °58374 | 153 |
| 35 | °93599 | 160 | °09942 | 159 | °26231 | 158 | °42466 | 158 | °58643 | 158 |
| 36 | °93872 | 164 | °10214 | 163 | °26502 | 162 | °42736 | 162 | °58912 | 162 |
| 37 | °94145 | 169 | °10486 | 168 | °26773 | 167 | °43006 | 167 | °59181 | 167 |
| 38 | °94418 | 173 | °10758 | 172 | °27044 | 171 | °43276 | 171 | °59450 | 171 |
| 39 | °94690 | 178 | °11030 | 177 | °27315 | 176 | °43546 | 176 | °59719 | 175 |
| 40 | °94962 | 182 | °11302 | 181 | °27586 | 180 | °43816 | 180 | °59988 | 179 |
| 41 | 6°95235 | 187 | 7°11574 | 186 | 7°27857 | 185 | 7°44086 | 185 | 7°60257 | 184 |
| 42 | °95508 | 191 | °11846 | 190 | °28128 | 189 | °44356 | 189 | °60526 | 188 |
| 43 | °95781 | 196 | °12117 | 195 | °28399 | 194 | °44626 | 194 | °60795 | 193 |
| 44 | °96054 | 200 | °12388 | 199 | °28670 | 198 | °44896 | 198 | °61064 | 197 |
| 45 | °96326 | 204 | °12660 | 203 | °28941 | 202 | °45166 | 203 | °61333 | 202 |
| 46 | °96599 | 209 | °12932 | 208 | °29212 | 207 | °45436 | 207 | °61602 | 206 |
| 47 | °96871 | 213 | °13204 | 212 | °29483 | 211 | °45706 | 212 | °61871 | 211 |
| 48 | °97144 | 217 | °13476 | 216 | °29754 | 215 | °45976 | 216 | °62140 | 215 |
| 49 | °97417 | 222 | °13748 | 221 | °30025 | 220 | °46246 | 221 | °62409 | 220 |
| 50 | °97690 | 227 | °14020 | 226 | °30296 | 225 | °46516 | 225 | °62678 | 224 |
| 51 | 6°97962 | 231 | 7°14291 | 230 | 7°30566 | 229 | 7°46786 | 230 | 7°62947 | 229 |
| 52 | °98234 | 236 | °14562 | 235 | °30836 | 234 | °47056 | 234 | °63216 | 233 |
| 53 | °98507 | 241 | °14834 | 240 | °31107 | 239 | °47325 | 239 | °63485 | 238 |
| 54 | °98780 | 245 | °15106 | 244 | °31378 | 243 | °47594 | 243 | °63754 | 242 |
| 55 | °99052 | 250 | °15378 | 249 | °31649 | 248 | °47864 | 248 | °64023 | 247 |
| 56 | °99324 | 254 | °15650 | 253 | °31920 | 252 | °48134 | 252 | °64292 | 251 |
| 57 | °99597 | 258 | °15921 | 257 | °32191 | 256 | °48404 | 257 | °64561 | 256 |
| 58 | °99870 | 263 | °16192 | 262 | °32462 | 261 | °48674 | 261 | °64830 | 260 |
| 59 | 7°00142 | 268 | °16464 | 267 | °32732 | 266 | °48944 | 266 | °65098 | 265 |
| 60 | °00414 | 273 | °16736 | 272 | °33002 | 271 | °49214 | 270 | °65366 | 269 |

| Min. | 45° | Parts for " | 46° | Parts for " | 47° | Parts for " | 48° | Parts for " | 49° | Parts for " |
|------|---------|-------------------|---------|-------------------|---------|-------------------|---------|-------------------|---------|-------------------|
| 0 | 7°65366 | 0 | 7°81462 | 0 | 7°97498 | 0 | 8°13474 | 0 | 8°29386 | 0 |
| 1 | *65635 | 5 | *81730 | 5 | *97765 | 4 | *13739 | 4 | *29651 | 4 |
| 2 | *65904 | 9 | *81998 | 9 | *98032 | 8 | *14004 | 8 | *29916 | 8 |
| 3 | *66173 | 14 | *82266 | 14 | *98299 | 13 | *14270 | 13 | *30181 | 13 |
| 4 | *66442 | 18 | *82534 | 18 | *98566 | 17 | *14536 | 17 | *30446 | 17 |
| 5 | *66711 | 23 | *82801 | 23 | *98832 | 22 | *14802 | 22 | *30710 | 22 |
| 6 | *66980 | 27 | *83068 | 27 | *99098 | 26 | *15068 | 26 | *30974 | 26 |
| 7 | *67248 | 31 | *83336 | 31 | *99365 | 30 | *15333 | 30 | *31239 | 30 |
| 8 | *67516 | 36 | *83604 | 36 | *99632 | 35 | *15598 | 35 | *31504 | 35 |
| 9 | *67785 | 40 | *83872 | 40 | *99899 | 39 | *15864 | 39 | *31768 | 39 |
| 10 | *68054 | 45 | *84140 | 45 | 8°00166 | 44 | *16130 | 44 | *32032 | 44 |
| 11 | 7°68322 | 50 | 7°84407 | 50 | 8°00432 | 49 | 8°16396 | 49 | 8°32297 | 49 |
| 12 | *68590 | 54 | *84674 | 54 | *00698 | 53 | *16662 | 53 | *32562 | 53 |
| 13 | *68859 | 59 | *84942 | 59 | *00965 | 58 | *16927 | 58 | *32826 | 58 |
| 14 | *69128 | 63 | *85210 | 63 | *01232 | 62 | *17192 | 62 | *33090 | 62 |
| 15 | *69396 | 67 | *85477 | 67 | *01498 | 66 | *17458 | 66 | *33355 | 66 |
| 16 | *69664 | 71 | *85744 | 71 | *01764 | 70 | *17724 | 70 | *33620 | 70 |
| 17 | *69933 | 76 | *86012 | 76 | *02031 | 75 | *17989 | 75 | *33884 | 75 |
| 18 | *70202 | 80 | *86280 | 80 | *02298 | 79 | *18254 | 79 | *34148 | 79 |
| 19 | *70470 | 85 | *86547 | 85 | *02564 | 84 | *18519 | 84 | *34413 | 84 |
| 20 | *70738 | 89 | *86814 | 89 | *02830 | 88 | *18784 | 88 | *34678 | 88 |
| 21 | 7°71007 | 94 | 7°87082 | 94 | 8°03096 | 93 | 8°19050 | 93 | 8°34942 | 93 |
| 22 | *71276 | 98 | *87350 | 98 | *03362 | 97 | *19316 | 97 | *35206 | 97 |
| 23 | *71544 | 103 | *87617 | 103 | *03629 | 102 | *19581 | 102 | *35470 | 102 |
| 24 | *71812 | 107 | *87884 | 107 | *03896 | 106 | *19846 | 106 | *35734 | 106 |
| 25 | *72080 | 112 | *88151 | 112 | *04162 | 111 | *20111 | 111 | *35998 | 111 |
| 26 | *72348 | 116 | *88418 | 116 | *04428 | 115 | *20376 | 115 | *36262 | 115 |
| 27 | *72617 | 121 | *88686 | 121 | *04694 | 120 | *20642 | 120 | *36527 | 120 |
| 28 | *72886 | 125 | *88954 | 125 | *04960 | 124 | *20908 | 124 | *36792 | 124 |
| 29 | *73154 | 130 | *89221 | 130 | *05227 | 129 | *21173 | 128 | *37056 | 128 |
| 30 | *73422 | 134 | *89488 | 134 | *05494 | 133 | *21438 | 132 | *37320 | 132 |
| 31 | 7°73690 | 139 | 7°89755 | 139 | 8°05760 | 138 | 8°21703 | 137 | 8°37584 | 137 |
| 32 | *73958 | 143 | *90022 | 143 | *06026 | 142 | *21968 | 141 | *37848 | 141 |
| 33 | *74226 | 148 | *90289 | 148 | *06292 | 147 | *22233 | 146 | *38112 | 146 |
| 34 | *74494 | 152 | *90556 | 152 | *06558 | 151 | *22498 | 150 | *38376 | 150 |
| 35 | *74763 | 157 | *90824 | 157 | *06824 | 156 | *22763 | 155 | *38640 | 155 |
| 36 | *75032 | 161 | *91092 | 161 | *07090 | 160 | *23028 | 159 | *38904 | 159 |
| 37 | *75300 | 166 | *91359 | 166 | *07356 | 165 | *23294 | 164 | *39168 | 164 |
| 38 | *75568 | 170 | *91626 | 170 | *07622 | 169 | *23560 | 168 | *39432 | 168 |
| 39 | *75836 | 175 | *91893 | 174 | *07889 | 173 | *23825 | 172 | *39696 | 172 |
| 40 | *76104 | 179 | *92160 | 178 | *08156 | 177 | *24090 | 176 | *39960 | 176 |
| 41 | 7°76372 | 184 | 7°92427 | 183 | 8°08422 | 182 | 8°24355 | 181 | 8°40224 | 181 |
| 42 | *76640 | 188 | *92694 | 187 | *08688 | 186 | *24620 | 185 | *40488 | 185 |
| 43 | *76908 | 193 | *92961 | 192 | *08954 | 191 | *24885 | 190 | *40752 | 190 |
| 44 | *77176 | 197 | *93228 | 196 | *09220 | 195 | *25150 | 194 | *41016 | 194 |
| 45 | *77444 | 202 | *93495 | 201 | *09486 | 200 | *25415 | 199 | *41280 | 199 |
| 46 | *77712 | 206 | *93762 | 205 | *09752 | 204 | *25680 | 203 | *41544 | 203 |
| 47 | *77980 | 211 | *94029 | 210 | *10018 | 209 | *25944 | 208 | *41808 | 208 |
| 48 | *78248 | 215 | *94296 | 214 | *10284 | 213 | *26208 | 212 | *42072 | 212 |
| 49 | *78516 | 219 | *94563 | 218 | *10550 | 217 | *26473 | 216 | *42336 | 216 |
| 50 | *78784 | 223 | *94830 | 222 | *10816 | 221 | *26738 | 220 | *42600 | 220 |
| 51 | 7°79052 | 228 | 7°95097 | 227 | 8°11081 | 226 | 8°27003 | 225 | 8°42863 | 224 |
| 52 | *79320 | 232 | *95364 | 231 | *11346 | 230 | *27268 | 229 | *43126 | 228 |
| 53 | *79588 | 237 | *95631 | 236 | *11612 | 235 | *27533 | 234 | *43390 | 233 |
| 54 | *79856 | 241 | *95898 | 240 | *11878 | 239 | *27798 | 238 | *43654 | 237 |
| 55 | *80124 | 246 | *96165 | 245 | *12144 | 244 | *28063 | 243 | *43918 | 242 |
| 56 | *80392 | 250 | *96432 | 249 | *12410 | 248 | *28328 | 247 | *44182 | 246 |
| 57 | *80660 | 255 | *96698 | 254 | *12676 | 253 | *28593 | 252 | *44446 | 251 |
| 58 | *80928 | 259 | *96964 | 258 | *12942 | 257 | *28858 | 257 | *44710 | 256 |
| 59 | *81194 | 264 | *97231 | 263 | *13208 | 262 | *29122 | 261 | *44973 | 260 |
| 60 | *81462 | 268 | *97498 | 267 | *13474 | 266 | *29386 | 265 | *45236 | 264 |

| Min. | 50° | Parts for " | 51° | Parts for " | 52° | Parts for " | 53° | Parts for " | 54° | Parts for " |
|------|---------|-------------------|---------|-------------------|---------|-------------------|---------|-------------------|---------|-------------------|
| 0 | 8°45236 | 0 | 8°61022 | 0 | 8°76742 | 0 | 8°92396 | 0 | 9°07982 | 0 |
| 1 | *45500 | 4 | *61285 | 4 | *77004 | 4 | *92656 | 4 | *08241 | 4 |
| 2 | *45764 | 8 | *61548 | 8 | *77266 | 8 | *92916 | 8 | *08500 | 8 |
| 3 | *46028 | 13 | *61810 | 13 | *77527 | 13 | *93176 | 12 | *08759 | 12 |
| 4 | *46292 | 17 | *62072 | 17 | *77788 | 17 | *93436 | 16 | *09018 | 16 |
| 5 | *46555 | 22 | *62335 | 22 | *78049 | 22 | *93697 | 21 | *09277 | 21 |
| 6 | *46818 | 26 | *62598 | 26 | *78310 | 26 | *93958 | 25 | *09536 | 25 |
| 7 | *47082 | 30 | *62860 | 30 | *78572 | 30 | *94218 | 29 | *09795 | 29 |
| 8 | *47346 | 35 | *63122 | 35 | *78834 | 35 | *94478 | 34 | *10054 | 34 |
| 9 | *47609 | 39 | *63384 | 39 | *79095 | 39 | *94738 | 38 | *10313 | 38 |
| 10 | *47872 | 44 | *63646 | 44 | *79356 | 44 | *94998 | 43 | *10572 | 43 |
| 11 | 8°48135 | 49 | 8°63909 | 48 | 8°79617 | 48 | 8°95258 | 47 | 9°10813 | 47 |
| 12 | *48398 | 53 | *64172 | 52 | *79878 | 52 | *95518 | 51 | *11090 | 51 |
| 13 | *48662 | 58 | *64434 | 57 | *80139 | 57 | *95778 | 56 | *11349 | 56 |
| 14 | *48926 | 62 | *64696 | 61 | *80400 | 61 | *96038 | 60 | *11608 | 60 |
| 15 | *49189 | 66 | *64958 | 65 | *80662 | 65 | *96298 | 64 | *11867 | 64 |
| 16 | *49452 | 70 | *65220 | 69 | *80924 | 69 | *96558 | 68 | *12126 | 68 |
| 17 | *49716 | 75 | *65483 | 74 | *81185 | 74 | *96818 | 73 | *12385 | 73 |
| 18 | *49980 | 79 | *65746 | 78 | *81446 | 78 | *97078 | 77 | *12644 | 77 |
| 19 | *50243 | 84 | *66008 | 83 | *81707 | 83 | *97338 | 82 | *12902 | 82 |
| 20 | *50506 | 88 | *66270 | 87 | *81968 | 87 | *97598 | 86 | *13160 | 86 |
| 21 | 8°50769 | 93 | 8°66532 | 92 | 8°82229 | 92 | 8°97858 | 91 | 9°13419 | 91 |
| 22 | *51032 | 97 | *66794 | 96 | *82490 | 96 | *98118 | 95 | *13678 | 95 |
| 23 | *51295 | 102 | *67056 | 101 | *82751 | 101 | *98378 | 100 | *13937 | 100 |
| 24 | *51558 | 106 | *67318 | 105 | *83012 | 105 | *98638 | 104 | *14196 | 104 |
| 25 | *51822 | 111 | *67580 | 110 | *83273 | 110 | *98898 | 109 | *14455 | 109 |
| 26 | *52086 | 115 | *67842 | 114 | *83534 | 114 | *99158 | 114 | *14714 | 113 |
| 27 | *52349 | 120 | *68104 | 119 | *83795 | 119 | *99418 | 118 | *14972 | 118 |
| 28 | *52612 | 124 | *68366 | 123 | *84056 | 123 | *99678 | 122 | *15230 | 122 |
| 29 | *52875 | 128 | *68628 | 127 | *84317 | 127 | *99937 | 126 | *15489 | 126 |
| 30 | *53138 | 132 | *68890 | 131 | *84578 | 131 | 9°00196 | 130 | *15748 | 130 |
| 31 | 8°53401 | 136 | 8°69152 | 135 | 8°84839 | 135 | 9°00456 | 134 | 9°16007 | 134 |
| 32 | *53664 | 140 | *69414 | 139 | *85100 | 139 | *00716 | 138 | *16266 | 138 |
| 33 | *53927 | 145 | *69676 | 144 | *85360 | 144 | *00976 | 143 | *16524 | 143 |
| 34 | *54190 | 149 | *69938 | 148 | *85620 | 148 | *01236 | 147 | *16782 | 147 |
| 35 | *54453 | 154 | *70200 | 153 | *85881 | 153 | *01496 | 152 | *17041 | 152 |
| 36 | *54716 | 158 | *70462 | 157 | *86142 | 157 | *01756 | 156 | *17300 | 156 |
| 37 | *54979 | 163 | *70724 | 162 | *86403 | 162 | *02015 | 161 | *17558 | 161 |
| 38 | *55242 | 167 | *70986 | 166 | *86664 | 166 | *02274 | 165 | *17816 | 165 |
| 39 | *55505 | 171 | *71248 | 170 | *86925 | 170 | *02534 | 169 | *18075 | 169 |
| 40 | *55768 | 175 | *71510 | 174 | *87186 | 174 | *02794 | 173 | *18334 | 173 |
| 41 | 8°56031 | 180 | 8°71772 | 179 | 8°87446 | 178 | 9°03053 | 177 | 9°18592 | 177 |
| 42 | *56294 | 184 | *72034 | 183 | *87706 | 182 | *03312 | 181 | *18850 | 181 |
| 43 | *56557 | 189 | *72295 | 188 | *87967 | 187 | *03572 | 186 | *19108 | 186 |
| 44 | *56820 | 193 | *72556 | 192 | *88228 | 191 | *03832 | 190 | *19366 | 190 |
| 45 | *57082 | 198 | *72818 | 197 | *88489 | 196 | *04091 | 195 | *19625 | 195 |
| 46 | *57344 | 202 | *73080 | 201 | *88750 | 200 | *04350 | 199 | *19884 | 199 |
| 47 | *57607 | 207 | *73342 | 206 | *89010 | 205 | *04610 | 204 | *20142 | 204 |
| 48 | *57870 | 211 | *73604 | 210 | *89270 | 209 | *04870 | 208 | *20400 | 208 |
| 49 | *58133 | 215 | *73865 | 214 | *89531 | 213 | *05129 | 212 | *20658 | 212 |
| 50 | *58396 | 219 | *74126 | 218 | *89792 | 217 | *05388 | 216 | *20916 | 216 |
| 51 | 8°58659 | 223 | 8°74388 | 222 | 8°90052 | 221 | 9°05648 | 220 | 9°21174 | 220 |
| 52 | *58922 | 227 | *74650 | 226 | *90312 | 225 | *05908 | 224 | *21432 | 224 |
| 53 | *59184 | 232 | *74912 | 231 | *90573 | 230 | *06167 | 229 | *21690 | 229 |
| 54 | *59446 | 236 | *75174 | 235 | *90834 | 234 | *06426 | 233 | *21948 | 233 |
| 55 | *59709 | 241 | *75435 | 240 | *91094 | 239 | *06685 | 238 | *22207 | 238 |
| 56 | *59972 | 245 | *75696 | 244 | *91354 | 243 | *06944 | 242 | *22466 | 242 |
| 57 | *60235 | 250 | *75958 | 249 | *91615 | 248 | *07203 | 247 | *22724 | 247 |
| 58 | *60498 | 255 | *76220 | 254 | *91876 | 253 | *07462 | 252 | *22982 | 251 |
| 59 | *60760 | 259 | *76481 | 258 | *92136 | 257 | *07722 | 256 | *23240 | 255 |
| 60 | *61022 | 263 | *76742 | 262 | *92396 | 261 | *07982 | 260 | *23498 | 259 |

| Min. | 55° | Parts for " | 56° | Parts for " | 57° | Parts for " | 58° | Parts for " | 59° | Parts for " |
|------|---------|-------------------|---------|-------------------|---------|-------------------|---------|-------------------|----------|-------------------|
| 0 | 9°23498 | 0 | 9°38944 | 0 | 9°54318 | 0 | 9°69620 | 0 | 9°84848 | 0 |
| 1 | *23756 | 4 | *39200 | 4 | *54573 | 4 | *69874 | 4 | *85101 | 4 |
| 2 | *24014 | 8 | *39456 | 8 | *54828 | 8 | *70128 | 8 | *85354 | 8 |
| 3 | *24272 | 12 | *39713 | 12 | *55084 | 12 | *70382 | 12 | *85607 | 12 |
| 4 | *24530 | 16 | *39970 | 16 | *55340 | 16 | *70636 | 16 | *85860 | 16 |
| 5 | *24788 | 21 | *40227 | 21 | *55596 | 21 | *70891 | 21 | *86113 | 21 |
| 6 | *25046 | 25 | *40484 | 25 | *55852 | 25 | *71146 | 25 | *86366 | 25 |
| 7 | *25303 | 29 | *40741 | 29 | *56107 | 29 | *71400 | 29 | *86619 | 29 |
| 8 | *25560 | 34 | *40998 | 34 | *56362 | 34 | *71654 | 34 | *86872 | 34 |
| 9 | *25818 | 38 | *41254 | 38 | *56617 | 38 | *71908 | 38 | *87125 | 38 |
| 10 | *26076 | 43 | *41510 | 43 | *56872 | 42 | *72162 | 42 | *87378 | 42 |
| 11 | 9°26334 | 47 | 9°41767 | 47 | 9°57128 | 47 | 9°72416 | 47 | 9°87631 | 46 |
| 12 | *26592 | 51 | *42024 | 51 | *57384 | 51 | *72670 | 51 | *87884 | 50 |
| 13 | *26850 | 56 | *42281 | 55 | *57639 | 55 | *72925 | 55 | *88137 | 54 |
| 14 | *27108 | 60 | *42538 | 59 | *57894 | 59 | *73180 | 59 | *88390 | 58 |
| 15 | *27366 | 64 | *42794 | 63 | *58150 | 63 | *73434 | 63 | *88643 | 62 |
| 16 | *27624 | 68 | *43050 | 67 | *58406 | 67 | *73688 | 67 | *88896 | 66 |
| 17 | *27881 | 73 | *43307 | 72 | *58661 | 72 | *73942 | 72 | *89149 | 71 |
| 18 | *28138 | 77 | *43564 | 76 | *58916 | 76 | *74196 | 76 | *89402 | 75 |
| 19 | *28396 | 82 | *43820 | 81 | *59171 | 81 | *74450 | 81 | *89654 | 80 |
| 20 | *28654 | 86 | *44076 | 85 | *59426 | 85 | *74704 | 85 | *89906 | 84 |
| 21 | 9°28912 | 90 | 9°44332 | 89 | 9°59681 | 89 | 9°74958 | 89 | 9°90159 | 89 |
| 22 | *29170 | 94 | *44588 | 93 | *59936 | 93 | *75212 | 93 | *90412 | 92 |
| 23 | *29427 | 99 | *44845 | 98 | *60192 | 97 | *75466 | 97 | *90665 | 96 |
| 24 | *29684 | 103 | *45102 | 102 | *60448 | 101 | *75720 | 101 | *90918 | 100 |
| 25 | *29942 | 108 | *45358 | 107 | *60703 | 106 | *75974 | 106 | *91170 | 105 |
| 26 | *30200 | 112 | *45614 | 111 | *60958 | 110 | *76228 | 110 | *91422 | 109 |
| 27 | *30457 | 117 | *45870 | 116 | *61213 | 115 | *76481 | 115 | *91675 | 114 |
| 28 | *30714 | 121 | *46126 | 120 | *61468 | 119 | *76734 | 119 | *91928 | 118 |
| 29 | *30972 | 125 | *46383 | 124 | *61723 | 123 | *76988 | 123 | *92181 | 122 |
| 30 | *31230 | 129 | *46640 | 128 | *61978 | 127 | *77242 | 127 | *92434 | 126 |
| 31 | 9°31487 | 133 | 9°46896 | 132 | 9°62233 | 131 | 9°77496 | 131 | 9°92686 | 130 |
| 32 | *31744 | 137 | *47152 | 136 | *62488 | 135 | *77750 | 135 | *92938 | 134 |
| 33 | *32001 | 142 | *47408 | 141 | *62743 | 140 | *78004 | 139 | *93191 | 138 |
| 34 | *32258 | 146 | *47664 | 145 | *62998 | 144 | *78258 | 143 | *93444 | 142 |
| 35 | *32516 | 151 | *47920 | 150 | *63253 | 149 | *78512 | 148 | *93696 | 147 |
| 36 | *32774 | 155 | *48176 | 154 | *63508 | 153 | *78766 | 152 | *93948 | 151 |
| 37 | *33031 | 160 | *48432 | 159 | *63763 | 158 | *79019 | 157 | *94200 | 156 |
| 38 | *33288 | 164 | *48688 | 163 | *64018 | 162 | *79272 | 161 | *94452 | 160 |
| 39 | *33545 | 168 | *48944 | 167 | *64272 | 166 | *79526 | 165 | *94705 | 164 |
| 40 | *33802 | 172 | *49200 | 171 | *64526 | 170 | *79780 | 169 | *94958 | 168 |
| 41 | 9°34059 | 176 | 9°49456 | 175 | 9°64781 | 174 | 9°80033 | 173 | 9°95210 | 172 |
| 42 | *34316 | 180 | *49712 | 179 | *65036 | 178 | *80286 | 177 | *95462 | 176 |
| 43 | *34574 | 185 | *49968 | 184 | *65291 | 183 | *80540 | 182 | *95714 | 180 |
| 44 | *34832 | 189 | *50224 | 188 | *65546 | 187 | *80794 | 186 | *95966 | 184 |
| 45 | *35089 | 194 | *50480 | 193 | *65801 | 192 | *81047 | 191 | *96219 | 189 |
| 46 | *35346 | 198 | *50736 | 197 | *66056 | 196 | *81300 | 195 | *96472 | 193 |
| 47 | *35603 | 203 | *50992 | 202 | *66310 | 201 | *81554 | 200 | *96724 | 198 |
| 48 | *35860 | 207 | *51248 | 206 | *66564 | 205 | *81808 | 204 | *96976 | 202 |
| 49 | *36117 | 211 | *51504 | 210 | *66819 | 209 | *82061 | 208 | *97228 | 206 |
| 50 | *36374 | 215 | *51760 | 214 | *67074 | 213 | *82314 | 212 | *97480 | 210 |
| 51 | 9°36631 | 219 | 9°52016 | 218 | 9°67329 | 217 | 9°82568 | 216 | 9°97732 | 214 |
| 52 | *36888 | 223 | *52272 | 222 | *67584 | 221 | *82822 | 220 | *97984 | 218 |
| 53 | *37145 | 227 | *52528 | 226 | *67838 | 225 | *83075 | 224 | *98236 | 222 |
| 54 | *37402 | 231 | *52784 | 230 | *68092 | 229 | *83328 | 228 | *98488 | 226 |
| 55 | *37659 | 236 | *53039 | 235 | *68347 | 234 | *83581 | 233 | *98740 | 231 |
| 56 | *37916 | 240 | *53294 | 239 | *68602 | 238 | *83834 | 237 | *98992 | 235 |
| 57 | *38173 | 245 | *53550 | 244 | *68856 | 243 | *84087 | 242 | *99244 | 240 |
| 58 | *38430 | 249 | *53806 | 248 | *69110 | 247 | *84340 | 246 | *99496 | 244 |
| 59 | *38687 | 253 | *54062 | 252 | *69365 | 251 | *84594 | 250 | *99748 | 248 |
| 60 | *38944 | 257 | *54318 | 256 | *69620 | 255 | *84848 | 254 | 10.00000 | 252 |

TABLE L.—Tables showing the length in feet of a degree, minute, and second of latitude and longitude, for every ten minutes of the quadrant. Based on the Ordnance Geodetical Tables, compression $\frac{1}{294}$. By Robert C. Carrington, F.R.G.S., F.A.S.L.

| LATITUDE. | | | | LONGITUDE. | | | |
|-----------|---------------------|---------|---------|------------|---------------------|---------|---------|
| Latitud. | Length in Feet of a | | | Latitude | Length in Feet of a | | |
| | Degree. | Minute. | Second. | | Degree. | Minute. | Second. |
| 0° 0' | 362755.6 | 6045.93 | 100.77 | 0° 0' | 365233.7 | 6087.23 | 101.454 |
| 10 | 362755.6 | 6045.93 | 100.77 | 10 | 365232.1 | 6087.20 | 101.453 |
| 20 | 362755.7 | 6045.93 | 100.77 | 20 | 365227.5 | 6087.13 | 101.452 |
| 30 | 362755.9 | 6045.93 | 100.77 | 30 | 365219.9 | 6087.00 | 101.450 |
| 40 | 362756.1 | 6045.93 | 100.77 | 40 | 365209.1 | 6086.82 | 101.447 |
| 50 | 362756.4 | 6045.94 | 100.77 | 50 | 365195.3 | 6086.59 | 101.443 |
| 1° 0' | 362756.7 | 6045.94 | 100.77 | 1° 0' | 365178.4 | 6086.31 | 101.438 |
| 10 | 362757.1 | 6045.95 | 100.77 | 10 | 365158.5 | 6085.98 | 101.433 |
| 20 | 362757.6 | 6045.96 | 100.77 | 20 | 365135.5 | 6085.59 | 101.427 |
| 30 | 362758.1 | 6045.97 | 100.77 | 30 | 365109.4 | 6085.16 | 101.419 |
| 40 | 362758.7 | 6045.98 | 100.77 | 40 | 365080.2 | 6084.67 | 101.411 |
| 50 | 362759.4 | 6045.99 | 100.77 | 50 | 365048.0 | 6084.13 | 101.402 |
| 2° 0' | 362760.1 | 6046.00 | 100.77 | 2° 0' | 365012.7 | 6083.54 | 101.392 |
| 10 | 362760.9 | 6046.01 | 100.77 | 10 | 364974.3 | 6082.91 | 101.381 |
| 20 | 362761.7 | 6046.03 | 100.77 | 20 | 364932.9 | 6082.22 | 101.370 |
| 30 | 362762.6 | 6046.04 | 100.77 | 30 | 364888.4 | 6081.47 | 101.358 |
| 40 | 362763.6 | 6046.06 | 100.77 | 40 | 364840.8 | 6080.68 | 101.345 |
| 50 | 362764.6 | 6046.08 | 100.77 | 50 | 364790.2 | 6079.84 | 101.331 |
| 3° 0' | 362765.7 | 6046.09 | 100.77 | 3° 0' | 364736.5 | 6078.94 | 101.316 |
| 10 | 362766.9 | 6046.11 | 100.77 | 10 | 364679.8 | 6078.00 | 101.300 |
| 20 | 362768.1 | 6046.13 | 100.77 | 20 | 364619.9 | 6077.00 | 101.283 |
| 30 | 362769.4 | 6046.16 | 100.77 | 30 | 364557.0 | 6075.95 | 101.266 |
| 40 | 362770.7 | 6046.18 | 100.77 | 40 | 364491.1 | 6074.85 | 101.248 |
| 50 | 362772.1 | 6046.20 | 100.77 | 50 | 364422.1 | 6073.70 | 101.228 |
| 4° 0' | 362773.6 | 6046.23 | 100.77 | 4° 0' | 364350.0 | 6072.50 | 101.206 |
| 10 | 362775.1 | 6046.25 | 100.77 | 10 | 364274.9 | 6071.25 | 101.187 |
| 20 | 362776.7 | 6046.28 | 100.77 | 20 | 364196.7 | 6069.95 | 101.166 |
| 30 | 362778.3 | 6046.30 | 100.77 | 30 | 364115.4 | 6068.59 | 101.143 |
| 40 | 362780.0 | 6046.33 | 100.77 | 40 | 364031.1 | 6067.19 | 101.120 |
| 50 | 362781.8 | 6046.36 | 100.77 | 50 | 363943.7 | 6065.73 | 101.096 |

| LATITUDE. | | | | LONGITUDE. | | | |
|-----------|---------------------|---------|---------|------------|---------------------|---------|---------|
| Latitude. | Length in Feet of a | | | Latitude. | Length in Feet of a | | |
| | Degree. | Minute. | Second. | | Degree. | Minute. | Second. |
| 5° 0' | 362783.6 | 6046.39 | 100.77 | 5° 0' | 363853.2 | 6064.22 | 101.070 |
| 10 | 362785.5 | 6046.42 | 100.77 | 10 | 363759.7 | 6062.66 | 101.044 |
| 20 | 362787.5 | 6046.46 | 100.77 | 20 | 363663.2 | 6061.05 | 101.018 |
| 30 | 362789.5 | 6046.49 | 100.77 | 30 | 363563.5 | 6059.39 | 100.990 |
| 40 | 362791.6 | 6046.53 | 100.78 | 40 | 363460.9 | 6057.68 | 100.961 |
| 50 | 362793.7 | 6046.56 | 100.78 | 50 | 363355.1 | 6055.92 | 100.932 |
| 6° 0' | 362795.9 | 6046.60 | 100.78 | 6° 0' | 363246.3 | 6054.11 | 100.902 |
| 10 | 362798.2 | 6046.64 | 100.78 | 10 | 363134.5 | 6052.24 | 100.871 |
| 20 | 362800.5 | 6046.68 | 100.78 | 20 | 363019.6 | 6050.33 | 100.839 |
| 30 | 362802.9 | 6046.72 | 100.78 | 30 | 362901.7 | 6048.36 | 100.806 |
| 40 | 362805.4 | 6046.76 | 100.78 | 40 | 362780.7 | 6046.35 | 100.772 |
| 50 | 362807.9 | 6046.80 | 100.78 | 50 | 362656.6 | 6044.28 | 100.738 |
| 7° 0' | 362810.4 | 6046.84 | 100.78 | 7° 0' | 362529.5 | 6042.16 | 100.703 |
| 10 | 362813.1 | 6046.89 | 100.78 | 10 | 362399.4 | 6039.99 | 100.667 |
| 20 | 362815.2 | 6046.93 | 100.78 | 20 | 362266.2 | 6037.77 | 100.630 |
| 30 | 362818.5 | 6046.98 | 100.78 | 30 | 362130.0 | 6035.50 | 100.592 |
| 40 | 362821.3 | 6047.02 | 100.78 | 40 | 361990.7 | 6033.18 | 100.553 |
| 50 | 362824.2 | 6047.07 | 100.78 | 50 | 361848.4 | 6030.81 | 100.513 |
| 8° 0' | 362827.1 | 6047.12 | 100.79 | 8° 0' | 361703.0 | 6028.38 | 100.473 |
| 10 | 362830.1 | 6047.17 | 100.79 | 10 | 361554.6 | 6025.91 | 100.432 |
| 20 | 362833.2 | 6047.22 | 100.79 | 20 | 361403.2 | 6023.39 | 100.390 |
| 30 | 362836.3 | 6047.27 | 100.79 | 30 | 361248.7 | 6020.81 | 100.347 |
| 40 | 362839.4 | 6047.32 | 100.79 | 40 | 361091.2 | 6018.19 | 100.303 |
| 50 | 362842.7 | 6047.38 | 100.79 | 50 | 360930.6 | 6015.51 | 100.258 |
| 9° 0' | 362846.0 | 6047.43 | 100.79 | 9° 0' | 360767.0 | 6012.78 | 100.213 |
| 10 | 362849.3 | 6047.49 | 100.79 | 10 | 360607.4 | 6010.01 | 100.167 |
| 20 | 362852.7 | 6047.55 | 100.79 | 20 | 360430.7 | 6007.18 | 100.120 |
| 30 | 362856.2 | 6047.60 | 100.79 | 30 | 360258.0 | 6004.30 | 100.072 |
| 40 | 362859.7 | 6047.66 | 100.79 | 40 | 360082.3 | 6001.37 | 100.023 |
| 50 | 362863.3 | 6047.72 | 100.80 | 50 | 359903.5 | 5998.39 | 99.973 |
| 10° 0' | 362866.9 | 6047.78 | 100.80 | 10° 0' | 359721.7 | 5995.36 | 99.923 |
| 10 | 362870.7 | 6047.85 | 100.80 | 10 | 359536.7 | 5992.28 | 99.871 |
| 20 | 362874.4 | 6047.91 | 100.80 | 20 | 359349.1 | 5989.15 | 99.819 |
| 30 | 362878.2 | 6047.97 | 100.80 | 30 | 359158.3 | 5985.97 | 99.766 |
| 40 | 362882.1 | 6048.04 | 100.80 | 40 | 358964.4 | 5982.74 | 99.712 |
| 50 | 362886.1 | 6048.10 | 100.80 | 50 | 358767.5 | 5979.46 | 99.658 |

| LATITUDE. | | | | LONGITUDE. | | | |
|-----------|---------------------|---------|---------|------------|---------------------|---------|---------|
| Latitude. | Length in Feet of a | | | Latitude. | Length in Feet of a | | |
| | Degree. | Minute. | Second. | | Degree. | Minute. | Second. |
| 11° 0' | 362890.1 | 6048.17 | 100.80 | 11° 0' | 358567.6 | 5976.13 | 99.602 |
| 10 | 362894.1 | 6048.23 | 100.80 | 10 | 358364.7 | 5972.75 | 99.546 |
| 20 | 362898.2 | 6048.30 | 100.80 | 20 | 358158.7 | 5969.31 | 99.489 |
| 30 | 362902.4 | 6048.37 | 100.81 | 30 | 357949.8 | 5965.83 | 99.431 |
| 40 | 362906.6 | 6048.44 | 100.81 | 40 | 357737.8 | 5962.30 | 99.372 |
| 50 | 362910.9 | 6048.52 | 100.81 | 50 | 357522.8 | 5958.71 | 99.312 |
| 12° 0' | 362915.2 | 6048.59 | 100.81 | 12° 0' | 357304.8 | 5955.08 | 99.251 |
| 10 | 362919.6 | 6048.66 | 100.81 | 10 | 357083.9 | 5951.40 | 99.190 |
| 20 | 362924.1 | 6048.74 | 100.81 | 20 | 356859.9 | 5947.67 | 99.128 |
| 30 | 362928.6 | 6048.81 | 100.81 | 30 | 356632.9 | 5943.88 | 99.065 |
| 40 | 362933.2 | 6048.89 | 100.81 | 40 | 356402.9 | 5940.05 | 99.001 |
| 50 | 362937.8 | 6048.96 | 100.82 | 50 | 356169.9 | 5936.17 | 98.936 |
| 13° 0' | 362942.5 | 6049.04 | 100.82 | 13° 0' | 355933.9 | 5932.23 | 98.871 |
| 10 | 362947.2 | 6049.12 | 100.82 | 10 | 355694.9 | 5928.25 | 98.804 |
| 20 | 362952.0 | 6049.20 | 100.82 | 20 | 355452.9 | 5924.22 | 98.737 |
| 30 | 362956.9 | 6049.28 | 100.82 | 30 | 355207.9 | 5920.13 | 98.669 |
| 40 | 362961.8 | 6049.36 | 100.82 | 40 | 354959.9 | 5916.00 | 98.600 |
| 50 | 362966.8 | 6049.45 | 100.82 | 50 | 354709.0 | 5911.82 | 98.530 |
| 14° 0' | 362971.8 | 6049.53 | 100.83 | 14° 0' | 354455.1 | 5907.59 | 98.460 |
| 10 | 362976.9 | 6049.62 | 100.83 | 10 | 354198.1 | 5903.30 | 98.388 |
| 20 | 362982.0 | 6049.70 | 100.83 | 20 | 353938.2 | 5898.97 | 98.316 |
| 30 | 362987.2 | 6049.79 | 100.83 | 30 | 353675.3 | 5894.59 | 98.243 |
| 40 | 362992.4 | 6049.87 | 100.83 | 40 | 353409.4 | 5890.16 | 98.169 |
| 50 | 362997.7 | 6049.96 | 100.83 | 50 | 353140.6 | 5885.68 | 98.095 |
| 15° 0' | 363003.1 | 6050.05 | 100.83 | 15° 0' | 352868.8 | 5881.15 | 98.019 |
| 10 | 363008.5 | 6050.14 | 100.84 | 10 | 352594.1 | 5876.57 | 97.943 |
| 20 | 363013.9 | 6050.23 | 100.84 | 20 | 352316.3 | 5871.94 | 97.866 |
| 30 | 363019.4 | 6050.32 | 100.84 | 30 | 352035.6 | 5867.26 | 97.788 |
| 40 | 363025.0 | 6050.42 | 100.84 | 40 | 351751.9 | 5862.53 | 97.709 |
| 50 | 363030.6 | 6050.51 | 100.84 | 50 | 351465.3 | 5857.76 | 97.629 |
| 16° 0' | 363036.3 | 6050.61 | 100.84 | 16° 0' | 351175.7 | 5852.93 | 97.549 |
| 10 | 363042.0 | 6050.70 | 100.84 | 10 | 350883.1 | 5848.05 | 97.468 |
| 20 | 363047.8 | 6050.80 | 100.85 | 20 | 350587.6 | 5843.13 | 97.386 |
| 30 | 363053.6 | 6050.89 | 100.85 | 30 | 350289.1 | 5838.15 | 97.303 |
| 40 | 363059.3 | 6050.99 | 100.85 | 40 | 349987.7 | 5833.13 | 97.219 |
| 50 | 363065.4 | 6051.09 | 100.85 | 50 | 349683.4 | 5828.06 | 97.134 |

| LATITUDE. | | | | LONGITUDE. | | | |
|-----------|---------------------|---------|---------|------------|---------------------|---------|---------|
| Latitude | Length in Feet of a | | | Latitude. | Length in Feet of a | | |
| | Degree. | Minute. | Second. | | Degree. | Minute. | Second. |
| 17° 0' | 363071.4 | 6051.19 | 100.85 | 17° 0' | 349376.0 | 5822.93 | 97.049 |
| 10 | 363077.4 | 6051.29 | 100.85 | 10 | 349065.8 | 5817.76 | 96.963 |
| 20 | 363083.5 | 6051.39 | 100.86 | 20 | 348752.6 | 5812.54 | 96.876 |
| 30 | 363089.7 | 6051.50 | 100.86 | 30 | 348436.5 | 5807.28 | 96.788 |
| 40 | 363095.9 | 6051.60 | 100.86 | 40 | 348117.4 | 5801.96 | 96.699 |
| 50 | 363102.1 | 6051.70 | 100.86 | 50 | 347795.4 | 5796.59 | 96.610 |
| 18° 0' | 363108.4 | 6051.81 | 100.86 | 18° 0' | 347470.5 | 5791.18 | 96.520 |
| 10 | 363114.8 | 6051.91 | 100.87 | 10 | 347142.6 | 5785.71 | 96.429 |
| 20 | 363121.2 | 6052.02 | 100.87 | 20 | 346811.8 | 5780.20 | 96.337 |
| 30 | 363127.6 | 6052.13 | 100.87 | 30 | 346478.1 | 5774.64 | 96.244 |
| 40 | 363134.1 | 6052.24 | 100.87 | 40 | 346141.5 | 5769.03 | 96.150 |
| 50 | 363140.7 | 6052.35 | 100.87 | 50 | 345801.9 | 5763.37 | 96.056 |
| 19° 0' | 363147.3 | 6052.46 | 100.87 | 19° 0' | 345459.5 | 5757.66 | 95.961 |
| 10 | 363153.9 | 6052.57 | 100.88 | 10 | 345114.1 | 5751.90 | 95.865 |
| 20 | 363160.6 | 6052.68 | 100.88 | 20 | 344765.8 | 5746.10 | 95.768 |
| 30 | 363167.4 | 6052.79 | 100.88 | 30 | 344414.6 | 5740.24 | 95.671 |
| 40 | 363174.2 | 6052.90 | 100.88 | 40 | 344060.6 | 5734.34 | 95.572 |
| 50 | 363181.0 | 6053.02 | 100.88 | 50 | 343703.6 | 5728.39 | 95.473 |
| 20° 0' | 363187.9 | 6053.13 | 100.89 | 20° 0' | 343343.7 | 5722.40 | 95.373 |
| 10 | 363194.8 | 6053.25 | 100.89 | 10 | 342980.9 | 5716.35 | 95.272 |
| 20 | 363201.8 | 6053.36 | 100.89 | 20 | 342615.2 | 5710.25 | 95.171 |
| 30 | 363208.8 | 6053.48 | 100.89 | 30 | 342246.7 | 5704.11 | 95.069 |
| 40 | 363215.9 | 6053.60 | 100.89 | 40 | 341875.2 | 5697.92 | 94.965 |
| 50 | 363223.1 | 6053.72 | 100.90 | 50 | 341500.9 | 5691.68 | 94.861 |
| 21° 0' | 363230.2 | 6053.84 | 100.90 | 21° 0' | 341123.7 | 5685.40 | 94.756 |
| 10 | 363237.5 | 6053.96 | 100.90 | 10 | 340743.6 | 5679.06 | 94.651 |
| 20 | 363244.7 | 6054.08 | 100.90 | 20 | 340360.6 | 5672.68 | 94.545 |
| 30 | 363252.1 | 6054.20 | 100.90 | 30 | 339974.8 | 5666.25 | 94.438 |
| 40 | 363259.4 | 6054.32 | 100.91 | 40 | 339586.1 | 5659.77 | 94.330 |
| 50 | 363266.8 | 6054.45 | 100.91 | 50 | 339194.5 | 5653.24 | 94.221 |
| 22° 0' | 363274.3 | 6054.57 | 100.91 | 22° 0' | 338800.1 | 5646.67 | 94.111 |
| 10 | 363281.8 | 6054.70 | 100.91 | 10 | 338402.8 | 5640.05 | 94.001 |
| 20 | 363289.3 | 6054.82 | 100.91 | 20 | 338002.7 | 5633.38 | 93.890 |
| 30 | 363296.9 | 6054.95 | 100.92 | 30 | 337599.7 | 5626.66 | 93.778 |
| 40 | 363304.6 | 6055.08 | 100.92 | 40 | 337193.9 | 5619.90 | 93.665 |
| 50 | 363312.2 | 6055.20 | 100.92 | 50 | 336785.2 | 5613.09 | 93.551 |

| LATITUDE. | | | | LONGITUDE. | | | |
|-----------|---------------------|---------|---------|------------|---------------------|---------|---------|
| Latitude. | Length in Feet of a | | | Latitude. | Length in Feet of a | | |
| | Degree. | Minute. | Second. | | Degree. | Minute. | Second. |
| 23° 0' | 363320.0 | 6055.33 | 100.92 | 23° 0' | 336373.6 | 5606.23 | 93.437 |
| 10 | 363327.7 | 6055.46 | 100.92 | 10 | 335959.3 | 5599.32 | 93.322 |
| 20 | 363335.5 | 6055.59 | 100.93 | 20 | 335542.1 | 5592.37 | 93.206 |
| 30 | 363343.4 | 6055.72 | 100.93 | 30 | 335122.0 | 5585.37 | 93.089 |
| 40 | 363351.3 | 6055.86 | 100.93 | 40 | 334699.2 | 5578.32 | 92.972 |
| 50 | 363359.2 | 6055.99 | 100.93 | 50 | 334273.5 | 5571.23 | 92.854 |
| 24° 0' | 363367.2 | 6056.12 | 100.94 | 24° 0' | 333845.0 | 5564.08 | 92.735 |
| 10 | 363375.2 | 6056.25 | 100.94 | 10 | 333413.7 | 5556.89 | 92.615 |
| 20 | 363383.3 | 6056.39 | 100.94 | 20 | 332979.5 | 5549.66 | 92.494 |
| 30 | 363391.4 | 6056.52 | 100.94 | 30 | 332542.6 | 5542.38 | 92.373 |
| 40 | 363399.6 | 6056.66 | 100.94 | 40 | 332102.8 | 5535.05 | 92.251 |
| 50 | 363407.8 | 6056.80 | 100.95 | 50 | 331660.3 | 5527.67 | 92.128 |
| 25° 0' | 363416.0 | 6056.93 | 100.95 | 25° 0' | 331214.9 | 5520.25 | 92.004 |
| 10 | 363424.3 | 6057.07 | 100.95 | 10 | 330766.7 | 5512.78 | 91.879 |
| 20 | 363432.6 | 6057.21 | 100.95 | 20 | 330315.8 | 5505.26 | 91.754 |
| 30 | 363440.9 | 6057.35 | 100.96 | 30 | 329862.0 | 5497.70 | 91.628 |
| 40 | 363449.3 | 6057.49 | 100.96 | 40 | 329405.5 | 5490.09 | 91.502 |
| 50 | 363457.7 | 6057.63 | 100.96 | 50 | 328946.2 | 5482.44 | 91.374 |
| 26° 0' | 363466.2 | 6057.77 | 100.96 | 26° 0' | 328484.1 | 5474.74 | 91.245 |
| 10 | 363474.7 | 6057.91 | 100.97 | 10 | 328019.2 | 5466.99 | 91.116 |
| 20 | 363483.3 | 6058.06 | 100.97 | 20 | 327551.6 | 5459.19 | 90.987 |
| 30 | 363491.9 | 6058.20 | 100.97 | 30 | 327081.2 | 5451.35 | 90.856 |
| 40 | 363500.5 | 6058.34 | 100.97 | 40 | 326608.0 | 5443.47 | 90.724 |
| 50 | 363509.2 | 6058.49 | 100.97 | 50 | 326132.1 | 5435.54 | 90.592 |
| 27° 0' | 363517.9 | 6058.63 | 100.98 | 27° 0' | 325653.4 | 5427.56 | 90.459 |
| 10 | 363526.6 | 6058.78 | 100.98 | 10 | 325171.9 | 5419.53 | 90.326 |
| 20 | 363535.4 | 6058.92 | 100.98 | 20 | 324687.7 | 5411.46 | 90.191 |
| 30 | 363544.2 | 6059.07 | 100.98 | 30 | 324200.8 | 5403.35 | 90.056 |
| 40 | 363553.0 | 6059.22 | 100.99 | 40 | 323711.2 | 5395.19 | 89.920 |
| 50 | 363561.9 | 6059.37 | 100.99 | 50 | 323218.8 | 5386.98 | 89.783 |
| 28° 0' | 363570.8 | 6059.51 | 100.99 | 28° 0' | 322723.6 | 5378.73 | 89.645 |
| 10 | 363579.8 | 6059.66 | 100.99 | 10 | 322225.7 | 5370.43 | 89.507 |
| 20 | 363588.8 | 6059.81 | 101.00 | 20 | 321725.1 | 5362.09 | 89.368 |
| 30 | 363597.8 | 6059.96 | 101.00 | 30 | 321221.8 | 5353.70 | 89.228 |
| 40 | 363606.8 | 6060.11 | 101.00 | 40 | 320715.8 | 5345.26 | 89.088 |
| 50 | 363615.9 | 6060.27 | 101.00 | 50 | 320207.1 | 5336.78 | 88.946 |

| LATITUDE. | | | | LONGITUDE. | | | |
|-----------|---------------------|---------|---------|------------|---------------------|---------|---------|
| Latitude. | Length in Feet of a | | | Latitude. | Length in Feet of a | | |
| | Degree. | Minute. | Second. | | Degree. | Minute. | Second. |
| 29° 0' | 363625.0 | 6060.42 | 101.01 | 29° 0' | 319695.6 | 5328.26 | 88.804 |
| 10 | 363634.2 | 6060.57 | 101.01 | 10 | 319181.5 | 5319.69 | 88.661 |
| 20 | 363643.4 | 6060.72 | 101.01 | 20 | 318664.6 | 5311.08 | 88.518 |
| 30 | 363652.6 | 6060.88 | 101.01 | 30 | 318145.1 | 5302.42 | 88.374 |
| 40 | 363661.9 | 6061.03 | 101.02 | 40 | 317622.8 | 5293.71 | 88.229 |
| 50 | 363671.2 | 6061.19 | 101.02 | 50 | 317097.9 | 5284.97 | 88.083 |
| 30° 0' | 363680.5 | 6061.34 | 101.02 | 30° 0' | 316570.3 | 5276.17 | 87.936 |
| 10 | 363689.9 | 6061.50 | 101.03 | 10 | 316040.0 | 5267.33 | 87.789 |
| 20 | 363699.3 | 6061.66 | 101.03 | 20 | 315507.0 | 5258.45 | 87.641 |
| 30 | 363708.7 | 6061.81 | 101.03 | 30 | 314971.4 | 5249.52 | 87.492 |
| 40 | 363718.1 | 6061.97 | 101.03 | 40 | 314433.1 | 5240.55 | 87.343 |
| 50 | 363727.6 | 6062.13 | 101.04 | 50 | 313892.1 | 5231.54 | 87.192 |
| 31° 0' | 363737.1 | 6062.29 | 101.04 | 31° 0' | 313348.5 | 5222.48 | 87.041 |
| 10 | 363746.7 | 6062.45 | 101.04 | 10 | 312802.2 | 5213.57 | 86.889 |
| 20 | 363756.2 | 6062.60 | 101.04 | 20 | 312253.3 | 5204.22 | 86.737 |
| 30 | 363765.8 | 6062.76 | 101.05 | 30 | 311701.7 | 5195.03 | 86.584 |
| 40 | 363775.4 | 6062.92 | 101.05 | 40 | 311147.5 | 5185.79 | 86.430 |
| 50 | 363785.1 | 6063.09 | 101.05 | 50 | 310590.7 | 5176.51 | 86.275 |
| 32° 0' | 363794.8 | 6063.25 | 101.05 | 32° 0' | 310031.2 | 5167.19 | 86.119 |
| 10 | 363804.5 | 6063.41 | 101.06 | 10 | 309469.1 | 5157.82 | 85.963 |
| 20 | 363814.2 | 6063.57 | 101.06 | 20 | 308904.4 | 5148.41 | 85.807 |
| 30 | 363824.0 | 6063.73 | 101.06 | 30 | 308337.1 | 5138.95 | 85.649 |
| 40 | 363833.8 | 6063.90 | 101.07 | 40 | 307767.2 | 5129.45 | 85.491 |
| 50 | 363843.6 | 6064.06 | 101.07 | 50 | 307194.6 | 5119.91 | 85.332 |
| 33° 0' | 363853.5 | 6064.23 | 101.07 | 33° 0' | 306619.5 | 5110.33 | 85.172 |
| 10 | 363863.4 | 6064.39 | 101.07 | 10 | 306041.7 | 5100.70 | 85.011 |
| 20 | 363873.3 | 6064.56 | 101.08 | 20 | 305461.4 | 5091.02 | 84.850 |
| 30 | 363883.2 | 6064.72 | 101.08 | 30 | 304878.5 | 5081.31 | 84.688 |
| 40 | 363893.1 | 6064.89 | 101.08 | 40 | 304293.0 | 5071.55 | 84.526 |
| 50 | 363903.1 | 6065.05 | 101.08 | 50 | 303704.9 | 5061.75 | 84.362 |
| 34° 0' | 363913.1 | 6065.22 | 101.09 | 34° 0' | 303114.2 | 5051.90 | 84.198 |
| 10 | 363923.1 | 6065.39 | 101.09 | 10 | 302521.0 | 5042.02 | 84.034 |
| 20 | 363933.2 | 6065.55 | 101.09 | 20 | 301925.2 | 5032.09 | 83.868 |
| 30 | 363943.2 | 6065.72 | 101.10 | 30 | 301326.8 | 5022.11 | 83.702 |
| 40 | 363953.3 | 6065.89 | 101.10 | 40 | 300725.9 | 5012.10 | 83.535 |
| 50 | 363963.4 | 6066.06 | 101.10 | 50 | 300122.4 | 5002.04 | 83.367 |

| LATITUDE. | | | | LONGITUDE. | | | |
|---------------|---------------------|---------|---------|---------------|---------------------|---------|---------|
| Latitude. | Length in Feet of a | | | Latitude. | Length in Feet of a | | |
| | Degree. | Minute. | Second. | | Degree. | Minute. | Second. |
| 35° 0' | 363973·6 | 6066·23 | 101·10 | 35° 0' | 299516·4 | 4991·94 | 83·199 |
| 10 | 363983·7 | 6066·40 | 101·11 | 10 | 298907·8 | 4981·80 | 83·030 |
| 20 | 363933·9 | 6066·57 | 101·11 | 20 | 298296·8 | 4971·61 | 82·860 |
| 30 | 364004·1 | 6066·74 | 101·11 | 30 | 297683·1 | 4961·38 | 82·690 |
| 40 | 364014·3 | 6066·91 | 101·12 | 40 | 297067·0 | 4951·12 | 82·519 |
| 50 | 364024·6 | 6067·08 | 101·12 | 50 | 296448·4 | 4940·81 | 82·347 |
| 36° 0' | 364034·9 | 6067·25 | 101·12 | 36° 0' | 295827·2 | 4930·45 | 82·174 |
| 10 | 364045·1 | 6067·42 | 101·12 | 10 | 295203·5 | 4920·06 | 82·001 |
| 20 | 364055·4 | 6067·59 | 101·13 | 20 | 294577·3 | 4909·62 | 81·827 |
| 30 | 364065·8 | 6067·76 | 101·13 | 30 | 293948·7 | 4899·15 | 81·652 |
| 40 | 364076·1 | 6067·94 | 101·13 | 40 | 293317·5 | 4888·63 | 81·477 |
| 50 | 364086·4 | 6068·11 | 101·14 | 50 | 292683·8 | 4878·06 | 81·301 |
| 37° 0' | 364096·8 | 6068·28 | 101·14 | 37° 0' | 292047·7 | 4867·46 | 81·124 |
| 10 | 364107·2 | 6068·45 | 101·14 | 10 | 291409·0 | 4856·82 | 80·947 |
| 20 | 364117·6 | 6068·63 | 101·14 | 20 | 290767·9 | 4846·13 | 80·769 |
| 30 | 364128·1 | 6068·80 | 101·15 | 30 | 290124·4 | 4835·41 | 80·590 |
| 40 | 364138·5 | 6068·98 | 101·15 | 40 | 289418·3 | 4824·64 | 80·411 |
| 50 | 364149·0 | 6069·15 | 101·15 | 50 | 288829·8 | 4813·83 | 80·231 |
| 38° 0' | 364159·5 | 6069·33 | 101·16 | 38° 0' | 288178·9 | 4802·98 | 80·050 |
| 10 | 364170·0 | 6069·50 | 101·16 | 10 | 287525·5 | 4792·09 | 79·868 |
| 20 | 364180·5 | 6069·68 | 101·16 | 20 | 286869·7 | 4781·16 | 79·686 |
| 30 | 364191·0 | 6069·85 | 101·16 | 30 | 286211·4 | 4770·19 | 79·503 |
| 40 | 364201·5 | 6070·03 | 101·17 | 40 | 285550·7 | 4759·18 | 79·320 |
| 50 | 364212·1 | 6070·20 | 101·17 | 50 | 284887·0 | 4748·13 | 79·136 |
| 39° 0' | 364222·6 | 6070·38 | 101·17 | 39° 0' | 284222·0 | 4737·03 | 78·951 |
| 10 | 364233·2 | 6070·55 | 101·18 | 10 | 283554·0 | 4725·90 | 78·765 |
| 20 | 364243·8 | 6070·73 | 101·18 | 20 | 242883·7 | 4714·73 | 78·579 |
| 30 | 364254·4 | 6070·91 | 101·18 | 30 | 282210·9 | 4703·52 | 78·392 |
| 40 | 364265·1 | 6071·09 | 101·18 | 40 | 281535·8 | 4692·26 | 78·204 |
| 50 | 364275·7 | 6071·27 | 101·19 | 50 | 280858·2 | 4680·97 | 78·016 |
| 40° 0' | 364286·3 | 6071·44 | 101·19 | 40° 0' | 280178·2 | 4669·64 | 77·827 |
| 10 | 364297·0 | 6071·62 | 101·19 | 10 | 279495·9 | 4658·27 | 77·638 |
| 20 | 364307·7 | 6071·80 | 101·20 | 20 | 278811·2 | 4646·85 | 77·448 |
| 30 | 364318·3 | 6071·97 | 101·20 | 30 | 278124·1 | 4635·40 | 77·257 |
| 40 | 364329·0 | 6072·15 | 101·20 | 40 | 277434·7 | 4623·91 | 77·065 |
| 50 | 364339·7 | 6072·33 | 101·21 | 50 | 276742·9 | 4612·38 | 76·873 |

| LATITUDE. | | | | LONGITUDE. | | | |
|-----------|---------------------|---------|---------|------------|---------------------|---------|---------|
| Latitude. | Length in Feet of a | | | Latitude. | Length in Feet of a | | |
| | Degree. | Minut. | Second. | | Degree. | Minute. | Second. |
| 41° 0' | 364350.4 | 6072.51 | 101.21 | 41° 0' | 276048.7 | 4600.81 | 76.680 |
| 10 | 364361.1 | 6072.69 | 101.21 | 10 | 275352.2 | 4589.20 | 76.480 |
| 20 | 364371.9 | 6072.87 | 101.21 | 20 | 274653.4 | 4577.56 | 76.293 |
| 30 | 364382.6 | 6073.04 | 101.22 | 30 | 273952.2 | 4565.87 | 76.093 |
| 40 | 364393.4 | 6073.22 | 101.22 | 40 | 273248.7 | 4554.75 | 75.902 |
| 50 | 364404.1 | 6073.40 | 101.22 | 50 | 272542.9 | 4542.38 | 75.706 |
| 42° 0' | 364414.9 | 6073.58 | 101.23 | 42° 0' | 271834.7 | 4530.58 | 75.509 |
| 10 | 364425.6 | 6073.76 | 101.23 | 10 | 271124.3 | 4518.74 | 75.312 |
| 20 | 364436.4 | 6073.94 | 101.23 | 20 | 270411.5 | 4506.86 | 75.114 |
| 30 | 364447.2 | 6074.12 | 101.24 | 30 | 269696.4 | 4494.94 | 74.916 |
| 40 | 364458.0 | 6074.30 | 101.24 | 40 | 268979.1 | 4482.99 | 74.717 |
| 50 | 364468.8 | 6074.48 | 101.24 | 50 | 268259.5 | 4470.99 | 74.517 |
| 43° 0' | 364479.6 | 6074.66 | 101.24 | 43° 0' | 267537.5 | 4458.96 | 74.316 |
| 10 | 364490.4 | 6074.84 | 101.25 | 10 | 266813.3 | 4446.89 | 74.115 |
| 20 | 364501.2 | 6075.02 | 101.25 | 20 | 266086.8 | 4434.78 | 73.913 |
| 30 | 364512.0 | 6075.20 | 101.25 | 30 | 265358.1 | 4422.64 | 73.711 |
| 40 | 364522.8 | 6075.38 | 101.26 | 40 | 264627.1 | 4410.45 | 73.508 |
| 50 | 364533.6 | 6075.56 | 101.26 | 50 | 263893.8 | 4398.23 | 73.304 |
| 44° 0' | 364544.4 | 6075.74 | 101.26 | 44° 0' | 263158.3 | 4385.97 | 73.100 |
| 10 | 364555.2 | 6075.92 | 101.27 | 10 | 262420.5 | 4373.68 | 72.895 |
| 20 | 364566.1 | 6076.10 | 101.27 | 20 | 261680.6 | 4361.34 | 72.689 |
| 30 | 364576.9 | 6076.28 | 101.27 | 30 | 260938.4 | 4348.97 | 72.483 |
| 40 | 364587.7 | 6076.46 | 101.27 | 40 | 260193.9 | 4336.57 | 72.276 |
| 50 | 364598.5 | 6076.64 | 101.28 | 50 | 259447.3 | 4324.12 | 72.069 |
| 45° 0' | 364609.4 | 6076.82 | 101.28 | 45° 0' | 258698.4 | 4311.64 | 71.861 |
| 10 | 364620.2 | 6077.00 | 101.28 | 10 | 257947.3 | 4299.12 | 71.652 |
| 20 | 364631.0 | 6077.18 | 101.29 | 20 | 257194.1 | 4286.57 | 71.443 |
| 30 | 364641.9 | 6077.37 | 101.29 | 30 | 256438.6 | 4273.98 | 71.233 |
| 40 | 364652.7 | 6077.55 | 101.29 | 40 | 255681.0 | 4261.35 | 71.022 |
| 50 | 364663.5 | 6077.73 | 101.30 | 50 | 254921.2 | 4248.69 | 70.811 |
| 46° 0' | 364674.4 | 6077.91 | 101.30 | 46° 0' | 254159.2 | 4235.99 | 70.600 |
| 10 | 364685.2 | 6078.09 | 101.30 | 10 | 253395.0 | 4223.25 | 70.388 |
| 20 | 364696.0 | 6078.27 | 101.30 | 20 | 252628.7 | 4210.48 | 70.175 |
| 30 | 364706.8 | 6078.45 | 101.31 | 30 | 251860.2 | 4197.67 | 69.961 |
| 40 | 364717.7 | 6078.63 | 101.31 | 40 | 251089.6 | 4184.83 | 69.747 |
| 50 | 364728.5 | 6078.81 | 101.31 | 50 | 250316.8 | 4171.95 | 69.532 |

| LATITUDE. | | | | LONGITUDE | | | |
|-----------|---------------------|---------|---------|-----------|---------------------|---------|---------|
| Latitude. | Length in Feet of a | | | Latitude. | Length in Feet of a | | |
| | Degree. | Minute. | Second. | | Degree. | Minute. | Second. |
| 47° 0' | 364739.3 | 6078.99 | 101.32 | 47° 0' | 249541.9 | 4159.03 | 69.317 |
| 10 | 364750.1 | 6079.17 | 101.32 | 10 | 248764.9 | 4146.08 | 69.101 |
| 20 | 364760.9 | 6079.35 | 101.32 | 20 | 247985.8 | 4133.10 | 68.885 |
| 30 | 364771.7 | 6079.53 | 101.33 | 30 | 247204.5 | 4120.08 | 68.668 |
| 40 | 364782.5 | 6079.71 | 101.33 | 40 | 246421.2 | 4107.02 | 68.450 |
| 50 | 364793.3 | 6079.89 | 101.33 | 50 | 245635.8 | 4093.93 | 68.232 |
| 48° 0' | 364804.1 | 6080.07 | 101.33 | 48° 0' | 244848.2 | 4080.80 | 68.013 |
| 10 | 364814.9 | 6080.25 | 101.34 | 10 | 244058.5 | 4067.64 | 67.794 |
| 20 | 364825.6 | 6080.43 | 101.34 | 20 | 243266.8 | 4054.45 | 67.574 |
| 30 | 364836.4 | 6080.61 | 101.34 | 30 | 242473.0 | 4041.22 | 67.353 |
| 40 | 364847.1 | 6080.79 | 101.35 | 40 | 241677.1 | 4027.95 | 67.132 |
| 50 | 364857.9 | 6080.97 | 101.35 | 50 | 240879.2 | 4014.65 | 66.911 |
| 49° 0' | 364868.6 | 6081.14 | 101.35 | 49° 0' | 240079.2 | 4001.32 | 66.689 |
| 10 | 364879.4 | 6081.32 | 101.36 | 10 | 239277.1 | 3987.95 | 66.466 |
| 20 | 364890.1 | 6081.50 | 101.36 | 20 | 238473.1 | 3974.55 | 66.242 |
| 30 | 364900.8 | 6081.68 | 101.36 | 30 | 237667.0 | 3961.12 | 66.018 |
| 40 | 364911.5 | 6081.86 | 101.36 | 40 | 236858.9 | 3947.65 | 65.794 |
| 50 | 364922.2 | 6082.04 | 101.37 | 50 | 236048.7 | 3934.15 | 65.569 |
| 50° 0' | 364932.9 | 6082.22 | 101.37 | 50° 0' | 235236.5 | 3920.61 | 65.343 |
| 10 | 364943.6 | 6082.39 | 101.37 | 10 | 234422.3 | 3907.04 | 65.117 |
| 20 | 364954.2 | 6082.57 | 101.38 | 20 | 233606.1 | 3893.44 | 64.890 |
| 30 | 364964.9 | 6082.75 | 101.38 | 30 | 232787.9 | 3879.80 | 64.663 |
| 40 | 364975.5 | 6082.93 | 101.38 | 40 | 231967.8 | 3866.13 | 64.435 |
| 50 | 364986.2 | 6083.10 | 101.38 | 50 | 231145.7 | 3852.43 | 64.207 |
| 51° 0' | 364996.8 | 6083.28 | 101.39 | 51° 0' | 230321.4 | 3838.69 | 63.978 |
| 10 | 365007.4 | 6083.46 | 101.39 | 10 | 229495.3 | 3824.92 | 63.749 |
| 20 | 365018.0 | 6083.63 | 101.39 | 20 | 228667.2 | 3811.12 | 63.519 |
| 30 | 365028.6 | 6083.81 | 101.40 | 30 | 227837.2 | 3797.29 | 63.288 |
| 40 | 365039.1 | 6083.99 | 101.40 | 40 | 227005.3 | 3783.42 | 63.057 |
| 50 | 365049.7 | 6084.16 | 101.40 | 50 | 226171.4 | 3769.52 | 62.825 |
| 52° 0' | 365060.2 | 6084.34 | 101.41 | 52° 0' | 225335.5 | 3755.59 | 62.593 |
| 10 | 365070.7 | 6084.51 | 101.41 | 10 | 224497.7 | 3741.63 | 62.360 |
| 20 | 365081.2 | 6084.69 | 101.41 | 20 | 223658.1 | 3727.64 | 62.127 |
| 30 | 365091.7 | 6084.86 | 101.41 | 30 | 222816.5 | 3713.61 | 61.893 |
| 40 | 365102.2 | 6085.04 | 101.42 | 40 | 221973.0 | 3699.55 | 61.659 |
| 50 | 365112.7 | 6085.21 | 101.42 | 50 | 221127.6 | 3685.46 | 61.424 |

| ● LATITUDE. | | | | LONGITUDE. | | | |
|-------------|---------------------|---------|----------|------------|---------------------|---------|---------|
| Latitude. | Length in Feet of a | | | Latitude. | Length in Feet of a | | |
| | Degree. | Minute. | Sec ond. | | Degree. | Minute. | Second. |
| 53° 0' | 365123·1 | 6085·39 | 101·42 | 53° 0' | 220280·3 | 3671·34 | 61·189 |
| 10 | 365133·6 | 6085·56 | 101·43 | 10 | 219431·1 | 3657·19 | 60·953 |
| 20 | 365144·0 | 6085·73 | 101·43 | 20 | 218580·0 | 3643·00 | 60·717 |
| 30 | 365154·4 | 6085·91 | 101·43 | 30 | 217727·1 | 3628·79 | 60·480 |
| 40 | 365164·7 | 6086·08 | 101·43 | 40 | 216872·3 | 3614·54 | 60·242 |
| 50 | 365175·1 | 6086·25 | 101·44 | 50 | 216015·7 | 3600·26 | 60·004 |
| 54° 0' | 365185·4 | 6086·42 | 101·44 | 54° 0' | 215157·2 | 3585·95 | 59·766 |
| 10 | 365195·7 | 6086·60 | 101·44 | 10 | 214296·9 | 3571·62 | 59·527 |
| 20 | 365206·1 | 6086·77 | 101·45 | 20 | 213434·7 | 3557·25 | 59·287 |
| 30 | 365216·3 | 6086·94 | 101·45 | 30 | 212570·7 | 3542·85 | 59·047 |
| 40 | 365226·6 | 6087·11 | 101·45 | 40 | 211704·9 | 3528·42 | 58·807 |
| 50 | 365236·8 | 6087·28 | 101·45 | 50 | 210837·3 | 3513·96 | 58·566 |
| 55° 0' | 365247·0 | 6087·45 | 101·46 | 55° 0' | 209968·0 | 3499·47 | 58·324 |
| 10 | 365257·2 | 6087·62 | 101·46 | 10 | 209096·8 | 3484·95 | 58·082 |
| 20 | 365267·4 | 6087·79 | 101·46 | 20 | 208223·8 | 3470·40 | 57·840 |
| 30 | 365277·6 | 6088·96 | 101·47 | 30 | 207349·0 | 3455·82 | 57·597 |
| 40 | 365287·7 | 6088·13 | 101·47 | 40 | 206472·5 | 3441·21 | 57·353 |
| 50 | 365297·8 | 6088·30 | 101·47 | 50 | 205594·2 | 3426·57 | 57·109 |
| 56° 0' | 365307·9 | 6088·47 | 101·47 | 56° 0' | 204714·0 | 3411·90 | 56·865 |
| 10 | 365318·0 | 6088·63 | 101·48 | 10 | 203832·2 | 3397·20 | 56·620 |
| 20 | 365328·0 | 6088·80 | 101·48 | 20 | 202948·6 | 3382·48 | 56·375 |
| 30 | 365338·0 | 6088·97 | 101·48 | 30 | 202063·3 | 3367·72 | 56·129 |
| 40 | 365348·0 | 6089·13 | 101·49 | 40 | 201176·2 | 3352·94 | 55·882 |
| 50 | 365358·0 | 6089·30 | 101·49 | 50 | 200287·4 | 3338·12 | 55·635 |
| 57° 0' | 365367·9 | 6089·47 | 101·49 | 57° 0' | 199396·9 | 3323·28 | 55·388 |
| 10 | 365377·8 | 6089·63 | 101·49 | 10 | 198504·7 | 3308·41 | 55·140 |
| 20 | 365387·7 | 6089·80 | 101·50 | 20 | 197610·8 | 3293·51 | 54·892 |
| 30 | 365397·6 | 6089·96 | 101·50 | 30 | 196715·2 | 3278·59 | 54·643 |
| 40 | 365407·4 | 6090·12 | 101·50 | 40 | 195817·9 | 3263·63 | 54·394 |
| 50 | 365417·2 | 6090·29 | 101·50 | 50 | 194919·0 | 3248·65 | 54·144 |
| 58° 0' | 365427·0 | 6090·45 | 101·51 | 58° 0' | 194018·3 | 3233·64 | 53·643 |
| 10 | 365436·8 | 6090·61 | 101·51 | 10 | 193116·0 | 3218·60 | 53·643 |
| 20 | 365446·5 | 6090·78 | 101·51 | 20 | 192212·1 | 3203·54 | 53·392 |
| 30 | 365456·2 | 6090·94 | 101·52 | 30 | 191306·5 | 3188·44 | 53·141 |
| 40 | 365465·9 | 6091·11 | 101·52 | 40 | 190399·3 | 3173·32 | 52·889 |
| 50 | 365475·5 | 6091·26 | 101·52 | 50 | 189490·4 | 3158·17 | 52·636 |

| LATITUDE. | | | | LONGITUDE. | | | |
|-----------|---------------------|---------|---------|------------|---------------------|---------|---------|
| Latitude. | Length in Feet of a | | | Latitude. | Length in Feet of a | | |
| | Degree. | Minute. | Second. | | Degree. | Minute. | Second. |
| 59° 0' | 365485.1 | 6091.42 | 101.52 | 59° 0' | 188579.9 | 3143.00 | 52.383 |
| 10 | 365494.7 | 6091.58 | 101.53 | 10 | 187667.8 | 3127.80 | 52.130 |
| 20 | 365504.3 | 6091.74 | 101.53 | 20 | 186754.1 | 3112.57 | 51.876 |
| 30 | 365513.8 | 6091.90 | 101.53 | 30 | 185838.8 | 3097.31 | 51.622 |
| 40 | 365523.3 | 6092.06 | 101.53 | 40 | 184921.9 | 3082.03 | 51.367 |
| 50 | 365532.8 | 6092.21 | 101.54 | 50 | 184003.4 | 3066.72 | 51.112 |
| 60° 0' | 365542.2 | 6092.37 | 101.54 | 60° 0' | 183083.3 | 3051.59 | 50.856 |
| 10 | 365551.6 | 6092.53 | 101.54 | 10 | 182161.6 | 3036.03 | 50.600 |
| 20 | 365561.0 | 6092.68 | 101.54 | 20 | 181238.4 | 3020.64 | 50.344 |
| 30 | 365570.3 | 6092.84 | 101.55 | 30 | 180313.7 | 3005.23 | 50.087 |
| 40 | 365579.6 | 6092.99 | 101.55 | 40 | 179387.4 | 2989.79 | 49.830 |
| 50 | 365588.9 | 6093.15 | 101.55 | 50 | 178459.5 | 2974.33 | 49.572 |
| 61° 0' | 365598.1 | 6093.30 | 101.56 | 61° 0' | 177530.1 | 2958.84 | 49.314 |
| 10 | 365607.3 | 6093.46 | 101.56 | 10 | 176599.2 | 2943.32 | 49.055 |
| 20 | 365616.5 | 6093.61 | 101.56 | 20 | 175666.8 | 2927.78 | 48.796 |
| 30 | 365625.7 | 6093.76 | 101.56 | 30 | 174732.8 | 2912.21 | 48.537 |
| 40 | 365634.8 | 6093.91 | 101.57 | 40 | 173797.4 | 2896.62 | 48.277 |
| 50 | 365643.9 | 6094.07 | 101.57 | 50 | 172860.5 | 2881.01 | 48.017 |
| 62° 0' | 365652.9 | 6094.22 | 101.57 | 62° 0' | 171922.1 | 2865.37 | 47.750 |
| 10 | 365661.9 | 6094.37 | 101.57 | 10 | 170982.2 | 2849.70 | 47.495 |
| 20 | 365670.9 | 6094.52 | 101.58 | 20 | 170040.9 | 2834.02 | 47.234 |
| 30 | 365679.8 | 6094.66 | 101.58 | 30 | 169098.1 | 2818.30 | 46.972 |
| 40 | 365688.7 | 6094.81 | 101.58 | 40 | 168153.8 | 2802.56 | 46.709 |
| 50 | 365697.6 | 6094.96 | 101.58 | 50 | 167208.1 | 2786.80 | 46.447 |
| 63° 0' | 365706.4 | 6095.11 | 101.59 | 63° 0' | 166261.0 | 2771.01 | 45.184 |
| 10 | 365715.2 | 6095.25 | 101.59 | 10 | 165312.4 | 2755.21 | 45.920 |
| 20 | 365723.9 | 6095.40 | 101.59 | 20 | 164362.5 | 2739.38 | 45.656 |
| 30 | 365732.6 | 6095.54 | 101.59 | 30 | 163411.1 | 2723.52 | 45.392 |
| 40 | 365741.3 | 6095.69 | 101.59 | 40 | 162458.4 | 2707.64 | 45.127 |
| 50 | 365749.9 | 6095.83 | 101.60 | 50 | 161504.2 | 2691.74 | 44.862 |
| 64° 0' | 365758.5 | 6095.98 | 101.60 | 64° 0' | 160548.6 | 2675.81 | 44.587 |
| 10 | 365767.1 | 6096.12 | 101.60 | 10 | 159591.6 | 2659.86 | 44.331 |
| 20 | 365775.6 | 6096.26 | 101.60 | 20 | 158633.2 | 2643.89 | 44.065 |
| 30 | 365784.1 | 6096.40 | 101.61 | 30 | 157673.5 | 2627.90 | 43.798 |
| 40 | 365792.6 | 6096.54 | 101.61 | 40 | 156712.5 | 2611.88 | 43.531 |
| 50 | 365801.0 | 6096.68 | 101.61 | 50 | 155750.1 | 2595.84 | 43.264 |

| LATITUDE. | | | | LONGITUDE. | | | |
|-----------|---------------------|---------|---------|------------|---------------------|---------|---------|
| Latitude. | Length in Feet of a | | | Latitude. | Length in Feet of a | | |
| | Degree. | Minute. | Second. | | Degree. | Minute. | Second. |
| 65° 0' | 365809.3 | 6096.82 | 101.61 | 65° 0' | 154786.3 | 2579.77 | 42.996 |
| 10 | 365817.6 | 6096.96 | 101.62 | 10 | 153821.2 | 2563.69 | 42.728 |
| 20 | 365825.9 | 6097.10 | 101.62 | 20 | 152854.8 | 2547.58 | 42.460 |
| 30 | 365834.2 | 6097.24 | 101.62 | 30 | 151887.2 | 2531.45 | 42.191 |
| 40 | 365842.4 | 6097.37 | 101.62 | 40 | 150918.2 | 2515.30 | 41.922 |
| 50 | 365850.5 | 6097.51 | 101.63 | 50 | 149947.9 | 2499.13 | 41.652 |
| 66° 0' | 365858.6 | 6097.64 | 101.63 | 66° 0' | 148976.3 | 2482.94 | 41.382 |
| 10 | 365866.7 | 6097.78 | 101.63 | 10 | 148003.4 | 2466.72 | 41.112 |
| 20 | 365874.7 | 6097.91 | 101.63 | 20 | 147029.3 | 2450.49 | 40.841 |
| 30 | 365882.7 | 6098.05 | 101.63 | 30 | 146053.9 | 2434.23 | 40.570 |
| 40 | 365890.7 | 6098.18 | 101.64 | 40 | 145977.3 | 2417.96 | 40.299 |
| 50 | 365898.6 | 6098.31 | 101.64 | 50 | 144099.3 | 2401.66 | 40.028 |
| 67° 0' | 365906.4 | 6098.44 | 101.64 | 67° 0' | 143120.2 | 2385.34 | 39.756 |
| 10 | 365914.3 | 6098.57 | 101.64 | 10 | 142139.8 | 2369.00 | 39.483 |
| 20 | 365922.0 | 6098.70 | 101.65 | 20 | 141158.2 | 2352.64 | 39.211 |
| 30 | 365929.8 | 6098.83 | 101.65 | 30 | 140175.4 | 2336.26 | 38.938 |
| 40 | 365937.4 | 6098.96 | 101.65 | 40 | 139191.4 | 2319.86 | 38.664 |
| 50 | 365945.1 | 6099.09 | 101.65 | 50 | 138206.1 | 2303.44 | 38.390 |
| 68° 0' | 365952.7 | 6099.21 | 101.65 | 68° 0' | 137219.7 | 2287.00 | 38.116 |
| 10 | 365960.2 | 6099.34 | 101.66 | 10 | 136232.1 | 2270.54 | 37.842 |
| 20 | 365967.7 | 6099.46 | 101.66 | 20 | 135243.3 | 2254.06 | 37.568 |
| 30 | 365975.2 | 6099.59 | 101.66 | 30 | 134253.4 | 2237.56 | 37.293 |
| 40 | 365982.6 | 6099.71 | 101.66 | 40 | 133262.3 | 2221.04 | 37.017 |
| 50 | 365989.9 | 6099.83 | 101.66 | 50 | 132270.1 | 2204.50 | 36.742 |
| 69° 0' | 365997.3 | 6099.96 | 101.67 | 69° 0' | 131276.7 | 2187.95 | 36.466 |
| 10 | 366004.5 | 6100.08 | 101.67 | 10 | 130282.2 | 2171.37 | 36.190 |
| 20 | 366011.7 | 6100.20 | 101.67 | 20 | 129286.6 | 2154.78 | 35.913 |
| 30 | 366018.9 | 6100.32 | 101.67 | 30 | 128289.9 | 2138.17 | 35.636 |
| 40 | 366026.1 | 6100.44 | 101.67 | 40 | 127292.1 | 2121.54 | 35.359 |
| 50 | 366033.1 | 6100.55 | 101.68 | 50 | 126293.2 | 2104.89 | 35.082 |
| 70° 0' | 366040.2 | 6100.67 | 101.68 | 70° 0' | 125293.2 | 2088.22 | 34.804 |
| 10 | 366047.2 | 6100.79 | 101.68 | 10 | 124292.1 | 2071.54 | 34.526 |
| 20 | 366054.1 | 6100.90 | 101.68 | 20 | 123289.9 | 2054.83 | 34.247 |
| 30 | 366061.0 | 6101.02 | 101.68 | 30 | 122286.7 | 2038.11 | 33.968 |
| 40 | 366067.8 | 6101.13 | 101.69 | 40 | 121282.4 | 2021.37 | 33.690 |
| 50 | 366074.6 | 6101.24 | 101.69 | 50 | 120277.1 | 2004.62 | 33.410 |

| LATITUDE. | | | | LONGITUDE. | | | |
|---------------|---------------------|----------|---------|---------------|---------------------|---------|---------|
| Latitude. | Length in Feet of a | | | Latitude. | Length in Feet of a | | |
| | Degree. | Min ute. | Second. | | Degree. | Minute. | Second. |
| 71° 0' | 366081.3 | 6101.36 | 101.69 | 71° 0' | 119270.7 | 1987.85 | 33.131 |
| 10 | 366088.0 | 6101.47 | 101.69 | 10 | 118263.3 | 1971.06 | 32.851 |
| 20 | 366094.6 | 6101.58 | 101.69 | 20 | 117254.9 | 1954.25 | 32.571 |
| 30 | 366101.2 | 6101.69 | 101.69 | 30 | 116245.6 | 1937.43 | 32.290 |
| 40 | 366107.8 | 6101.80 | 101.70 | 40 | 115235.2 | 1920.59 | 32.009 |
| 50 | 366114.3 | 6101.91 | 101.70 | 50 | 114223.8 | 1903.73 | 31.729 |
| 72° 0' | 366120.7 | 6102.01 | 101.70 | 72° 0' | 113211.4 | 1886.86 | 31.448 |
| 10 | 366127.1 | 6102.12 | 101.70 | 10 | 112198.0 | 1869.97 | 31.166 |
| 20 | 366133.4 | 6102.22 | 101.70 | 20 | 111183.7 | 1853.06 | 30.884 |
| 30 | 366139.7 | 6102.33 | 101.71 | 30 | 110168.4 | 1836.14 | 30.602 |
| 40 | 366145.9 | 6102.43 | 101.71 | 40 | 109132.2 | 1819.20 | 30.320 |
| 50 | 366153.1 | 6102.54 | 101.71 | 50 | 108135.0 | 1802.25 | 30.038 |
| 73° 0' | 366158.2 | 6102.64 | 101.71 | 73° 0' | 107116.9 | 1785.28 | 29.755 |
| 10 | 366164.3 | 6102.74 | 101.71 | 10 | 106098.0 | 1768.30 | 29.472 |
| 20 | 366170.3 | 6102.84 | 101.71 | 20 | 105077.9 | 1751.30 | 29.189 |
| 30 | 366176.3 | 6102.94 | 101.72 | 30 | 104057.0 | 1734.28 | 28.905 |
| 40 | 366182.2 | 6103.04 | 101.72 | 40 | 103035.3 | 1717.26 | 28.621 |
| 50 | 366188.1 | 6103.14 | 101.72 | 50 | 102012.8 | 1700.21 | 28.337 |
| 74° 0' | 366193.9 | 6103.23 | 101.73 | 74° 0' | 100989.1 | 1683.15 | 28.053 |
| 10 | 366199.6 | 6103.33 | 101.73 | 10 | 99964.7 | 1666.08 | 27.768 |
| 20 | 366205.3 | 6103.42 | 101.73 | 20 | 98939.5 | 1648.99 | 27.483 |
| 30 | 366211.0 | 6103.52 | 101.73 | 30 | 97913.4 | 1631.89 | 27.198 |
| 40 | 366216.6 | 6103.61 | 101.73 | 40 | 96886.5 | 1614.78 | 26.913 |
| 50 | 366222.1 | 6103.70 | 101.73 | 50 | 95858.7 | 1597.65 | 26.627 |
| 75° 0' | 366227.6 | 6103.79 | 101.73 | 75° 0' | 94830.1 | 1580.50 | 26.342 |
| 10 | 366233.0 | 6103.88 | 101.73 | 10 | 93800.6 | 1563.34 | 26.056 |
| 20 | 366238.4 | 6103.97 | 101.73 | 20 | 92730.4 | 1546.17 | 25.770 |
| 30 | 366243.7 | 6104.06 | 101.73 | 30 | 91739.4 | 1528.99 | 25.483 |
| 40 | 366249.0 | 6104.15 | 101.74 | 40 | 90707.6 | 1511.79 | 25.196 |
| 50 | 366254.2 | 6104.24 | 101.74 | 50 | 89675.0 | 1494.58 | 24.901 |
| 76° 0' | 366259.6 | 6104.32 | 101.74 | 76° 0' | 88641.6 | 1477.36 | 24.623 |
| 10 | 366264.4 | 6104.41 | 101.74 | 10 | 87607.4 | 1460.12 | 24.335 |
| 20 | 366269.5 | 6104.49 | 101.74 | 20 | 86572.5 | 1442.88 | 24.048 |
| 30 | 366274.5 | 6104.58 | 101.74 | 30 | 85536.9 | 1425.62 | 23.760 |
| 40 | 366279.4 | 6104.66 | 101.74 | 40 | 84500.5 | 1408.34 | 23.472 |
| 50 | 366284.3 | 6104.74 | 101.75 | 50 | 83463.4 | 1391.06 | 23.184 |

| LATITUDE. | | | | LONGITUDE. | | | |
|---------------|---------------------|---------|---------|---------------|---------------------|---------|---------|
| Latitude. | Length in Feet of a | | | Latitude. | Length in Feet of a | | |
| | Degree. | Minute. | Second. | | Degree. | Minute. | Second. |
| 77° 0' | 366289·1 | 6104·82 | 101·75 | 77° 0' | 82425·6 | 1373·76 | 22·896 |
| 10 | 366293·8 | 6104·90 | 101·75 | 10 | 81387·0 | 1356·45 | 22·108 |
| 20 | 366298·5 | 6104·98 | 101·75 | 20 | 80347·8 | 1339·13 | 22·319 |
| 30 | 366303·1 | 6105·05 | 101·75 | 30 | 79307·9 | 1321·80 | 22·030 |
| 40 | 366307·7 | 6105·13 | 101·75 | 40 | 78267·3 | 1304·46 | 21·741 |
| 50 | 366312·3 | 6105·21 | 101·75 | 50 | 77226·0 | 1287·10 | 21·452 |
| 78° 0' | 366316·7 | 6105·28 | 101·75 | 78° 0' | 76184·0 | 1269·73 | 21·162 |
| 79° 0' | 366342·3 | 6105·71 | 101·76 | 79° 0' | 69918·8 | 1165·31 | 19·422 |
| 80° 0' | 366365·8 | 6106·10 | 101·77 | 80° 0' | 63631·8 | 1060·53 | 17·676 |
| 81° 0' | 366387·1 | 6106·45 | 101·77 | 81° 0' | 57325·2 | 955·42 | 15·924 |
| 82° 0' | 366406·3 | 6106·77 | 101·78 | 82° 0' | 51000·6 | 850·01 | 14·167 |
| 83° 0' | 366423·2 | 6107·05 | 101·78 | 83° 0' | 44660·3 | 744·34 | 12·406 |
| 84° 0' | 366438·0 | 6107·30 | 101·79 | 84° 0' | 38306·1 | 638·44 | 10·641 |
| 85° 0' | 366450·5 | 6107·51 | 101·79 | 85° 0' | 31939·9 | 532·33 | 8·872 |
| 86° 0' | 366460·7 | 6107·68 | 101·79 | 86° 0' | 25563·9 | 426·07 | 7·101 |
| 87° 0' | 366468·7 | 6107·81 | 101·80 | 87° 0' | 19179·8 | 319·66 | 5·328 |
| 88° 0' | 366474·4 | 6107·91 | 101·80 | 88° 0' | 12789·9 | 213·17 | 3·553 |
| 89° 0' | 366477·9 | 6107·97 | 101·80 | 89° 0' | 6395·9 | 106·60 | 1·777 |
| 90° 0' | 366479·0 | 6107·98 | 101·80 | 90° 0' | 0·0 | 0·0 | 0·0 |

TABLE M.—For computing the reduction to the meridian in seconds, or $\frac{\text{Vers H. A.}}{\sin 1''}$.

[illegible]

| 50m | 58m | 57m | 56m | 55m | 54m | 53m | 52m | 51m | 50m | 49m | 48m | 47m | 46m | 45m | 44m | 43m | 42m | 41m | 40m | 39m | 38m | 37m | 36m | 35m | s. |
|-----|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|----|
| 30 | 0.5.4.412.3.24.0.39.8.59.4.183.0.110.4.414.8.177.2.216.4.259.6.306.6.357.7.412.7.471.5.534.3.601.0.671.6.746.1.824.6.907.0.993.2.1083.3.1177.5.1271.8.1368.8.1470.9.1589.8.183.4.1915.9.2100.9.2299.8.2509.8.2734.8.2974.8.3234.8.3514.8.3814.8.4134.8.4474.8.4834.8.5214.8.5614.8.6034.8.6474.8.6934.8.7414.8.7914.8.8434.8.8974.8.9534.8.1013.8.1073.8.1133.8.1193.8.1253.8.1313.8.1373.8.1433.8.1493.8.1553.8.1613.8.1673.8.1733.8.1793.8.1853.8.1913.8.1973.8.2033.8.2093.8.2153.8.2213.8.2273.8.2333.8.2393.8.2453.8.2513.8.2573.8.2633.8.2693.8.2753.8.2813.8.2873.8.2933.8.2993.8.3053.8.3113.8.3173.8.3233.8.3293.8.3353.8.3413.8.3473.8.3533.8.3593.8.3653.8.3713.8.3773.8.3833.8.3893.8.3953.8.4013.8.4073.8.4133.8.4193.8.4253.8.4313.8.4373.8.4433.8.4493.8.4553.8.4613.8.4673.8.4733.8.4793.8.4853.8.4913.8.4973.8.5033.8.5093.8.5153.8.5213.8.5273.8.5333.8.5393.8.5453.8.5513.8.5573.8.5633.8.5693.8.5753.8.5813.8.5873.8.5933.8.5993.8.6053.8.6113.8.6173.8.6233.8.6293.8.6353.8.6413.8.6473.8.6533.8.6593.8.6653.8.6713.8.6773.8.6833.8.6893.8.6953.8.7013.8.7073.8.7133.8.7193.8.7253.8.7313.8.7373.8.7433.8.7493.8.7553.8.7613.8.7673.8.7733.8.7793.8.7853.8.7913.8.7973.8.8033.8.8093.8.8153.8.8213.8.8273.8.8333.8.8393.8.8453.8.8513.8.8573.8.8633.8.8693.8.8753.8.8813.8.8873.8.8933.8.8993.8.9053.8.9113.8.9173.8.9233.8.9293.8.9353.8.9413.8.9473.8.9533.8.9593.8.9653.8.9713.8.9773.8.9833.8.9893.8.9953.8.1001.8.1007.8.1013.8.1019.8.1025.8.1031.8.1037.8.1043.8.1049.8.1055.8.1061.8.1067.8.1073.8.1079.8.1085.8.1091.8.1097.8.1103.8.1109.8.1115.8.1121.8.1127.8.1133.8.1139.8.1145.8.1151.8.1157.8.1163.8.1169.8.1175.8.1181.8.1187.8.1193.8.1199.8.1205.8.1211.8.1217.8.1223.8.1229.8.1235.8.1241.8.1247.8.1253.8.1259.8.1265.8.1271.8.1277.8.1283.8.1289.8.1295.8.1301.8.1307.8.1313.8.1319.8.1325.8.1331.8.1337.8.1343.8.1349.8.1355.8.1361.8.1367.8.1373.8.1379.8.1385.8.1391.8.1397.8.1403.8.1409.8.1415.8.1421.8.1427.8.1433.8.1439.8.1445.8.1451.8.1457.8.1463.8.1469.8.1475.8.1481.8.1487.8.1493.8.1499.8.1505.8.1511.8.1517.8.1523.8.1529.8.1535.8.1541.8.1547.8.1553.8.1559.8.1565.8.1571.8.1577.8.1583.8.1589.8.1595.8.1601.8.1607.8.1613.8.1619.8.1625.8.1631.8.1637.8.1643.8.1649.8.1655.8.1661.8.1667.8.1673.8.1679.8.1685.8.1691.8.1697.8.1703.8.1709.8.1715.8.1721.8.1727.8.1733.8.1739.8.1745.8.1751.8.1757.8.1763.8.1769.8.1775.8.1781.8.1787.8.1793.8.1799.8.1805.8.1811.8.1817.8.1823.8.1829.8.1835.8.1841.8.1847.8.1853.8.1859.8.1865.8.1871.8.1877.8.1883.8.1889.8.1895.8.1901.8.1907.8.1913.8.1919.8.1925.8.1931.8.1937.8.1943.8.1949.8.1955.8.1961.8.1967.8.1973.8.1979.8.1985.8.1991.8.1997.8.2003.8.2009.8.2015.8.2021.8.2027.8.2033.8.2039.8.2045.8.2051.8.2057.8.2063.8.2069.8.2075.8.2081.8.2087.8.2093.8.2099.8.2105.8.2111.8.2117.8.2123.8.2129.8.2135.8.2141.8.2147.8.2153.8.2159.8.2165.8.2171.8.2177.8.2183.8.2189.8.2195.8.2201.8.2207.8.2213.8.2219.8.2225.8.2231.8.2237.8.2243.8.2249.8.2255.8.2261.8.2267.8.2273.8.2279.8.2285.8.2291.8.2297.8.2303.8.2309.8.2315.8.2321.8.2327.8.2333.8.2339.8.2345.8.2351.8.2357.8.2363.8.2369.8.2375.8.2381.8.2387.8.2393.8.2399.8.2405.8.2411.8.2417.8.2423.8.2429.8.2435.8.2441.8.2447.8.2453.8.2459.8.2465.8.2471.8.2477.8.2483.8.2489.8.2495.8.2501.8.2507.8.2513.8.2519.8.2525.8.2531.8.2537.8.2543.8.2549.8.2555.8.2561.8.2567.8.2573.8.2579.8.2585.8.2591.8.2597.8.2603.8.2609.8.2615.8.2621.8.2627.8.2633.8.2639.8.2645.8.2651.8.2657.8.2663.8.2669.8.2675.8.2681.8.2687.8.2693.8.2699.8.2705.8.2711.8.2717.8.2723.8.2729.8.2735.8.2741.8.2747.8.2753.8.2759.8.2765.8.2771.8.2777.8.2783.8.2789.8.2795.8.2801.8.2807.8.2813.8.2819.8.2825.8.2831.8.2837.8.2843.8.2849.8.2855.8.2861.8.2867.8.2873.8.2879.8.2885.8.2891.8.2897.8.2903.8.2909.8.2915.8.2921.8.2927.8.2933.8.2939.8.2945.8.2951.8.2957.8.2963.8.2969.8.2975.8.2981.8.2987.8.2993.8.2999.8.3005.8.3011.8.3017.8.3023.8.3029.8.3035.8.3041.8.3047.8.3053.8.3059.8.3065.8.3071.8.3077.8.3083.8.3089.8.3095.8.3101.8.3107.8.3113.8.3119.8.3125.8.3131.8.3137.8.3143.8.3149.8.3155.8.3161.8.3167.8.3173.8.3179.8.3185.8.3191.8.3197.8.3203.8.3209.8.3215.8.3221.8.3227.8.3233.8.3239.8.3245.8.3251.8.3257.8.3263.8.3269.8.3275.8.3281.8.3287.8.3293.8.3299.8.3305.8.3311.8.3317.8.3323.8.3329.8.3335.8.3341.8.3347.8.3353.8.3359.8.3365.8.3371.8.3377.8.3383.8.3389.8.3395.8.3401.8.3407.8.3413.8.3419.8.3425.8.3431.8.3437.8.3443.8.3449.8.3455.8.3461.8.3467.8.3473.8.3479.8.3485.8.3491.8.3497.8.3503.8.3509.8.3515.8.3521.8.3527.8.3533.8.3539.8.3545.8.3551.8.3557.8.3563.8.3569.8.3575.8.3581.8.3587.8.3593.8.3599.8.3605.8.3611.8.3617.8.3623.8.3629.8.3635.8.3641.8.3647.8.3653.8.3659.8.3665.8.3671.8.3677.8.3683.8.3689.8.3695.8.3701.8.3707.8.3713.8.3719.8.3725.8.3731.8.3737.8.3743.8.3749.8.3755.8.3761.8.3767.8.3773.8.3779.8.3785.8.3791.8.3797.8.3803.8.3809.8.3815.8.3821.8.3827.8.3833.8.3839.8.3845.8.3851.8.3857.8.3863.8.3869.8.3875.8.3881.8.3887.8.3893.8.3899.8.3905.8.3911.8.3917.8.3923.8.3929.8.3935.8.3941.8.3947.8.3953.8.3959.8.3965.8.3971.8.3977.8.3983.8.3989.8.3995.8.4001.8.4007.8.4013.8.4019.8.4025.8.4031.8.4037.8.4043.8.4049.8.4055.8.4061.8.4067.8.4073.8.4079.8.4085.8.4091.8.4097.8.4103.8.4109.8.4115.8.4121.8.4127.8.4133.8.4139.8.4145.8.4151.8.4157.8.4163.8.4169.8.4175.8.4181.8.4187.8.4193.8.4199.8.4205.8.4211.8.4217.8.4223.8.4229.8.4235.8.4241.8.4247.8.4253.8.4259.8.4265.8.4271.8.4277.8.4283.8.4289.8.4295.8.4301.8.4307.8.4313.8.4319.8.4325.8.4331.8.4337.8.4343.8.4349.8.4355.8.4361.8.4367.8.4373.8.4379.8.4385.8.4391.8.4397.8.4403.8.4409.8.4415.8.4421.8.4427.8.4433.8.4439.8.4445.8.4451.8.4457.8.4463.8.4469.8.4475.8.4481.8.4487.8.4493.8.4499.8.4505.8.4511.8.4517.8.4523.8.4529.8.4535.8.4541.8.4547.8.4553.8.4559.8.4565.8.4571.8.4577.8.4583.8.4589.8.4595.8.4601.8.4607.8.4613.8.4619.8.4625.8.4631.8.4637.8.4643.8.4649.8.4655.8.4661.8.4667.8.4673.8.4679.8.4685.8.4691.8.4697.8.4703.8.4709.8.4715.8.4721.8.4727.8.4733.8.4739.8.4745.8.4751.8.4757.8.4763.8.4769.8.4775.8.4781.8.4787.8.4793.8.4799.8.4805.8.4811.8.4817.8.4823.8.4829.8.4835.8.4841.8.4847.8.4853.8.4859.8.4865.8.4871.8.4877.8.4883.8.4889.8.4895.8.4901.8.4907.8.4913.8.4919.8.4925.8.4931.8.4937.8.4943.8.4949.8.4955.8.4961.8.4967.8.4973.8.4979.8.4985.8.4991.8.4997.8.5003.8.5009.8.5015.8.5021.8.5027.8.5033.8.5039.8.5045.8.5051.8.5057.8.5063.8.5069.8.5075.8.5081.8.5087.8.5093.8.5099.8.5105.8.5111.8.5117.8.5123.8.5129.8.5135.8.5141.8.5147.8.5153.8.5159.8.5165.8.5171.8.5177.8.5183.8.5189.8.5195.8.5201.8.5207.8.5213.8.5219.8.5225.8.5231.8.5237.8.5243.8.5249.8.5255.8.5261.8.5267.8.5273.8.5279.8.5285.8.5291.8.5297.8.5303.8.5309.8.5315.8.5321.8.5327.8.5333.8.5339.8.5345.8.5351.8.5357.8.5363.8.5369.8.5375.8.5381.8.5387.8.5393.8.5399.8.5405.8.5411.8.5417.8.5423.8.5429.8.5435.8.5441.8.5447.8.5453.8.5459.8.5465.8.5471.8.5477.8.5483.8.5489.8.5495.8.5501.8.5507.8.5513.8.5519.8.5525.8.5531.8.5537.8.5543.8.5549.8.5555.8.5561.8.5567.8.5573.8.5579.8.5585.8.5591.8.5597.8.5603.8.5609.8.5615.8.5621.8.5627.8.5633.8.5639.8.5645.8.5651.8.5657.8.5663.8.5669.8.5675.8.5681.8.5687.8.5693.8.5699.8.5705.8.5711.8.5717.8.5723.8.5729.8.5735.8.5741.8.5747.8.5753.8.5759.8.5765.8.5771.8.5777.8.5783.8.5789.8.5795.8.5801.8.5807.8.5813.8.5819.8.5825.8.5831.8.5837.8.5843.8.5849.8.5855.8.5861.8.5867.8.5873.8.5879.8.5885.8.5891.8.5897.8.5903.8.5909.8.5915.8.5921.8.5927.8.5933.8.5939.8.5945.8.5951.8.5957.8.5963.8.5969.8.5975.8.5981.8.5987.8.5993.8.5999.8.6005.8.6011.8.6017.8.6023.8.6029.8.6035.8.6041.8.6047.8.6053.8.6059.8.6065.8.6071.8.6077.8.6083.8.6089.8.6095.8.6101.8.6107.8.6113.8.6119.8.6125.8.6131.8.6137.8.6143.8.6149.8.6155.8.6161.8.6167.8.6173.8.6179.8.6185.8.6191.8.6197.8.6203.8.6209.8.6215.8.6221.8.6227.8.6233.8.6239.8.6245.8.6251.8.6257.8.6263.8.6269.8.6275.8.6281.8.6287.8.6293.8.6299.8.6305.8.6311.8.6317.8.6323.8.6329.8.6335.8.6341.8.6347.8.6353.8.6359.8.6365.8.6371.8.6377.8.6383.8.6389.8.6395.8.6401.8.6407.8.6413.8.6419.8.6425.8.6431.8.6437.8.6443.8.6449.8.6455.8.6461.8.6467.8.6473.8.6479.8.6485.8.6491.8.6497.8.6503.8.6509.8.6515.8.6521.8.6527.8.6533.8.6539.8.6545.8.6551.8.6557.8.6563.8.6569.8.6575.8.6581.8.6587.8.6593.8.6599.8.6605.8.6611.8.6617.8.6623.8.6629.8.6635.8.6641.8.6647.8.6653.8.6659.8.6665.8.6671.8.6677.8.6683.8.6689.8.6695.8.6701.8.6707.8.6713.8.6719.8.6725.8.6731.8.6737.8.6743.8.6749.8.6755.8.6761.8.6767.8.6773.8.6779.8.6785.8.6791.8.6797.8.6803.8.6809.8.6815.8.6821.8.6827.8.6833.8.6839.8.6845.8.6851.8.6857.8.6863.8.6869.8.6875.8.6881.8.6887.8.6893.8.6899.8.6905.8.6911.8.6917.8.6923.8.6929.8.6935.8.6941.8.6947.8.6953.8.6959.8.6965.8.6971.8.6977.8.6983.8.6989.8.6995.8.7001.8.7007.8.7013.8.7019.8.7025.8.7031.8.7037.8.7043.8.7049.8.7055.8.7061.8.7067.8.7073.8.7079.8.7085.8.7091.8.7097.8.7103.8.7109.8.7115.8.7121.8.7127.8.7133.8.7139.8.7145.8.7151.8.7157.8.7163.8.7169.8.7175.8.7181.8.7187.8.7193.8.7199.8.7205.8.7211.8.7217.8.7223.8.7229.8.7235.8.7241.8.7247.8.7253.8.7259.8.7265.8.7271.8.7277.8.7283.8.7289.8.7295.8.7301.8.7307.8.7313.8.7319.8.7325.8.7331.8.7337.8.7343.8.7349.8.7355.8.7361.8.7367.8.7373.8.7379.8.7385.8.7391.8.7397.8.7403.8.7409.8.7415.8.7421.8.7427.8.7433.8.7439.8.7445.8.7451.8.7457.8.7463.8.7469.8.7475.8.7481.8.7487.8.7493.8.7499.8.7505.8.7511.8.7517.8.7523.8.7529.8.7535.8.7541.8.7547.8.7553.8.7559.8.7565.8.7571.8.7577.8.7583.8.7589.8.7595.8.7601.8.7607.8.7613.8.7619.8.7625.8.7631.8.7637.8.7643.8.7649.8.7655.8.7661.8.7667.8.7673.8.7679.8.7685.8.7691.8.7697.8.7703.8.7709.8.7715.8.7721.8.7727.8.7733.8.7739.8.7745.8.7751.8.7757.8.7763.8.7769.8.7775.8.7781.8.7787.8.7793.8.7799.8.7805.8.7811.8.7817.8.7823.8.7829.8.7835.8.7841.8.7847.8.7853.8.7859.8.7865.8.7871.8.7877.8.7883.8.7889.8.7895.8.7901.8.7907.8.7913.8.7919.8.7925.8.7931.8.7937.8.7943.8.7949.8.7955.8.7961.8.7967.8.7973.8.7979.8.7985.8.7991.8.7997.8.8003.8.8009.8.8015.8.8021.8.8027.8.8033.8.8039.8.8045.8.8051.8.8057.8.8063.8.8069.8.8075.8.8081.8.8087.8.8093.8.8099.8.8105.8.8111.8.8117.8.8123.8.8129.8.8135.8.8141.8.8147.8.8153.8.8159.8.8165.8.8171.8.8177.8.8183.8.8189.8.8195.8.8201.8.8207.8.8213.8.8219.8.8225.8.8231.8.8237.8.8243.8.8249.8.8255.8.8261.8.8267.8.8273.8.8279.8.8285.8.8291.8.8297.8.8303.8.8309.8.8315.8.8321.8.8327.8.8333.8.8339.8.8345.8.8351.8.8357.8.8363.8.8369.8.8375.8.8381.8.8387.8.8393.8.8399.8.8405.8.8411.8.8417.8.8423.8.8429.8.8435.8.8441.8.8447.8.8453.8.8459.8.8465.8.8471.8.8477.8.8483.8.8489.8.8495.8.8501.8.8507.8.8513.8.8519.8.8525.8.8531.8.8537.8.8543.8.8549.8.8555.8.8561.8.8567.8.8573.8.8579.8.8585.8.8591.8.8597.8.8603.8.8609.8.8615.8.8621.8.8627.8.8633.8.8639.8.8645.8.8651.8.8657.8.8663.8.8669.8.8675.8.8681.8.8687.8.8693.8.8699.8.8705.8.8711.8.8717.8.8723.8.8729.8.8735.8.8741.8.8747.8.8753.8.8759.8.8765.8.8771.8.8777.8.8783.8.8789.8.8795.8.8801.8.8807.8.8813.8.8819.8.8825.8.8831.8.8837.8.8843.8.8849.8.8855.8.8861.8.8867.8.8873.8.8879.8.8885.8.8891.8.8897.8.8903.8.8909.8.8915.8.8921.8.8927.8.8933.8.8939.8.8945.8.8951.8.8957.8.8963.8.8969.8.8975.8.8981.8.8987.8.8993.8.8999.8.9005.8.9011.8.9017.8.9023.8.9029.8.9035.8.9041.8.9047.8.9053.8.9059.8.9065.8.9071.8.9077.8.9083.8.9089.8.9095.8.9101.8.9107.8.9113.8.9119.8.9125.8.9131.8.9137.8.9143.8.9149.8.9155.8.9161.8.9167.8.9173.8.9179.8.9185.8.9191.8.9197.8.9203.8.9209.8.9215.8.9221.8.9227.8.9233.8.9239.8.9245.8.9251.8.9257.8.9263.8.9269.8.9275.8.9281.8.9287.8.9293.8.9299.8.9305.8.9311.8.9317.8.9323.8.9329.8.9335.8.9341.8.9347.8.9353.8.9359.8.9365.8.9371.8.9377.8.9383.8.9389.8.9395.8.9401.8.9407.8.9413.8.9419.8.9425.8.9431.8.9437.8.9443.8.9449.8.9455.8.9461.8.9467.8.9473.8.9479.8.9485.8.9491.8.9497.8.9503.8.9509.8.9515.8.9521.8.9527.8.9533.8.9539.8.9545.8.9551.8.9557.8.9563.8.9569.8.9575.8.9581.8.9587.8.9593.8.9599.8.9605.8.9611.8.9617.8.9623.8.9629.8.9635.8.9641.8.9647.8.9653.8.9659.8.9665.8.9671.8.9677.8.9683.8.9689.8.9695.8.9701.8.9707.8.9713.8.9719.8.9725.8.9731.8.9737.8.9743.8.9749.8.9755.8.9761.8.9767.8.9773.8.9779.8.9785.8.9791.8.9797.8.9803.8.9809.8.9815.8.9821.8.9827.8.9833.8.9839.8.9845.8.9851.8.9857.8.9863.8.9869.8.9875.8.9881.8.9887.8.9893.8.9899.8.9905.8.9911.8.9917.8.9923.8.9929.8.9935.8.9941.8.9947.8.9953.8.9959.8.9965.8.9971.8.9977.8.9983.8.9989.8.9995.8.1001.8.1007.8.1013.8.1019.8.1025.8.1031.8.1037.8.1043.8.1049.8.1055.8.1061.8.1067.8.1073.8.1079.8.1085.8.1091.8.1097.8.1103.8.1109.8.1115.8.1121 | | | | | | | | | | | | | | | | | | | | | | | | |

TABLE N.—*Dip Table for calculation of Heights to 55 miles.*Dip in feet = $0.8815 d^2$ (in miles).

| Dist. | Dip in feet. | Dist. | Dip in feet. | Dist. | Dip in feet. | Dist. | Dip in feet. | Dist. | Dip in feet. |
|-------|--------------|-------|--------------|-------|--------------|-------|--------------|-------|--------------|
| 1 | 0.9 | 13 | 149 | 19 | 318 | 25 | 551 | 34 | 1019 |
| 1½ | 2.0 | 13¼ | 155 | 19¼ | 327 | 25¼ | 562 | 35 | 1086 |
| 2 | 3.5 | 13½ | 161 | 19½ | 335 | 25½ | 573 | 36 | 1142 |
| 2½ | 5.5 | 13¾ | 167 | 19¾ | 344 | 25¾ | 584 | 37 | 1206 |
| 3 | 7.9 | 14 | 173 | 20 | 353 | 26 | 596 | 38 | 1273 |
| 3½ | 10.8 | 14½ | 179 | 20½ | 361 | 26½ | 607 | 39 | 1340 |
| 4 | 14.1 | 14¾ | 185 | 20¾ | 370 | 26¾ | 619 | 40 | 1410 |
| 4½ | 17.8 | 15 | 192 | 21 | 379 | 27 | 631 | 41 | 1482 |
| 5 | 22.0 | 15¼ | 198 | 21¼ | 388 | 27¼ | 643 | 42 | 1555 |
| 5½ | 26.6 | 15½ | 205 | 21½ | 398 | 27½ | 655 | 43 | 1630 |
| 6 | 31.7 | 15¾ | 212 | 21¾ | 407 | 27¾ | 667 | 44 | 1706 |
| 6½ | 37.2 | 16 | 219 | 22 | 417 | 28 | 679 | 45 | 1785 |
| 7 | 43.1 | 16¼ | 226 | 22¼ | 427 | 28¼ | 691 | 46 | 1865 |
| 7½ | 49.6 | 16½ | 233 | 22½ | 436 | 28½ | 703 | 47 | 1947 |
| 8 | 56.4 | 16¾ | 240 | 22¾ | 446 | 28¾ | 716 | 48 | 2031 |
| 8½ | 63.7 | 17 | 247 | 23 | 456 | 29 | 729 | 49 | 2116 |
| 9 | 71 | 17¼ | 255 | 23¼ | 466 | 29¼ | 741 | 50 | 2204 |
| 9½ | 79 | 17½ | 262 | 23½ | 476 | 29½ | 754 | 51 | 2292 |
| 10 | 88 | 17¾ | 270 | 23¾ | 486 | 29¾ | 767 | 52 | 2383 |
| 10½ | 97 | 18 | 278 | 24 | 497 | 30 | 780 | 53 | 2474 |
| 11 | 107 | 18¼ | 286 | 24¼ | 507 | 30¼ | 793 | 54 | 2570 |
| 11½ | 117 | 18½ | 294 | 24½ | 518 | 30½ | 847 | 55 | 2670 |
| 12 | 127 | 18¾ | 302 | 24¾ | 529 | 30¾ | 902 | | |
| 12½ | 138 | 19 | 310 | 25 | 540 | 31 | 960 | | |

TABLE O.—Angles subtended by various lengths at different distances.

Angle in secs. = $\frac{\text{feet subtended} \times 34}{\text{distance in nautical miles}}$.

| Feet. | Distance in Miles Nautical. | | | | | | | | | | | | | | |
|-------|-----------------------------|---------------|-------|----------------|----------------|----------------|------|----------------|------|----------------|------|----------------|------|----------------|------|
| | $\frac{1}{2}$ | $\frac{3}{4}$ | 1 | $1\frac{1}{4}$ | $1\frac{1}{2}$ | $1\frac{3}{4}$ | 2 | $2\frac{1}{2}$ | 3 | $3\frac{1}{2}$ | 4 | $4\frac{1}{2}$ | 5 | $5\frac{1}{2}$ | 8 |
| 1 | 2'16 | 1'08 | 0'44 | 0'28 | 0'22 | 0'20 | 0'17 | 0'14 | 0'11 | 0'10 | 0'8 | 0'8 | 0'7 | 0'6 | 0'04 |
| 2 | 4'32 | 2'16 | 1'32 | 0'56 | 0'46 | 0'40 | 0'34 | 0'26 | 0'23 | 0'20 | 0'17 | 0'15 | 0'13 | 0'12 | 0'08 |
| 3 | 6'48 | 3'24 | 2'16 | 1'20 | 1'08 | 1'00 | 0'51 | 0'40 | 0'34 | 0'30 | 0'25 | 0'22 | 0'20 | 0'19 | 0'12 |
| 4 | 9'04 | 4'32 | 3'02 | 1'48 | 1'31 | 1'20 | 1'08 | 0'54 | 0'45 | 0'40 | 0'34 | 0'30 | 0'27 | 0'25 | 0'17 |
| 5 | 11'20 | 5'40 | 3'46 | 2'16 | 1'53 | 1'39 | 1'25 | 1'08 | 0'57 | 0'49 | 0'42 | 0'38 | 0'34 | 0'31 | 0'21 |
| 6 | 13'36 | 6'48 | 4'32 | 2'43 | 2'16 | 1'59 | 1'42 | 1'22 | 1'08 | 0'59 | 0'51 | 0'46 | 0'41 | 0'37 | 0'26 |
| 7 | 15'48 | 7'54 | 5'16 | 3'10 | 2'38 | 2'16 | 1'59 | 1'36 | 1'19 | 1'08 | 0'59 | 0'53 | 0'48 | 0'43 | 0'30 |
| 8 | 18'08 | 9'04 | 6'04 | 3'38 | 3'02 | 2'36 | 2'16 | 1'50 | 1'31 | 1'18 | 1'08 | 1'00 | 0'55 | 0'49 | 0'34 |
| 9 | 20'24 | 10'12 | 6'48 | 4'05 | 3'24 | 2'56 | 2'33 | 2'02 | 1'42 | 1'28 | 1'16 | 1'08 | 1'01 | 0'55 | 0'38 |
| 10 | 22'36 | 11'18 | 7'33 | 4'32 | 3'47 | 3'14 | 2'49 | 2'16 | 1'53 | 1'37 | 1'24 | 1'17 | 1'08 | 0'56 | 0'42 |
| 11 | 24'52 | 12'26 | 8'19 | 5'00 | 4'09 | 3'34 | 3'06 | 2'30 | 2'05 | 1'47 | 1'32 | 1'25 | 1'15 | 1'08 | 0'46 |
| 12 | 27'08 | 13'34 | 9'04 | 5'28 | 4'32 | 3'54 | 3'23 | 2'44 | 2'16 | 1'57 | 1'41 | 1'31 | 1'22 | 1'14 | 0'51 |
| 13 | 29'24 | 14'42 | 9'49 | 5'56 | 4'54 | 4'12 | 3'40 | 2'58 | 2'27 | 2'06 | 1'50 | 1'38 | 1'29 | 1'20 | 0'55 |
| 14 | 31'40 | 15'50 | 10'34 | 6'24 | 5'17 | 4'32 | 3'57 | 3'12 | 2'38 | 2'16 | 1'58 | 1'45 | 1'36 | 1'26 | 0'59 |
| 15 | 33'56 | 16'58 | 11'20 | 6'48 | 5'40 | 4'52 | 4'15 | 3'24 | 2'50 | 2'26 | 2'07 | 1'52 | 1'42 | 1'33 | 1'04 |

TABLE P.—*Table of Distances at which Objects can be Seen at Sea, according to their respective Elevations and the Elevation of the Eye of the Observer.*

| Height in Feet. | Distance in English or Statute Miles. | Distance in Geographical or Nautical Miles. | Height in Feet. | Distance in English or Statute Miles. | Distance in Geographical or Nautical Miles. |
|-----------------------|--|--|-----------------------|--|--|
| 5 | 2° 958 | 2° 565 | 100 | 13° 228 | 11° 47 |
| 10 | 4° 184 | 3° 628 | 110 | 13° 874 | 12° 03 |
| 15 | 5° 123 | 4° 443 | 120 | 14° 490 | 12° 56 |
| 20 | 5° 916 | 5° 130 | 130 | 15° 083 | 13° 08 |
| 25 | 6° 614 | 5° 736 | 140 | 15° 652 | 13° 57 |
| 30 | 7° 245 | 6° 283 | 150 | 16° 201 | 14° 22 |
| 35 | 7° 826 | 6° 787 | 200 | 18° 708 | 16° 22 |
| 40 | 8° 366 | 7° 255 | 250 | 20° 916 | 18° 14 |
| 45 | 8° 874 | 7° 696 | 300 | 22° 912 | 19° 87 |
| 50 | 9° 354 | 8° 112 | 350 | 24° 748 | 21° 46 |
| 55 | 9° 811 | 8° 509 | 400 | 26° 457 | 22° 54 |
| 60 | 10° 246 | 8° 886 | 450 | 28° 062 | 24° 33 |
| 65 | 10° 665 | 9° 249 | 500 | 29° 580 | 25° 65 |
| 70 | 11° 067 | 9° 598 | 550 | 36° 024 | 26° 90 |
| 75 | 11° 456 | 9° 935 | 600 | 32° 403 | 28° 10 |
| 80 | 11° 832 | 10° 26 | 650 | 33° 726 | 29° 25 |
| 85 | 12° 196 | 10° 57 | 700 | 35° 000 | 30° 28 |
| 90 | 12° 549 | 10° 88 | 800 | 37° 416 | 32° 45 |
| 95 | 12° 893 | 11° 18 | 900 | 39° 836 | 34° 54 |
| | | | 1000 | 41° 833 | 36° 28 |

Example.—A tower 150 feet high will be visible to an observer whose eye is elevated 15 feet above the water 19 nautical miles; thus, from the Table :
15 feet elevation distance visible 4.44 nautical miles.

| | | | | |
|-----|----|----|-------|--|
| 150 | .. | .. | 14.22 | |
| | | | <hr/> | |
| | | | 18.66 | |

From Admiralty Tables.

TABLE Q.—*True Depression or Distance of the Sea Horizon.*

| Height. | Dep. | Square. | Height. | Dep. | Square. | Height. | Dep. | Square. | Height. | Dep. | Square. |
|---------|------|---------|---------|------|---------|---------|------|---------|---------|-------|---------|
| ft. | ' | | ft. | ' | | ft. | ' | | ft. | ' | |
| 1' 1 | 1 | 1 | 3293 | 61 | 3721 | 12966 | 121 | 14641 | 181 | 32761 | |
| 3' 5 | 2 | 4 | 3403 | 62 | 3844 | 13183 | 122 | 14884 | 182 | 33124 | |
| 8' 0 | 3 | 9 | 3513 | 63 | 3969 | 13397 | 123 | 15129 | 183 | 33489 | |
| 14' 2 | 4 | 16 | 3624 | 64 | 4096 | 13615 | 124 | 15376 | 184 | 33856 | |
| 22' 1 | 5 | 25 | 3740 | 65 | 4225 | 13836 | 125 | 15625 | 185 | 34225 | |
| 31' 9 | 6 | 36 | 3855 | 66 | 4356 | 14061 | 126 | 15876 | 186 | 34596 | |
| 43' 3 | 7 | 49 | 3974 | 67 | 4489 | 14282 | 127 | 16129 | 187 | 34969 | |
| 56' 6 | 8 | 64 | 4093 | 68 | 4624 | 14502 | 128 | 16384 | 188 | 35344 | |
| 71' 7 | 9 | 81 | 4213 | 69 | 4761 | 14737 | 129 | 16641 | 189 | 35721 | |
| 88' 5 | 10 | 100 | 4337 | 70 | 4900 | 14970 | 130 | 16900 | 190 | 36100 | |
| 107 | 11 | 121 | 4461 | 71 | 5041 | 15197 | 131 | 17161 | 191 | 36481 | |
| 127 | 12 | 144 | 4587 | 72 | 5184 | 15429 | 132 | 17424 | 192 | 36864 | |
| 149 | 13 | 169 | 4716 | 73 | 5329 | 15664 | 133 | 17689 | 193 | 37249 | |
| 174 | 14 | 196 | 4846 | 74 | 5476 | 15901 | 134 | 17956 | 194 | 37636 | |
| 199 | 15 | 225 | 4976 | 75 | 5625 | 16139 | 135 | 18225 | 195 | 38025 | |
| 226 | 16 | 256 | 5112 | 76 | 5776 | 16380 | 136 | 18496 | 196 | 38416 | |
| 256 | 17 | 289 | 5249 | 77 | 5929 | 16622 | 137 | 18769 | 197 | 38809 | |
| 287 | 18 | 324 | 5385 | 78 | 6084 | 16866 | 138 | 19044 | 198 | 39204 | |
| 319 | 19 | 361 | 5524 | 79 | 6241 | 17111 | 139 | 19321 | 199 | 39601 | |
| 354 | 20 | 400 | 5665 | 80 | 6400 | 17362 | 140 | 19600 | 200 | 40000 | |
| 390 | 21 | 441 | 5808 | 81 | 6561 | 17608 | 141 | 19881 | 201 | 40401 | |
| 428 | 22 | 484 | 5952 | 82 | 6724 | 17860 | 142 | 20164 | 202 | 40804 | |
| 468 | 23 | 529 | 6098 | 83 | 6889 | 18111 | 143 | 20449 | 203 | 41209 | |
| 510 | 24 | 576 | 6246 | 84 | 7056 | 18366 | 144 | 20736 | 204 | 41616 | |
| 550 | 25 | 625 | 6394 | 85 | 7225 | 18622 | 145 | 21025 | 205 | 42025 | |
| 598 | 26 | 676 | 6547 | 86 | 7396 | 18878 | 146 | 21316 | 206 | 42436 | |
| 645 | 27 | 729 | 6700 | 87 | 7569 | 19140 | 147 | 21609 | 207 | 42849 | |
| 694 | 28 | 784 | 6855 | 88 | 7744 | 19401 | 148 | 21904 | 208 | 43264 | |
| 744 | 29 | 841 | 7012 | 89 | 7921 | 19664 | 149 | 22201 | 209 | 43681 | |
| 797 | 30 | 900 | 7172 | 90 | 8100 | 19930 | 150 | 22500 | 210 | 44100 | |
| 850 | 31 | 961 | 7332 | 91 | 8281 | 20197 | 151 | 22801 | 211 | 44521 | |
| 906 | 32 | 1024 | 7492 | 92 | 8464 | 20465 | 152 | 23104 | 212 | 44944 | |
| 964 | 33 | 1089 | 7656 | 93 | 8649 | 20736 | 153 | 23409 | 213 | 45369 | |
| 1023 | 34 | 1156 | 7824 | 94 | 8836 | 21008 | 154 | 23716 | 214 | 45796 | |
| 1084 | 35 | 1225 | 7987 | 95 | 9025 | 21282 | 155 | 24025 | 215 | 46225 | |
| 1147 | 36 | 1296 | 8158 | 96 | 9216 | 21558 | 156 | 24336 | 216 | 46656 | |
| 1211 | 37 | 1369 | 8330 | 97 | 9409 | 21836 | 157 | 24649 | 217 | 47089 | |
| 1278 | 38 | 1444 | 8504 | 98 | 9604 | 22115 | 158 | 24964 | 218 | 47524 | |
| 1346 | 39 | 1521 | 8678 | 99 | 9801 | 22397 | 159 | 25281 | 219 | 47961 | |
| 1416 | 40 | 1600 | 8852 | 100 | 10000 | 22680 | 160 | 25600 | 220 | 48400 | |
| 1487 | 41 | 1681 | 9032 | 101 | 10201 | 22964 | 161 | 25921 | 221 | 48841 | |
| 1561 | 42 | 1764 | 9210 | 102 | 10404 | 23251 | 162 | 26244 | 222 | 49284 | |
| 1636 | 43 | 1849 | 9393 | 103 | 10609 | 23540 | 163 | 26569 | 223 | 49729 | |
| 1713 | 44 | 1936 | 9577 | 104 | 10816 | 23830 | 164 | 26896 | 224 | 50176 | |
| 1792 | 45 | 2025 | 9760 | 105 | 11025 | 24121 | 165 | 27225 | 225 | 50625 | |
| 1872 | 46 | 2116 | 9951 | 106 | 11236 | 24415 | 166 | 27556 | 226 | 51076 | |
| 1954 | 47 | 2209 | 10135 | 107 | 11449 | 24711 | 167 | 27889 | 227 | 51529 | |
| 2039 | 48 | 2304 | 10325 | 108 | 11664 | 25008 | 168 | 28224 | 228 | 51984 | |
| 2124 | 49 | 2401 | 10518 | 109 | 11881 | 25307 | 169 | 28561 | 229 | 52441 | |
| 2212 | 50 | 2500 | 10712 | 110 | 12100 | 25608 | 170 | 28900 | 230 | 52900 | |
| 2301 | 51 | 2601 | 10908 | 111 | 12321 | 25911 | 171 | 29241 | 231 | 53361 | |
| 2393 | 52 | 2704 | 11105 | 112 | 12544 | 26215 | 172 | 29584 | 232 | 53824 | |
| 2485 | 53 | 2809 | 11304 | 113 | 12769 | 26521 | 173 | 29929 | 233 | 54289 | |
| 2581 | 54 | 2916 | 11506 | 114 | 12996 | 26829 | 174 | 30276 | 234 | 54756 | |
| 2677 | 55 | 3025 | 11700 | 115 | 13225 | 27139 | 175 | 30625 | 235 | 55225 | |
| 2775 | 56 | 3136 | 11913 | 116 | 13465 | 27451 | 176 | 30976 | 236 | 55696 | |
| 2875 | 57 | 3246 | 12120 | 117 | 13689 | 27764 | 177 | 31329 | 237 | 56169 | |
| 2977 | 58 | 3364 | 12328 | 118 | 13924 | 28079 | 178 | 31684 | 238 | 56644 | |
| 3081 | 59 | 3481 | 12538 | 119 | 14161 | 28396 | 179 | 32041 | 239 | 57121 | |
| 3186 | 60 | 3600 | 12749 | 120 | 14400 | 28715 | 180 | 32400 | 240 | 57600 | |

From Raper.

TABLE R.—*Angles subtended at different distances by a pole ten feet in length.*

| Yds. | Angle. | Yds. | Angle. | Yds. | Angle. | Yds. | Angle. |
|------|----------|------|---------|------|---------|------|---------|
| | ° ' " | | ° ' " | | ° ' " | | ° ' " |
| 17 | 11 25 20 | 92 | 2 05 00 | 167 | 1 08 46 | 242 | 0 47 26 |
| 18 | 10 23 20 | 93 | 2 02 50 | 168 | 1 08 00 | 243 | 0 47 08 |
| 20 | 9 00 00 | 95 | 2 00 40 | 170 | 1 07 24 | 245 | 0 46 48 |
| 22 | 8 47 48 | 97 | 1 58 30 | 172 | 1 06 44 | 247 | 0 46 28 |
| 23 | 8 10 18 | 98 | 1 56 30 | 173 | 1 06 04 | 248 | 0 46 08 |
| 25 | 7 37 54 | 101 | 1 54 30 | 175 | 1 05 30 | 250 | 0 45 48 |
| 27 | 7 09 10 | 102 | 1 52 40 | 177 | 1 04 40 | 252 | 0 45 30 |
| 28 | 6 45 48 | 103 | 1 50 54 | 178 | 1 04 16 | 253 | 0 45 12 |
| 30 | 6 21 36 | 105 | 1 49 00 | 180 | 1 03 40 | 255 | 0 44 54 |
| 32 | 6 01 32 | 107 | 1 47 20 | 182 | 1 03 04 | 257 | 0 44 38 |
| 33 | 5 43 28 | 108 | 1 45 44 | 183 | 1 02 30 | 258 | 0 44 20 |
| 35 | 5 27 10 | 110 | 1 44 10 | 185 | 1 01 56 | 260 | 0 44 02 |
| 37 | 5 12 20 | 112 | 1 42 20 | 187 | 1 01 22 | 262 | 0 43 50 |
| 38 | 4 58 44 | 113 | 1 41 00 | 188 | 1 00 50 | 263 | 0 43 36 |
| 40 | 4 46 16 | 115 | 1 39 40 | 190 | 1 00 18 | 265 | 0 43 14 |
| 42 | 4 34 50 | 117 | 1 38 14 | 192 | 0 59 46 | 267 | 0 43 00 |
| 43 | 4 24 20 | 118 | 1 36 50 | 193 | 0 59 16 | 268 | 0 42 42 |
| 45 | 4 14 00 | 120 | 1 35 30 | 195 | 0 58 46 | 270 | 0 42 26 |
| 47 | 4 05 30 | 122 | 1 34 10 | 197 | 0 58 16 | 272 | 0 42 11 |
| 48 | 3 57 00 | 123 | 1 32 50 | 198 | 0 57 48 | 273 | 0 41 55 |
| 50 | 3 49 06 | 125 | 1 31 40 | 200 | 0 57 18 | 275 | 0 41 40 |
| 52 | 3 41 42 | 127 | 1 30 30 | 202 | 0 56 48 | 277 | 0 41 25 |
| 53 | 3 34 50 | 128 | 1 29 16 | 203 | 0 56 20 | 278 | 0 41 10 |
| 55 | 3 28 20 | 130 | 1 28 10 | 205 | 0 55 54 | 280 | 0 40 55 |
| 57 | 5 22 10 | 132 | 1 27 00 | 207 | 0 55 26 | 382 | 0 40 41 |
| 58 | 3 16 24 | 133 | 1 26 00 | 208 | 0 55 00 | 283 | 0 40 26 |
| 60 | 3 10 50 | 135 | 1 24 50 | 210 | 0 54 34 | 285 | 0 40 12 |
| 62 | 3 05 40 | 137 | 1 23 50 | 212 | 0 54 16 | 287 | 0 39 58 |
| 63 | 3 01 00 | 138 | 1 22 50 | 213 | 0 54 00 | 288 | 0 39 44 |
| 65 | 2 56 14 | 140 | 1 21 50 | 215 | 0 53 26 | 290 | 0 39 30 |
| 67 | 2 51 40 | 142 | 1 20 50 | 217 | 0 52 54 | 292 | 0 39 17 |
| 68 | 2 47 40 | 143 | 1 20 00 | 218 | 0 52 28 | 293 | 0 39 04 |
| 70 | 2 43 40 | 145 | 1 19 00 | 220 | 0 52 04 | 295 | 0 38 50 |
| 72 | 2 40 00 | 147 | 1 18 08 | 222 | 0 51 42 | 297 | 0 38 37 |
| 73 | 2 36 14 | 148 | 1 17 16 | 223 | 0 51 20 | 298 | 0 38 24 |
| 75 | 2 32 44 | 150 | 1 16 22 | 225 | 0 50 56 | 300 | 0 38 11 |
| 77 | 2 29 30 | 152 | 1 15 30 | 227 | 0 50 34 | 302 | 0 37 59 |
| 78 | 2 26 20 | 153 | 1 14 44 | 228 | 0 50 12 | 303 | 0 37 47 |
| 80 | 2 23 14 | 155 | 1 14 00 | 230 | 0 49 50 | 305 | 0 37 34 |
| 82 | 2 20 16 | 157 | 1 13 08 | 232 | 0 49 28 | 307 | 0 37 22 |
| 83 | 2 17 30 | 158 | 1 12 20 | 233 | 0 49 06 | 308 | 0 37 10 |
| 85 | 2 14 50 | 160 | 1 11 40 | 235 | 0 48 44 | 310 | 0 36 58 |
| 87 | 2 12 10 | 162 | 1 11 00 | 237 | 0 48 22 | 312 | 0 36 46 |
| 88 | 2 09 44 | 163 | 1 10 10 | 238 | 0 48 02 | 313 | 0 36 34 |
| 90 | 2 07 20 | 165 | 1 09 30 | 240 | 0 47 44 | 315 | 0 36 23 |

TABLE S.—For converting Intervals of Time or Longitude into Decimals of a Day.

| Long. | Time. | Decimals of a Day. | Long. | Time. | Decimals of a Day. | Long. | Time. | Decimals of a Day. |
|-------|-------|--------------------|-------|-------|--------------------|-------|-------|--------------------|
| o | h | | o | m | | o | m | |
| 15 | 1 | *0417 | 0 15 | 1 | *0007 | 7 45 | 31 | *0215 |
| 30 | 2 | *0833 | 0 30 | 2 | *0014 | 8 0 | 32 | *0222 |
| 45 | 3 | *1250 | 0 45 | 3 | *0021 | 8 15 | 33 | *0229 |
| 60 | 4 | *1667 | 1 0 | 4 | *0028 | 8 30 | 34 | *0236 |
| 75 | 5 | *2083 | 1 15 | 5 | *0035 | 8 45 | 35 | *0243 |
| 90 | 6 | *2500 | 1 30 | 6 | *0042 | 9 0 | 36 | *0250 |
| 105 | 7 | *2917 | 1 45 | 7 | *0049 | 9 15 | 37 | *0257 |
| 120 | 8 | *3333 | 2 0 | 8 | *0056 | 9 30 | 38 | *0264 |
| 135 | 9 | *3750 | 2 15 | 9 | *0062 | 9 45 | 39 | *0271 |
| 150 | 10 | *4167 | 2 30 | 10 | *0069 | 10 0 | 40 | *0278 |
| 165 | 11 | *4583 | 2 45 | 11 | *0076 | 10 15 | 41 | *0285 |
| 180 | 12 | *5000 | 3 0 | 12 | *0083 | 10 30 | 42 | *0292 |
| 195 | 13 | *5417 | 3 15 | 13 | *0090 | 10 45 | 43 | *0299 |
| 210 | 14 | *5833 | 3 30 | 14 | *0097 | 11 0 | 44 | *0306 |
| 225 | 15 | *6250 | 3 45 | 15 | *0104 | 11 15 | 45 | *0312 |
| 240 | 16 | *6667 | 4 0 | 16 | *0111 | 11 30 | 46 | *0319 |
| 255 | 17 | *7083 | 4 15 | 17 | *0118 | 11 45 | 47 | *0326 |
| 270 | 18 | *7500 | 4 30 | 18 | *0125 | 12 0 | 48 | *0333 |
| 285 | 19 | *7917 | 4 45 | 19 | *0132 | 12 15 | 49 | *0340 |
| 300 | 20 | *8333 | 5 0 | 20 | *0139 | 12 30 | 50 | *0347 |
| 315 | 21 | *8750 | 5 15 | 21 | *0146 | 12 45 | 51 | *0354 |
| 330 | 22 | *9167 | 5 30 | 22 | *0153 | 13 0 | 52 | *0361 |
| 345 | 23 | *9583 | 5 45 | 23 | *0160 | 13 15 | 53 | *0368 |
| 360 | 24 | 1'0000 | 6 0 | 24 | *0167 | 13 30 | 54 | *0375 |
| | | | 6 15 | 25 | *0174 | 13 45 | 55 | *0382 |
| | | | 6 30 | 26 | *0181 | 14 0 | 56 | *0389 |
| | | | 6 45 | 27 | *0187 | 14 15 | 57 | *0396 |
| | | | 7 0 | 28 | *0194 | 14 30 | 58 | *0403 |
| | | | 7 15 | 29 | *0201 | 14 45 | 59 | *0410 |
| | | | 7 30 | 30 | *0208 | 15 0 | 60 | *0417 |

From Shadwell's "Chronometers."

TABLE T.—*Metrical and English Barometers.*

| Barometer Scales. | | Barometer Scales. | | Barometer Scales. | |
|-------------------|----------|-------------------|----------|-------------------|----------|
| Fr. Mill. | Eng. In. | Fr. Mill. | Eng. In. | Fr. Mill. | Eng. In. |
| 640 | 25·2 | 691 | 27·2 | 742 | 29·2 |
| 643 | 25·3 | 693 | 27·3 | 744 | 29·3 |
| 645 | 25·4 | 696 | 27·4 | 747 | 29·4 |
| 648 | 25·5 | 698 | 27·5 | 749 | 29·5 |
| 650 | 25·6 | 701 | 27·6 | 752 | 29·6 |
| 653 | 25·7 | 704 | 27·7 | 754 | 29·7 |
| 655 | 25·8 | 706 | 27·8 | 757 | 29·8 |
| 658 | 25·9 | 709 | 27·9 | 759 | 29·9 |
| 660 | 26·0 | 711 | 28·0 | 762 | 30·0 |
| 663 | 26·1 | 714 | 28·1 | 765 | 30·1 |
| 665 | 26·2 | 716 | 28·2 | 767 | 30·2 |
| 668 | 26·3 | 719 | 28·3 | 770 | 30·3 |
| 670 | 26·4 | 721 | 28·4 | 772 | 30·4 |
| 673 | 26·5 | 724 | 28·5 | 775 | 30·5 |
| 676 | 26·6 | 726 | 28·6 | 777 | 30·6 |
| 678 | 26·7 | 729 | 28·7 | 780 | 30·7 |
| 681 | 26·8 | 732 | 28·8 | 782 | 30·8 |
| 683 | 26·9 | 734 | 28·9 | 785 | 30·9 |
| 686 | 27·0 | 737 | 29·0 | 787 | 31·0 |
| 688 | 27·1 | 739 | 29·1 | | |

TABLE U.—*Corresponding Thermometers, Fahrenheit, Centigrade, Réaumur.*

| F. | C. | R. | F. | C. | R. | F. | C. | R. |
|----|-------|-------|----|------|------|-----|------|------|
| ° | | ° | ° | | ° | ° | | ° |
| 0 | -17.8 | -14.2 | 41 | 5.0 | 4.0 | 81 | 27.2 | 21.8 |
| 1 | -17.2 | -13.8 | 42 | 5.6 | 4.4 | 82 | 27.8 | 22.2 |
| 2 | -16.7 | -13.3 | 43 | 6.1 | 4.9 | 83 | 28.3 | 22.7 |
| 3 | -16.1 | -12.9 | 44 | 6.7 | 5.3 | 84 | 28.9 | 23.1 |
| 4 | -15.6 | -12.4 | 45 | 7.2 | 5.8 | 85 | 29.4 | 23.6 |
| 5 | -15.0 | -12.0 | 46 | 7.8 | 6.2 | 86 | 30.0 | 24.0 |
| 6 | -14.4 | -11.6 | 47 | 8.3 | 6.7 | 87 | 30.6 | 24.4 |
| 7 | -13.9 | -11.1 | 48 | 8.9 | 7.1 | 88 | 31.1 | 24.9 |
| 8 | -13.3 | -10.7 | 49 | 9.4 | 7.5 | 89 | 31.7 | 25.3 |
| 9 | -12.8 | -10.2 | 50 | 10.0 | 8.0 | 90 | 32.2 | 25.8 |
| 10 | -12.2 | -9.8 | 51 | 10.6 | 8.4 | 91 | 32.8 | 26.2 |
| 11 | -11.7 | -9.3 | 52 | 11.1 | 8.9 | 92 | 33.3 | 26.7 |
| 12 | -11.1 | -8.9 | 53 | 11.7 | 9.3 | 93 | 33.9 | 27.1 |
| 13 | -10.6 | -8.4 | 54 | 12.2 | 9.8 | 94 | 34.4 | 27.6 |
| 14 | -10.0 | -8.0 | 55 | 12.8 | 10.2 | 95 | 35.0 | 28.0 |
| 15 | -9.4 | -7.5 | 56 | 13.3 | 10.7 | 96 | 35.6 | 28.4 |
| 16 | -8.9 | -7.1 | 57 | 13.9 | 11.1 | 97 | 36.1 | 28.9 |
| 17 | -8.3 | -6.7 | 58 | 14.4 | 11.6 | 98 | 36.7 | 29.3 |
| 18 | -7.8 | -6.2 | 59 | 15.0 | 12.0 | 99 | 37.2 | 29.8 |
| 19 | -7.2 | -5.8 | 60 | 15.6 | 12.4 | 100 | 37.8 | 30.2 |
| 20 | -6.7 | -5.3 | 61 | 16.1 | 12.9 | 101 | 38.3 | 30.7 |
| 21 | -6.1 | -4.9 | 62 | 16.7 | 13.3 | 102 | 38.9 | 31.1 |
| 22 | -5.6 | -4.4 | 63 | 17.2 | 13.8 | 103 | 39.4 | 31.6 |
| 23 | -5.0 | -4.0 | 64 | 17.8 | 14.2 | 104 | 40.0 | 32.0 |
| 24 | -4.4 | -3.6 | 65 | 18.3 | 14.7 | 105 | 40.6 | 32.4 |
| 25 | -3.9 | -3.1 | 66 | 18.9 | 15.1 | 106 | 41.1 | 32.9 |
| 26 | -3.3 | -2.7 | 67 | 19.4 | 15.6 | 107 | 41.7 | 33.3 |
| 27 | -2.8 | -2.2 | 68 | 20.0 | 16.0 | 108 | 42.2 | 33.8 |
| 28 | -2.2 | -1.8 | 69 | 20.6 | 16.4 | 109 | 42.8 | 34.2 |
| 29 | -1.7 | -1.3 | 70 | 21.1 | 16.9 | 110 | 43.3 | 34.7 |
| 30 | -1.1 | -0.9 | 71 | 21.7 | 17.3 | 111 | 43.9 | 35.1 |
| 31 | -0.6 | -0.4 | 72 | 22.2 | 17.8 | 112 | 44.4 | 35.5 |
| 32 | 0 | 0 | 73 | 22.8 | 18.2 | 113 | 45.0 | 36.0 |
| 33 | 0.6 | 0.4 | 74 | 23.3 | 18.7 | 114 | 45.6 | 36.4 |
| 34 | 1.1 | 0.9 | 75 | 23.9 | 19.1 | 115 | 46.1 | 36.9 |
| 35 | 1.7 | 1.3 | 76 | 24.4 | 19.6 | 116 | 46.7 | 37.3 |
| 36 | 2.2 | 1.8 | 77 | 25.0 | 20.0 | 117 | 47.2 | 37.8 |
| 37 | 2.8 | 2.2 | 78 | 25.6 | 20.5 | 118 | 47.8 | 38.2 |
| 38 | 3.3 | 2.7 | 79 | 26.1 | 20.9 | 119 | 48.3 | 38.7 |
| 39 | 3.9 | 3.1 | 80 | 26.7 | 21.3 | 120 | 48.9 | 39.1 |
| 40 | 4.4 | 3.6 | | | | | | |

TABLE V.—*Measures used to express depths in Foreign Charts.*

| National Measure. | | Eng. Feet. | Eng. Fathoms. |
|-------------------|----------------|------------|---------------|
| French | Metre | 3·281 | 0·5468 |
| | Brasse | 5·329 | 0·8881 |
| Spanish | Braza | 5·492 | 0·9153 |
| Swedish | Fomn | 5·843 | 0·974 |
| Danish | Favn | 6·175 | 1·0292 |
| Norwegian | „ | 6·175 | 1·0292 |
| German | Faden | 5·906 | 0·984 |
| Dutch | Vaden | 5·575 | 0·929 |
| Russian | Marine Sashine | 6·000 | 1·000 |
| Portuguese | Braca | 6·004 | 1·000 |

APPENDIX W

To prove that Angle in seconds = $\frac{\text{number of feet subtended} \times 34}{\text{distance in sea miles}}$.

In cases involving small angles :

$$\text{Circular measure of an angle} = \frac{\text{arc it subtends}}{\text{radius of the arc}}.$$

Angular unit = Angle, whose circular measure is unity,

$$= \frac{360''}{2\pi} = 206267''.$$

And since angles are to one another as the arcs they subtend (Euc. VI.), and therefore as their circular measures

$$\frac{\theta}{206267''} = \frac{\text{circular measure of } \theta}{1},$$

$$\therefore \theta = 206267 \times \frac{\text{arc}}{\text{radius}}.$$

θ being small, chord may be substituted for arc.

$$\therefore \theta = 206267 \times \frac{\text{feet subtending } \theta''}{\text{distance in feet}},$$

$$= \frac{206267 \times \text{feet subtending } \theta''}{\text{distance in miles} \times 6080}.$$

$$\therefore \text{Angle in seconds} = \frac{\text{number of feet subtended} \times 34}{\text{distance in sea miles}} \text{ very nearly.}$$

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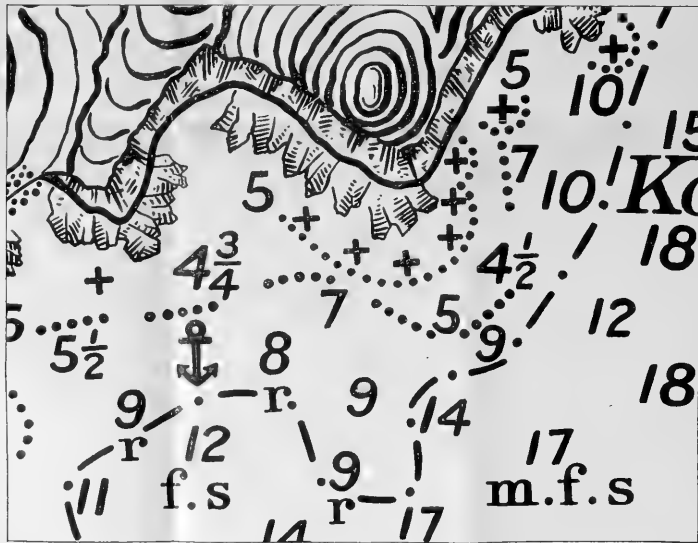
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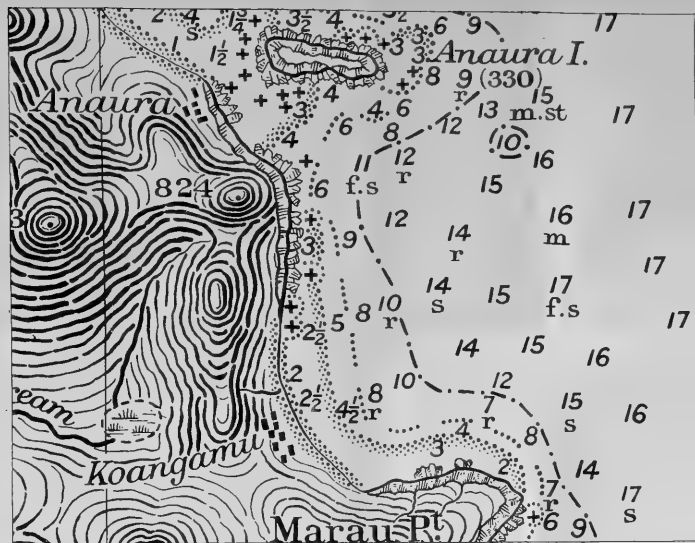
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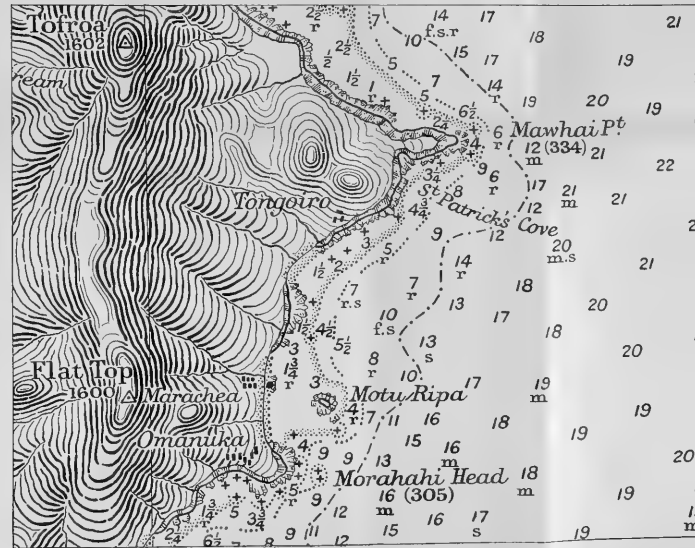
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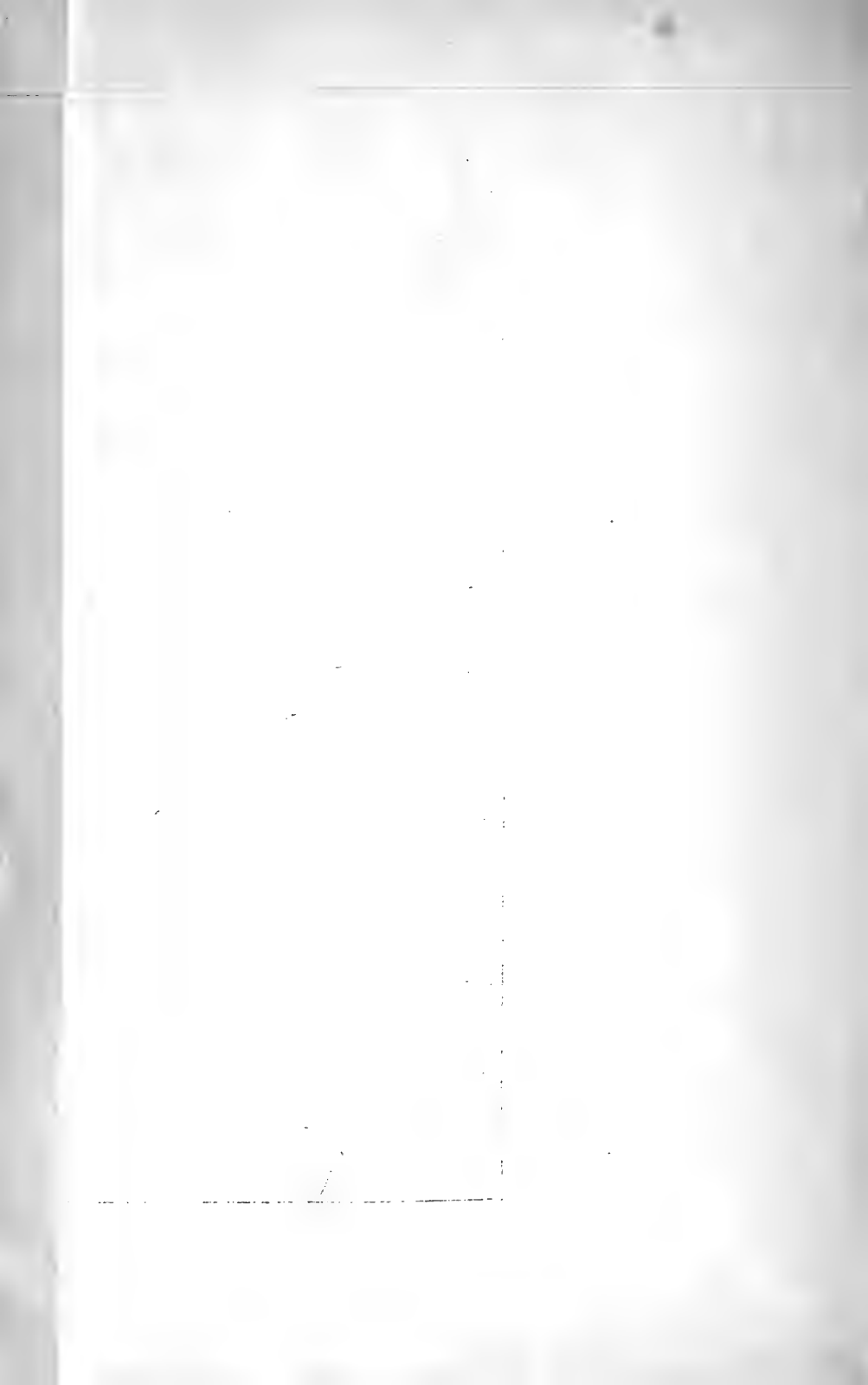


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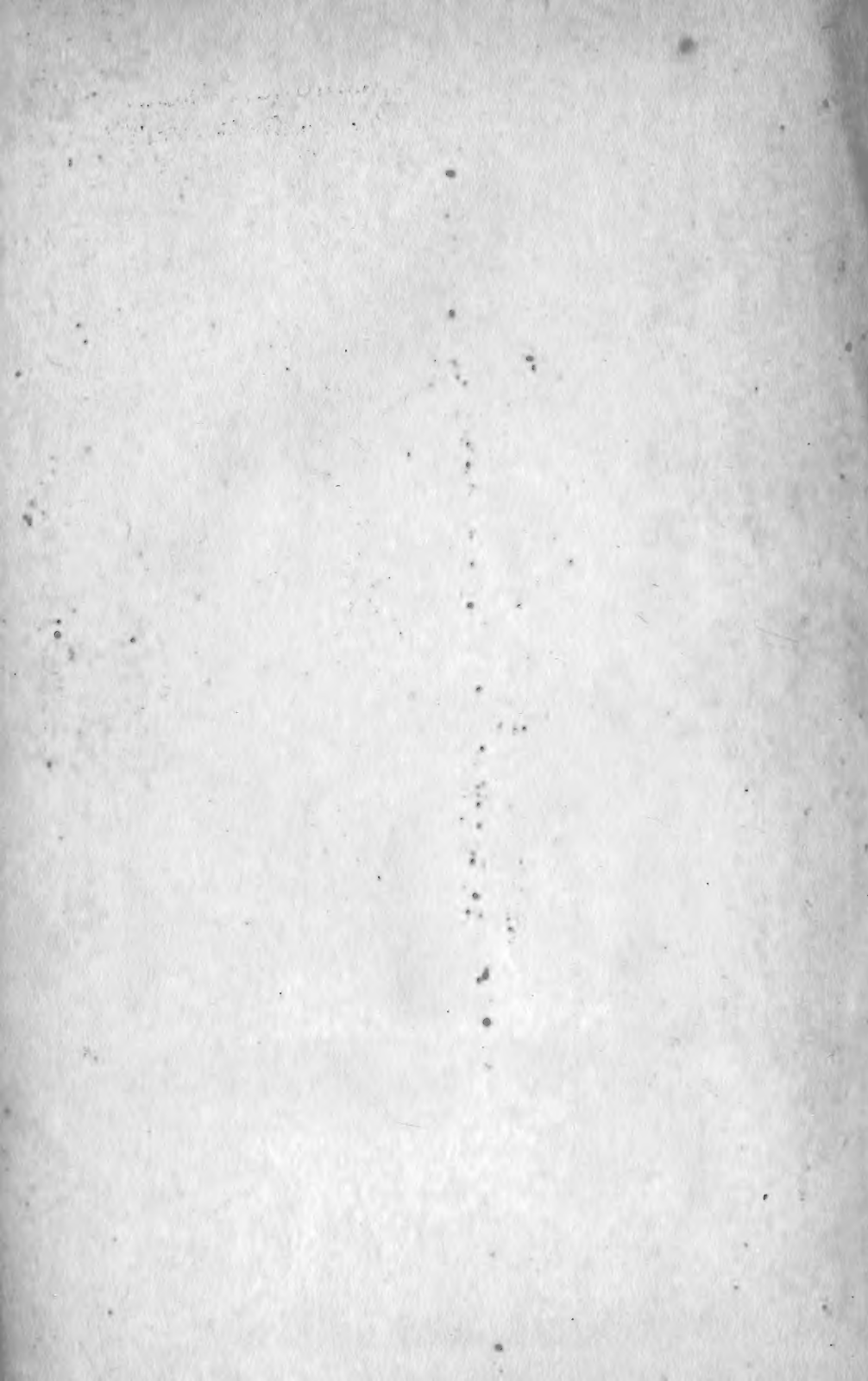
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